From Local Gestures to Global Art: Toward a *Total Work of Art* of Theory and Practice

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Abstract

This manifesto articulates the vision of a *total work of art* emerging from the convergence of artistic practice and category-theoretic mathematics. It weaves together the author's curatorial endeavors, theoretical investigations in sheaf theory and category theory, and formal research in multi-agent systems. By drawing analogies between local-to-global principles in mathematics and the assembly of meaning in art, it proposes a framework where conceptual rigor and creative expression unify. The text itself is composed as both a scholarly treatise and a poetic act, reflecting its thesis that knowledge and art can coalesce into a single, comprehensive form.

1 Introduction

In my journey as an artist and mathematician, I have sought to unify disparate domains of practice into a singular creative vision. The idea of a total work of art, guides this integration—an ideal where music, image, text, and even theory converge into one coherent experience. This manifesto outlines such a vision: it merges conceptual art and curatorial experiments with category theory and sheaf theory, and aligns them with formal research into multi-agent systems cohomology. Far from a polemic, the tone here is reflective and scholarly, yet poetic, befitting a manifesto that straddles the boundary of academic paper and art piece.

For context, consider the range of my practice. I have co-curated a group exhibition, ironically titled *Eric Schmid is an Idiot* (Detroit, 2017), featuring 85 artists in chaotic dialogue.[1] I have produced sound works that play with philosophical texts—for example, reciting mathematical lecture notes as a performance and publication (*Notes on Lecture Notes*, Edition Eric Schmid, 2024). Concurrently, I have authored mathematical notes on category theory and topos theory[2] and on sheaf theory and cohomology,[3] and formulated a doctoral research proposal on categorical approaches to economics and machine learning.[4] These activities are not isolated, but rather facets of a single pursuit of understanding and creation. This manifesto attempts to articulate the unifying principles behind them—a personal theory-of-everything in art and mathematics.

The sections that follow mirror both the logical and the lyrical. We begin with the local-vs-global dialectic of sheaf theory, move to the structural insights of category theory, and then to a cohomological understanding of collective systems. Each part interweaves mathematical concepts with examples from artistic practice, demonstrating how they inform one another. In this way, the manifesto itself becomes a synthesis: part theoretical paper, part conceptual artwork, aiming for a form of total work of art in its own right.

2 Local to Global: The Sheaf of Art

Many problems in mathematics exhibit a tension between local behavior and global behavior on a space. In topology and geometry, for instance, one can often define solutions or structures piecewise that fail to assemble into a valid whole. Sheaf theory provides a systematic framework to track data locally and determine how to "glue" or assemble this local data into global

[1] *Eric Schmid is an Idiot*, group exhibition at Cave (Detroit), Jan–Feb 2017, curated by What Pipeline and Kavita B. Schmid.

[2] Eric Schmid, *A Very Short Introduction to Topos Theory* (adapted from R. Pettigrew's notes), arXiv:2406.19409 [math.CT] (2024).

[3] Eric Schmid, *Introduction to Sheaf Theory and Sheaf Cohomology*, unpublished notes (2025).

[4] Eric Schmid, *Formalizing Cohomology for Multi-Agent Systems: A Categorical Approach to Economic and Reinforcement Learning Models*, PhD Research Proposal (Global Center for Advanced Studies, 2025).

structures.[5] The guiding question is, in essence, *when can local solutions or constructions be uniquely merged into a global one?*[6] This question resonates beyond mathematics. It can be asked of art: when can individual creative acts or components be unified into a coherent whole?

Consider the curatorial problem of assembling an exhibition or the compositional challenge of creating a multimedia artwork. Each artwork or component is like a *local section* defined on its own patch of meaning. The exhibition as a whole is the attempt to paste these sections together on a conceptual "space" (the gallery, the theme, the cultural context). The overlaps between pieces—points of intersection in theme, form, or content—are where consistency must be checked. If two works address a common idea (their domains intersect), do they do so harmoniously or in contradiction? The curator's task is akin to the sheaf gluing condition: ensure that on every overlap, the local definitions agree, so that a single coherent experience can cover the union.

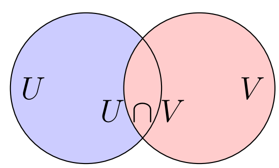

Figure 1: Overlapping local contexts (open sets U and V) sharing a common part $U \cap V$. A section (solution) on U and one on V must agree on $U \cap V$ to glue into a section on $U \cup V$. Analogously, artistic components that overlap in concept must align on their intersection to form a unified work.

In the exhibition *Eric Schmid is an Idiot*, dozens of independent artistic voices were brought together. Each artist's contribution can be seen as a local section on some region of the aesthetic manifold. The exhibition space X was covered by many conceptual "opens" U_i—each U_i an area of inquiry or style inhabited by one artist or a group. Where two works touched on a common idea, they overlapped ($U_i \cap U_j$), and one could compare their treatments. When works resonated, their local meanings agreed on overlap, yielding a larger coherent meaning on $U_i \cup U_j$. When works clashed or

[5] Schmid, *Introduction to Sheaf Theory and Sheaf Cohomology* (2025), Introduction, p. 1: "Sheaf theory provides a systematic framework to track data locally on a topological space and determine how to 'glue' or assemble this local data into global structures."

[6] Ibid., p. 1.

spoke at cross-purposes, one felt a discontinuity—an obstruction to gluing. In sheaf-theoretic terms, a failure to glue is measured by cohomology: certain mismatches or inconsistencies manifest as nontrivial cohomology classes, abstract indicators that the local data could not globally reconcile.

Sheaf cohomology thus formalizes the notion of an *obstruction* to unity. If every piece of an artwork or exhibition fit together perfectly (no contradiction or gap in the overlaps), one might say the cohomology vanishes—nothing stands in the way of a global section, a total work. In practice, however, the richest assemblies often have purposeful friction: a work may introduce a twist that resists complete integration. This is analogous to a sheaf cohomology class carrying meaningful information: the global whole has an added nuance or ambiguity precisely because not everything glued seamlessly. Rather than a flaw, such a gap can be a productive tension, an invitation to the viewer to resolve or dwell in the ambiguity. The total work of art ideal strives for a unified artwork, but it need not imply homogeneity or perfection of fit; it can embrace a patchwork of local textures, provided we are aware of how they connect and where they don't.

In short, the local-to-global principle underlies both the technical world of algebraic topology and the experiential world of art. By understanding an artwork as a kind of sheaf (of ideas, images, sounds, etc. spread across different contexts but tied together), we gain a language to discuss unity and disunity in creation. What cannot be articulated in one medium alone might be realized by the relationship between two or more media, just as a global section might only exist when we take multiple charts together. This perspective elevates curation and composition to the level of a mathematical art: a delicate process of aligning local truths to reveal (or deliberately obscure) a global truth.

3 Categories and Functors: The Structure of Transformation

If sheaf theory teaches us to mind the gap between local and global, category theory teaches us to mind the *relationships* themselves. Category theory is a mathematics of structure and connection: it studies abstract *objects* and the *morphisms* (arrows) between them. Rather than focusing on the internal details of objects, it emphasizes how objects relate and compose. In my practice, this has become a philosophical stance. An artwork can be seen not only as an isolated object but as a node in a network of transformations: an idea becomes a proposal, which becomes a performance, which becomes a documentation, and so on. Each stage is an object in some conceptual category, and the transitions are morphisms.

In formal terms, a category $\mathcal{C}$ consists of objects (say, $X, Y, Z \in \mathcal{C}$) and morphisms between them (functions, mappings, or arrows $f : X \to Y$). Morphisms can be composed (arrows chain together) and objects have identities (each object X has an identity arrow id_X that does nothing). This simple schema can encode remarkably complex situations. For example, consider a category of artistic media: one object might be TEXT, another IMAGE, another SOUND. A morphism from TEXT to IMAGE could represent a process of visual illustration of a text, whereas a morphism from IMAGE to TEXT could represent describing an image in words. If these processes can compose (perhaps chaining image-to-text to text-to-sound, etc.), we are capturing the idea that an artistic concept can cycle through forms while preserving some core structure.

A central notion in category theory is the *functor*: a mapping $F : \mathcal{C} \to \mathcal{D}$ between categories that carries objects to objects ($X \mapsto F(X)$) and morphisms to morphisms ($f : X \to Y$ gives $F(f) : F(X) \to F(Y)$), respecting compositions and identities. Functors preserve structure, translating patterns in one domain into another. My work *Notes on Lecture Notes* (2024) can be thought of in these terms. In that project, I took formal lecture notes on logic, category theory, and related fields, and transformed them into an artist's book and performance. There was a source category $\mathcal{C}$ of mathematical discourse (objects like THEOREM, PROOF, DEFINITION, with morphisms given by logical implication or textual reference), and a target category $\mathcal{D}$ of aesthetic presentations (objects like SPOKEN WORD, PRINTED PAGE, with morphisms like NARRATION or LAYOUT). The process of creating the piece was a functor $F : \mathcal{C} \to \mathcal{D}$: each theorem or definition in $\mathcal{C}$ was mapped to a recited or typographically designed element in $\mathcal{D}$. Crucially, the logical structure was preserved under this mapping—the order of exposition, the hierarchy of ideas, the referential structure all remained intact, merely transported to a new medium. In category language, F preserved composition: if a proof p follows a theorem t in the source (a composite $t \to p$), then $F(t)$ is followed by $F(p)$ in the performance, maintaining the narrative arrow $F(t) \to F(p)$. In this way, the functorial viewpoint helped ensure that the resulting artwork was not a disjoint collage of math fragments, but a structurally coherent translation of one discourse into another. The mathematics became art, yet remained mathematically informed art.

We can extend this lens further. Just as one can speak of a "space" of an exhibition in sheaf terms, one can speak of the entire enterprise of art-and-math as a category. One might imagine a category where objects are ideas (mathematical concepts, aesthetic themes, personal experiences) and morphisms are acts of translation or transformation (a proof, a painting, a poem, a performance, each transforming an idea into a different representational state). In this meta-category, composition of morphisms corresponds to the chaining of creative acts. For instance, an idea I transformed into a scholarly article and then adapted into an installation is one composite morphism (Idea $\to$ Article $\to$ Installation). Another route might be Idea $\to$ Drawing $\to$ Animation. If both routes start at the same I and end in, say, a similar concept expressed as Installation vs. Animation, we might ask: do these yield isomorphic results or fundamentally different ones? Category theory gives us a language for such questions of universality and equivalence across forms.

To push this vision to its limit, consider the concept of a *topos*. A topos is a special kind of category that behaves like a universe of sets: it has an internal logic, it contains a notion of truth values (a *subobject classifier*), and supports constructions like products, exponentials, and power-objects. In essence, a topos is a self-contained world of discourse in which one can do mathematics. Now, by analogy, an immersive artwork or a complex conceptual framework might be seen as a kind of topos. When I curate an ambitious exhibition or create a layered artwork, I am effectively setting up a miniature world with its own rules and internal logic. The viewers entering that space navigate according to those rules, just as a logician working in a topos navigates by the internal logic of that universe. In a large group exhibition, the curator might even provide an "axiomatic" basis (a statement of themes or questions) and each artwork is a proposition expressed in that system, with the subobject classifier being the criterion by which we judge if a given interpretation holds within the context. While this is a poetic metaphor, it highlights how category theory's broad abstractions can illuminate artistic creation: both involve constructing worlds (categories) and translating between them (functors).

We can illustrate the idea of synthesis of mediums with a simple diagram (Fig. 2). Multiple media or dimensions of an artwork feed into a singular whole. In category terms, one might think of this as a colimit: the total work is an amalgamation of its parts, the smallest common "super-object" to which all parts contribute. In more concrete terms, consider an opera: we have music, drama, and visual design each as components, and their fusion creates the opera. In my practice, I extend this principle: theoretical mathematics becomes another component to fuse with sound, text, and image. The result aimed for is a conceptual opera of sorts, where abstract theory and sensory experience play in concert.

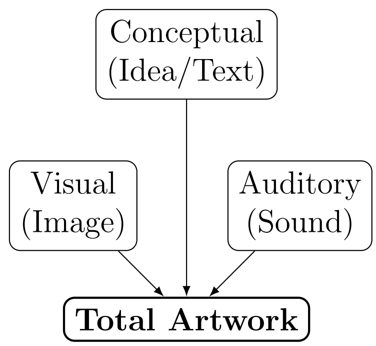

Figure 2: Components of a total artwork as a unified structure. Multiple media (visual, auditory, textual/conceptual) feed into a single total work of art. This can be seen as a diagram in a category, where different object types (media) are mapped via morphisms (creative processes) into one final object (the whole artwork). The total work thus represents a colimit of its constituent parts, containing all of them coherently.

In category theory, one often finds that diverse structures can be connected by universal properties. Dually, one finds dualities and adjoint pairs that link seemingly opposite concepts. I often sense this in art as well. Rigorous logical thinking and free artistic intuition might appear as opposites, but I experience them as adjoint processes: each informs and bounds the other. The logical provides frameworks that free intuition can then inhabit; the intuitive provides new ideas that the logical can formalize. Together they form a pair of functors between the category of raw inspiration and the category of structured knowledge, each left or right adjoint to the other in different contexts. Such analogies, while fanciful, have practical impact on how I balance analysis and creativity. They remind me that form and content, structure and chaos, exist in a symbiotic dance. Category theory's lesson is that what matters are the *maps* between things; in my work, the maps between art and math are where the real creation happens.

4 Cohomology of Collaboration: Emergence and Synthesis

Stepping back to a broader systems view, we find ourselves in a world of many agents: many participants, many interacting parts. Whether in an economy, a society, a swarm of learning algorithms, or a collaborative art project, we encounter complex multi-agent systems. A recurring theme is that local actions, decisions, or pieces of information, when taken together, produce global phenomena that no single agent fully intended or understands. In the language of my research: *local decisions made by individual agents with limited information collectively determine global system behavior. This mirrors precisely the domain where sheaf theory excels—analyzing how local data and constraints generate global structures.*[7] In an artistic collective or a large-scale project involving many creators or participants, much the same can be said: each person contributes based on a local perspective (their own ideas or role), and the overall result emerges from the interplay of those contributions.

My doctoral work formalizes this parallel by casting economic and reinforcement learning models as sheaf-theoretic structures. The aim is to reveal new insights by applying algebraic topology to multi-agent systems, effectively discovering the "shape" of information flow and misalignment in those systems. For instance, if no single trader in a market has global knowledge, but the market as a whole exhibits a pattern, we might model knowledge as a presheaf over a network of agents and then compute a cohomology class that represents the emergent market behavior (something like a price signal or a bubble) that is not attributable to any one agent alone. This is directly analogous to how in an art exhibition, the "atmosphere" or message of the whole may not be attributable to any one artwork, but arises from their combination. There is a kind of cohomological class to the exhibition as well—a metaphorical one residing in the gaps between what each piece says and what the ensemble says.

By pursuing a *categorical approach* to multi-agent systems, I have found a conceptual meeting point for different fields. Category theory becomes a lin-

[7] Eric Schmid, PhD Proposal (2025), p. 1: "Local decisions made by individual agents with limited information collectively determine global system behavior. This mirrors precisely the mathematical domain where sheaf theory excels – analyzing how local data and constraints generate global structures."

gua franca, allowing one to *bridge the often-disconnected worlds of theoretical computer science, economic modeling, and machine learning*, as noted in my proposal's abstract.[8] These same mathematical bridges can extend further into the humanities and arts. The formalism that lets us speak about finance and AI in one breath can also speak about cultural production and aesthetics. In all cases, we are dealing with systems of interacting components, and we care about what structures emerge at scale.

Practically, this research involves developing formalizations in a proof assistant (dependently typed languages like Agda/Coq) to ensure the mathematics is sound, and implementing algorithms that compute cohomology for concrete scenarios. There is an analogy here to artistic craft: just as an engineer or programmer ensures that a theoretical design works in a real system, an artist ensures that a conceptual idea works when instantiated in real materials or social settings. My dual engagement with code and concept has reinforced the idea that theory and practice must co-evolve. In one project, I might be writing code to calculate the cohomology of a communication network; in another, I might be organizing a participatory performance. Both are about orchestrating interactions and understanding the resultant whole. Both satisfy a similar intellectual aesthetic: a fascination with emergent complexity.

To illustrate, imagine simulating an art exhibition as a multi-agent system. Each artwork and each viewer is an agent; each agent has local data (the artwork has content, the viewer has interpretations). As viewers move through the exhibition, they share interpretations or react (interactions between agents). We could build a sheaf model where to each subset of agents (say a group discussing a piece) we assign the information they share, and on overlaps of groups the information must be consistent. The cohomology of this sheaf could, in principle, detect a kind of global inconsistency or novelty in how the exhibition is perceived—perhaps a theme that no single viewer articulated, but which is implicit in the collective narrative. While this is a playful thought experiment, it shows how thinking in terms of cohomology and category can lead to new ways of conceptualizing even the ephemeral qualities of art experiences.

In less technical terms, the categorical perspective urges us toward unity

[8] Schmid, PhD Proposal (2025), Abstract: "This approach bridges the often-disconnected worlds of theoretical computer science, economic modeling, and machine learning, with implementations allowing practical applications in these domains."

without uniformity. It suggests that underneath disparate activities there is a common structural quest: to find the language that connects local detail to global insight. Cohomology, in the manifesto sense, is the poetry of those connections—it is how we quantify the unspoken, how we acknowledge the unsolvable remainder. When I curated the chaotic 85-artist exhibition, I was enacting a kind of cohomological computation by hand: feeling out which combinations produce synergy and which leave unresolved dissonance. When I write a theorem or a program, I similarly search for elegant resolution, but also note the remainders that hint at deeper truth (like how certain equations have a remainder that becomes the next big question).

Thus, my formal research and my art practice are not two separate paths, but rather one path viewed at different scales. One operates in the realm of symbols and silicon, the other in the realm of people and perception. Yet both strive to illuminate how local parts make up wholes, and what new properties arise in the making.

5 Conclusion: Toward Synthesis

At the heart of this manifesto is a belief in unity—unity of knowledge and expression. The total work of art I seek is not just a mixture of artistic media, but a fusion of disciplines and modes of thought. It is a vision in which proving a theorem and curating an exhibition are seen as compatible acts of creation, where writing code and writing poetry inform each other, and where no frontier is fixed between art and science. The preceding sections have sketched how concepts from sheaf theory, category theory, and cohomology provide a scaffold for this vision, how they lend vocabulary and form to intuitions I have long had in my artistic work. Conversely, by treating art as a domain of insight, these mathematical ideas gain new interpretations and metaphors, potentially suggesting novel avenues for inquiry (for example, thinking of a social network's cohomology as the "ghost" of collective memory, or an art installation as a topos with its own internal logic).

The manifesto itself is intended as a *total work*: it is at once exposition and artifice. Its academic structure (with sections, footnotes, and diagrams) is real—each concept is as precise as in a research paper—but the assembly and tone are also aesthetic, inviting a reading that is not only for information but for experience. In embodying the interweaving it advocates, this text becomes a microcosm of my practice.

Looking forward, the path is one of continued synthesis. There are many local patches yet to explore: new mathematical frameworks (perhaps higher category theory, ∞-topoi, or derived functors) and new artistic experiments (perhaps performances that enact logical structures, or paintings generated by type theoretic programs). Each will be a local piece of the puzzle. The challenge and joy will be gluing them into the evolving global picture of a life's work. I expect obstructions along the way—indeed, cohomology assures me they will be there. But every obstruction is also an opportunity: a sign of something fundamentally new that does not reduce to what came before.

In closing, I return to the fundamental intuition: that the world can be seen as a collection of interconnected systems, and our role as thinkers or artists is to make sense of the connections. We might even say all human creative endeavors are objects in one grand category, and creativity itself consists in the morphisms between them, transforming one form of understanding into another. If so, then to create a *total work of art* is to embrace the richest network of morphisms possible—to let music transform philosophy, let mathematics inform music, let art elucidate science, until the very distinctions blur. In that blurring, new clarity may emerge: a realization of underlying unity. This manifesto is a step toward that unity, a testament to the possibility that one can live in many worlds at once, speaking across the divides, and in doing so, create something indivisible and whole.