Transcendental Mathematics

Homotopy Type Theory, Husserlian Constitution, and the Limits of Formalization

Unifying Phenomenology and Ontology through Homotopy Type Theory

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Dedicated to Alyssa, my mom Kavita and Tim Pierson.

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Part I

Introduction: The problem of bridging phenomenology and mathematical ontology

0.1 The phenomenological-ontological divide

Contemporary philosophy faces a fundamental aporia in the relationship between subjective experience and mathematical structure. Edmund Husserl's transcendental phenomenology grounds all knowledge in the constitutive activity of consciousness, establishing a rigorous science of subjective experience that brackets questions of being-in-itself. [1] Conversely, Alain Badiou's mathematical ontology asserts that mathematics *is* ontology, that set theory captures the formal conditions for presentation of being qua being, independent of any constituting subject. [2]

These positions appear irreconcilable. Husserl's phenomenology culminates in transcendental idealism, where objectivity emerges from intersubjective constitution. The lifeworld provides the foundational stratum upon which scientific idealizations are constructed. [3] Badiou explicitly breaks with all forms of idealism, rejecting the transcendental subject entirely. For Badiou, objects appear in worlds with degrees of existence determined by objective logical structures, not through constitution by consciousness. [4]

Yet both thinkers share a commitment to foundations: Husserl seeks transcendental foundations for logic itself, while Badiou identifies mathematics as the discourse on being. Both recognize that formal structures possess an objectivity that transcends psychological contingency. This suggests a deeper convergence beneath apparent opposition.

1. Edmund Husserl, *Formal and Transcendental Logic*, trans. Dorion Cairns (The Hague: Martinus Nijhoff, 1969), 40.
2. Alain Badiou, *Being and Event*, trans. Oliver Feltham (London: Continuum, 2005), 4.
3. Husserl, *The Crisis of European Sciences and Transcendental Phenomenology*, trans. David Carr (Evanston: Northwestern University Press, 1970), 121.
4. Badiou, *Logics of Worlds*, trans. Alberto Toscano (London: Continuum, 2009), 97.

0.2 The type-theoretic turn

Per Martin-Löf's intuitionistic type theory provides unexpected resources for bridging this divide. [5] Martin-Löf himself acknowledged phenomenological influences, particularly from Husserl and Brentano, in developing his meaning explanations for type theory. [6] Type theory makes judgments primitive rather than propositions, echoing Husserl's emphasis on acts of consciousness over abstract logical forms.

Moreover, Martin-Löf's constructive foundations resonate with phenomenological constitution. Types are not given complete but constructed through introduction and elimination rules, much as Husserl's intentional objects are constituted through founding and founded acts. [7]

Homotopy type theory extends Martin-Löf type theory by interpreting identity types as paths and types as spaces, revealing that intensional type theory has intrinsic homotopical structure. [8] The univalence axiom states that equivalent types are literally identical, embodying a principle of structural invariance that bridges extensional and intensional perspectives.

0.3 Thesis and roadmap

This work argues that homotopy type theory provides the framework to unify Husserl's transcendental phenomenology with Badiou's mathematical ontology into coherent transcendental philosophy. The key insight is that HoTT operates simultaneously at two levels: low-level phenomenological foundations capturing constitution and evidence, and high-level ontological structures revealing objective mathematical forms.

The subsequent chapters develop this synthesis systematically, culminating in a proposal for transcendental mathematics where mathematical structures emerge through interaction of constitutive activity and objective constraints.

5. Per Martin-Löf, *Intuitionistic Type Theory* (Naples: Bibliopolis, 1984).

6. Per Martin-Löf, "On the Meanings of the Logical Constants and the Justifications of the Logical Laws," *Nordic Journal of Philosophical Logic* 1, no. 1 (1996): 11-60.

7. Husserl, *Logical Investigations*, trans. J.N. Findlay (London: Routledge, 2001), Investigation VI, §43-52.

8. The Univalent Foundations Program, *Homotopy Type Theory: Univalent Foundations of Mathematics* (Princeton: Institute for Advanced Study, 2013).

Husserl's transcendental logic and intentionality

0.4 Intentionality as fundamental structure

Husserl's phenomenology begins with the insight that consciousness is always consciousness *of* something. Intentionality designates this essential feature: every mental act has directedness toward an object in a particular manner. [9] The intentional structure involves three correlative moments: the intentional act (noesis), the intentional object, and the intentional content (noema).

From a type-theoretic perspective, intentional structure has form of context-dependent judgment. An intentional act does not access objects simpliciter but objects-under-descriptions, objects-as-determined-in-certain-respects. This suggests formalization using dependent types, where the type of the object depends on the intentional mode.

0.5 Evidence and fulfillment

Husserl's theory of knowledge centers on evidence (Evidenz), understood not as subjective certainty but as fulfillment of intentions. [10] Intentions can be empty (merely signitive) or fulfilled (intuitive). Evidence arises through synthesis of coincidence between meant and given.

Type-theoretically, evidence corresponds to proof terms. The judgment $a : A$ asserts that term a provides evidence for type A. Empty intentions correspond to uninstantiated types; fulfillment corresponds to construction of canonical terms through introduction rules.

9. Husserl, *Logical Investigations*, V, §20.
10. Husserl, *Logical Investigations*, VI, §8.

0.6 Formal and transcendental logic

In *Formal and Transcendental Logic*, Husserl articulates the relationship between pure formal logic and transcendental phenomenology. [11] Formal logic stratifies into three levels: morphology of judgments, logic of non-contradiction, and logic of truth. The transition from formal to transcendental logic recognizes that logical forms require grounding in consciousness.

0.7 Categorial intuition and genetic phenomenology

Husserl's theory of categorial intuition explains how consciousness accesses ideal structures like mathematical objects. [12] Categorial acts are founded on sensuous intuition but constitute new intentional objects not reducible to underlying sensory objects. From a type-theoretic perspective, categorial intuition corresponds to formation of dependent types and identity types.

Genetic phenomenology investigates how complex intentional structures emerge from simpler ones through sedimentation and motivation.[13] Type theory provides formal analog through universe hierarchy and type formation rules.

11. Husserl, *Formal and Transcendental Logic*.
12. Husserl, *Logical Investigations*, VI, §§40-60.
13. Husserl, *Analyses Concerning Passive and Active Synthesis*, trans. Anthony Steinbock (Dordrecht: Kluwer, 2001).

Badiou's set-theoretic ontology and theory of appearing

0.8 Mathematics as ontology

Alain Badiou's fundamental thesis radically identifies mathematics with ontology: mathematics *is* the discourse on being qua being. [14] Set theory, specifically ZFC, captures the formal conditions for presentation of any existing thing. The claim rests on meta-ontological argument that ontology seeks to articulate what it means for something to be, independent of what it specifically is.

0.9 Situation, state, and event

Badiou distinguishes situation from state of situation.[15] The situation presents multiplicities; the state represents what is presented through meta-structure. Set-theoretically, if X is a set, the power set $\mathcal{P}(X)$ represents the state, creating fundamental excess by Cantor's theorem.

An event is supernumerary element that names itself, violating axiom of foundation.[16] Events are ontologically impossible within ZFC yet necessary for genuine novelty, rupturing situation's structure and forcing reorganization.

0.10 Truth procedures and forcing

A truth procedure is infinite process initiated by event through which a truth is constructed.[17] Badiou appropriates Paul Cohen's forcing technique: a truth is generic set,

14. Badiou, *Being and Event*, 6.
15. Badiou, *Being and Event*, Meditation 8.
16. Badiou, *Being and Event*, Meditation 17.
17. Badiou, *Being and Event*, Meditation 31.

subset of situation that is indiscernible within situation's language. This formalism captures how scientific and artistic truths emerge through processes that escape encyclopedic knowledge.

0.11 Appearing and logics of worlds

Logics of Worlds addresses being-there, appearing of beings in determinate worlds.[18] Each world has transcendental: complete Heyting algebra that indexes degrees of existence. Category theory provides formalism, with worlds as topoi having internal logic. This is objective phenomenology without constituting consciousness.

18. Badiou, *Logics of Worlds*, 97.

Martin-Löf type theory as phenomenological foundation

0.12 Judgments as acts of knowledge

Martin-Löf's 1996 paper radically reconceives judgment as any act of knowledge, any epistemic achievement. [19] Four basic judgment forms structure type theory: A type, $a : A$, $A = B$, and $a = b : A$. These are not propositions but acts of judging, aligning with Husserl's emphasis on evidence and givenness.

0.13 Evidence and meaning explanations

Martin-Löf's meaning explanations provide pre-mathematical semantics for type theory. [20] A type is given by specifying how canonical elements are formed and when they are equal. What does $a : A$ mean? Term a is method which, when executed, computes to canonical element of A. This computational semantics connects to Husserl's theory of fulfillment.

0.14 Dependent types and horizons

Dependent types provide formalization of Husserl's horizonal structure. A dependent type $B(x)$ indexed by $x : A$ captures how meaning of B depends on which x we consider. The telescoping context represents cumulative sedimentation characteristic of genetic phenomenology.

19. Martin-Löf, "On the Meanings of the Logical Constants," 12-15.
20. Martin-Löf, *Intuitionistic Type Theory*, 6-10.

0.15 Identity types and constructive foundations

Identity types $\mathsf{Id}_A(a, b)$ formalize equality as type rather than meta-theoretic relation. This structure illuminates Husserl's eidetic variation: identity proofs are paths between points, representing different routes of variation preserving essence. Martin-Löf type theory is constructive, aligning with phenomenological constitution where objects are constituted through explicit constructions.

Homotopy type theory as unifying framework

0.16 The homotopical interpretation

Homotopy type theory emerges from recognizing that intensional Martin-Löf type theory has intrinsic homotopy-theoretic semantics. [21] Types interpret as spaces, terms as points, and identity types as path spaces. This is *synthetic* homotopy theory, taking spaces and paths as primitive.

0.17 Univalence axiom

The univalence axiom states that identity of types is equivalent to equivalence of types. [22] An equivalence $f : A \simeq B$ is function that is invertible up to homotopy. Univalence asserts:

$$\mathsf{ua} : (A \simeq B) \simeq (A =_{\mathsf{Type}} B)$$

This means equivalent types are literally identical, capturing mathematical practice where isomorphic structures are freely substituted. Philosophically, univalence embodies structuralism: structural properties determine identity.[23]

21. Steve Awodey and Michael Warren, "Homotopy Theoretic Models of Identity Types," *Mathematical Proceedings of the Cambridge Philosophical Society* 146, no. 1 (2009): 45-55.
22. The Univalent Foundations Program, *Homotopy Type Theory*, 67-73.
23. Steve Awodey, "Structuralism, Invariance, and Univalence," *Philosophia Mathematica* 22, no. 1 (2014): 1-11.

0.18 Higher inductive types

Higher inductive types generalize inductive types by allowing constructors for paths as well as points. [24] The circle S^1 has one point and one non-trivial loop. Using HITs, Licata and Shulman proved $\pi_1(S^1) \cong \mathbb{Z}$ directly in HoTT. [25] HITs enable synthetic homotopy theory without set-theoretic reduction.

0.19 Unifying phenomenology and ontology

HoTT unifies Husserlian phenomenology and Badiousian ontology through its dual nature. Intensionally, identity types are proof-relevant with computational semantics and context-dependent meaning. Extensionally, univalence makes structural identity objective, with homotopical semantics providing geometric structure. These are not separate aspects but unified in HoTT's internal logic.

The key is that paths have dual status: they are both constructions (phenomenological) and objective structures (ontological). Univalence crystallizes this duality: equivalent structures are intersubstitutable in practice yet share objective properties. This resolves the tension between Husserl's constitutive phenomenology and Badiou's mathematical realism.

0.20 Bridging set theory and category theory

Badiou's progression from *Being and Event* (set theory) to *Logics of Worlds* (category theory) creates tension requiring resolution. Set theory provides intrinsic ontology; category theory provides extrinsic phenomenology. HoTT unifies these through its interpretation in $(\infty, 1)$-toposes, which generalize both set-theoretic models and categorical structures. [26]

The internal language of $(\infty, 1)$-toposes is precisely homotopy type theory. This means HoTT can express both set-theoretic reasoning (via h-sets at level 0) and categorical structures (via higher types). Badiou's two frameworks become special cases of more general type-theoretic foundation.

24. The Univalent Foundations Program, *Homotopy Type Theory*, Chapter 6.
25. Daniel Licata and Michael Shulman, "Calculating the Fundamental Group of the Circle in Homotopy Type Theory," *Proceedings of LICS* (2013): 223-232.
26. Michael Shulman, "Homotopy Type Theory: A Synthetic Approach to Higher Equalities," *Categories for the Working Philosopher* (2018): 283-315.

Active inference and categorical phenomenology

0.21 The free energy principle

Karl Friston's free energy principle states that adaptive biological systems minimize variational free energy. [27] Free energy is information-theoretic quantity that bounds surprise in sensory data given a generative model. Systems minimize free energy through perception (updating internal models) and action (changing sensory inputs).

This provides mathematical formalization of intentionality as system-environment coupling mediated by Markov blankets. A Markov blanket partitions states into internal (hidden states of system), external (environmental states), sensory (conveying environmental information), and active (influencing environment). This statistical boundary enables self-organizing systems to maintain integrity while engaging with world.

0.22 Active inference as embodied intentionality

Active inference implements hierarchical predictive processing that mirrors Husserl's genetic phenomenology.[28] Higher levels track slower, more abstract regularities; lower levels track fast dynamics. Prediction errors propagate upward; predictions propagate downward. This bidirectional flow captures noetic-noematic correlation.

The generative model corresponds to Husserl's intentional structures: hierarchically organized expectations about how sensory data are generated. Precision-weighting captures attention as optimization of gain control. Sensory attenuation filters self-generated inputs, corresponding to distinction between self and other.

27. Karl Friston, "The Free-Energy Principle: A Unified Brain Theory?," *Nature Reviews Neuroscience* 11 (2010): 127-138.
28. Jakob Hohwy, *The Predictive Mind* (Oxford: Oxford University Press, 2013).

Consciousness emerges from hierarchically deep inference about precision on the action model across multiple timescales. [29] Subjective valuation structures phenomenal selfhood: experience is organized by inference about how well organism minimizes expected free energy. World is encountered as matrix of possibilities for action.

0.23 Categorical formulations

Recent developments establish polynomial functors as the fundamental mathematical structure underlying adaptive systems and active inference.[30]

Polynomial functors provide the fundamental mathematical language for compositional systems and active inference.[31] A polynomial functor $p : \mathsf{Set} \to \mathsf{Set}$ has the form:

$$p = \sum_{i \in p(1)} y^{p[i]}$$

where $p(1)$ denotes the set of positions, $p[i]$ denotes the set of directions at position i, and y denotes the identity functor on Set. This notation emphasizes that a polynomial is entirely determined by its positions and directions: evaluating at any set X yields $p(X) = \sum_{i \in p(1)} X^{p[i]}$.

The category Poly has polynomial functors as objects and dependent lenses as morphisms. A lens $\varphi : p \to q$ consists of a pair (φ^*, φ_*) where $\varphi^* : p(1) \to q(1)$ maps positions forward and, for each position $i \in p(1)$, the function $\varphi_{*,i} : q[\varphi^*(i)] \to p[i]$ maps directions backward. This bidirectional structure captures interaction protocols: positions represent outputs while directions represent possible inputs.

Dynamical systems emerge as coalgebras of polynomial functors. A p-coalgebra consists of a state space S together with a morphism $\sigma : S \to p(S)$, which decomposes into a readout map $\sigma^* : S \to p(1)$ and an update map $\sigma_* : (s : S) \to (p[\sigma^*(s)] \to S)$. For active inference, the state s encodes beliefs, the readout $\sigma^*(s)$ generates observations, and the update $\sigma_*(s)$ specifies how actions transform beliefs. The Markov blanket structure emerges naturally: sensory states correspond to positions, active states to directions, and internal states to the coalgebra carrier.

29. George Deane, "Consciousness in Active Inference: Deep Self-Models, Other Minds, and the Challenge of Psychedelic-Induced Ego-Dissolution," *Neuroscience of Consciousness* 2021, no. 2 (2021): niab024.

30. Toby St Clere Smithe, "Mathematical Foundations for a Compositional Account of the Bayesian Brain," PhD thesis, University of Oxford (2023), arXiv:2212.12538.

31. David Spivak and Nelson Niu, *Polynomial Functors: A Mathematical Theory of Interaction* (2024), available at https://toposinstitute.github.io/poly.

Composition of polynomials provides hierarchical structure. Active inference hierarchies correspond to iterated compositions $p_1 \triangleleft p_2 \triangleleft \cdots \triangleleft p_n$. Each level modulates lower levels: the polynomial p_{i+1} provides the state space for level i, with precision weighting implemented through the backward maps. The composition product is associative but not commutative, reflecting the inherent asymmetry in hierarchical control.

0.24 Computational phenomenology

Computational phenomenology uses generative modeling to formalize phenomenological descriptions.[32] Husserlian time-consciousness (retention-protention structure) is formalized using Bayesian networks that integrate past and anticipate future. The present moment emerges as integrated continuity, not atomic instant.

Affective inference tracks how well organism minimizes expected free energy. [33] Increases in expected uncertainty produce unpleasure; decreases produce pleasure. Affective tone permeates perception via deep goal hierarchies, corresponding to Husserl's recognition that experience is always affectively colored.

This demonstrates that phenomenological structures admit rigorous mathematical formalization compatible with type-theoretic foundations. Active inference bridges first-person phenomenology and third-person neuroscience through information geometry.

0.25 Bridging phenomenology and computation

Active inference resolves apparent tension between phenomenological first-person perspective and computational third-person description. The free energy principle is neutral about ontology: it applies to any system with Markov blanket. But when system has hierarchically deep temporal models, consciousness emerges as intrinsic feature.

This aligns with dual-aspect monism: same process has physical aspect (free energy minimization) and phenomenal aspect (conscious experience). [34] Physiological and phenomenal are not separate substances but complementary descriptions of unified process.

Type theory provides formal framework for this unity. Computational semantics (proofs as programs) captures mechanistic aspect. Phenomenological semantics (judg-

32. Daniel Rudrauf et al., "A Mathematical Model of Embodied Consciousness," *Journal of Theoretical Biology* 428 (2017): 106-131.

33. Karl Friston and Christopher Frith, "A Duet for One," *Consciousness and Cognition* 36 (2015): 390-405.

34. Mark Solms and Karl Friston, "How and Why Consciousness Arises: Some Considerations from Physics and Physiology," *Journal of Consciousness Studies* 25, no. 5-6 (2018): 202-238.

ments as acts of knowledge) captures experiential aspect. These coexist in single formalism because type theory operates at both levels simultaneously.

Synthesis and conclusions

0.26 Transcendental mathematics

We propose transcendental mathematics as philosophical position where mathematical structures are neither Platonic abstracta independent of mind nor subjective constructions reducible to psychology. Instead, they emerge through interaction of constitutive activity and objective constraints, formalized in homotopy type theory.

This position reconciles Husserl and Badiou by recognizing that both are partially correct. Husserl rightly insists that mathematical objects require constitutive activity for access and understanding. We cannot think mathematical structures except through intentional acts that construct, grasp, and manipulate them. Genetic phenomenology reveals how complex mathematical concepts emerge from simpler intuitions through sedimentation.

Badiou rightly insists that mathematical structures possess objectivity transcending subjective constitution. Mathematical truths are not contingent on human psychology. Set theory captures formal conditions for presentation of being itself. Mathematical structures constrain what can be thought rather than being freely constructed.

HoTT synthesizes these insights. Types are constructed through formation rules (Husserlian constitution), yet they have objective homotopical structure (Badiousian realism). Identity proofs are constructed through evidence (phenomenological), yet they are paths in objective space (ontological). Univalence makes structural identity both subjective (depends on equivalence construction) and objective (determines literal identity).

0.27 The synthetic method

Synthetic mathematics takes mathematical structures as primitive rather than reducing them to set-theoretic constructions. Synthetic geometry treats points and lines as basic; synthetic homotopy theory treats spaces and paths as basic. HoTT provides synthetic foundations where types, terms, and judgments are primitive.

This synthetic approach aligns with phenomenological method. Phenomenology describes structures as they present themselves rather than reducing them to hypothetical substrates. Husserl's categorial intuition grasps states of affairs directly, not as constructions from sensory atoms. Similarly, HoTT grasps types directly as spaces, not as sets with additional structure.

The synthetic method avoids vicious regress. If mathematical objects are constructed from sets, what are sets constructed from? Set theory postulates sets as primitive, but then faces question of ontological status. HoTT breaks regress by taking type-theoretic judgments as primitive acts that require no further foundation. Judgments are what we do; they need no reduction.

0.28 Overcoming the correlation gap

Meillassoux argues that post-Kantian philosophy suffers from correlationism: inability to think being independent of relation to thought.[35] Every access to being is mediated through subject-object correlation. This makes ancestral statements (about time before consciousness) problematic.

HoTT overcomes correlationism without naive realism. Types have objective structure independent of whether anyone constructs them, yet they are accessible only through judgments. The resolution: judgments are not arbitrary but constrained by objective type structure. When we judge $a : A$, we are not creating this fact but recognizing it. The objectivity lies in what judgments are correct, not in eliminating judgments.

This parallels Husserl's transcendental idealism properly understood. Transcendental constitution does not create objects ex nihilo but provides conditions for their givenness. Objects transcend acts that constitute them because constitution is governed by eidetic laws, not arbitrary will. Similarly, type formation is governed by rules that constrain what can be constructed.

0.29 Mathematical structuralism realized

Mathematical structuralism holds that mathematical objects are positions in structures, individuated by structural relationships rather than intrinsic properties. [36] HoTT realizes

35. Meillassoux, *After Finitude*, 5.

36. Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford: Oxford University Press, 1997).

structuralism formally through univalence.

In set theory, isomorphic structures are distinct. The natural numbers constructed from empty set differ from those constructed via Zermelo ordinals, yet mathematically they are "the same." This forces structuralists into awkward positions: either embracing non-eliminative structuralism (structures as sui generis entities) or eliminative structuralism (no mathematical objects, only structures).

HoTT dissolves the dilemma. Univalence makes equivalent structures literally identical. There is no junk distinguishing structurally identical objects because identity is structural. Two constructions of natural numbers are identical if they are equivalent as ordered sets with successor operation. The particular construction method is propositionally irrelevant.

This validates mathematical practice. Mathematicians freely identify isomorphic groups, homeomorphic spaces, equivalent categories. Univalence formalizes this principle: mathematical identity is structural identity. The essence of mathematical objects lies in their structural properties, and univalence makes this essence constitutive of identity.

0.30 Consciousness and mathematical intuition

Active inference provides account of how finite consciousness accesses infinite mathematical structures. Mathematical intuition is not mysterious faculty but specialized application of hierarchical predictive processing. Abstract concepts emerge from embodied sensorimotor engagement through multiple levels of abstraction.

Basic arithmetic emerges from practices of collecting, comparing, ordering. Through sedimentation, these operations become idealized into formal operations on abstract numbers. Geometric intuition emerges from spatial navigation and manipulation. Through idealization, physical space becomes abstract geometric structure.

The hierarchical architecture of active inference explains how consciousness bootstraps from concrete to abstract. Lower levels track sensorimotor contingencies; middle levels track causal relationships; higher levels track abstract patterns. Mathematical concepts occupy highest levels, maximally removed from sensory particulars yet still grounded in embodied practice through chain of dependencies.

This resolves Kant's question about synthetic a priori knowledge. Mathematical knowledge is synthetic because it extends beyond definitions; it is a priori because it concerns necessary structures. The necessity derives not from Platonic realm or transcendental subject but from constraints on any system minimizing expected free energy in structured environment. Mathematical structures are patterns that emerge universally in systems engaging with world.

0.31 Truth and objectivity

Badiou's theory of truth as infinite generic procedure connects to HoTT through notion of identity type. A truth is indiscernible subset constructed through forcing. Type-theoretically, truths are elements of identity types that cannot be definitionally reduced but must be constructed through proof.

Scientific truth emerges through verification of connections between phenomena and theory. Each verification is local judgment extending our knowledge. The complete truth is never finished but unfolds through infinite procedure. In HoTT, this corresponds to construction of identity proof through sequence of rewrites and transports that cannot be definitionally collapsed.

Generic truths escape constructible language (Gödel's L). In HoTT, not all identity proofs are uniquely determined by endpoints. Higher homotopical structure means multiple paths exist, corresponding to different verification routes. The indiscernibility of generic truth corresponds to proof-relevance of identity types.

Yet truths are objective. Once constructed, identity proofs have determinate structure. Other mathematicians can verify the proof, follow the same path, reach same conclusion. Objectivity does not require uniqueness; it requires intersubjective reproducibility. HoTT's computational semantics ensures reproducibility: proofs execute deterministically to canonical forms.

0.32 The future of foundations

HoTT represents third generation of foundational systems after set theory and category theory. Set theory provided first rigorous foundations but privileges discrete, extensional structures. Category theory enabled structural mathematics but remained semi-formal. HoTT combines rigor of set theory with structuralism of category theory while adding computational content.

The future likely involves refinements addressing current limitations. Cubical type theory provides computational models of univalence, resolving decidability questions. [37] Modal type theories extend HoTT with modalities for necessity, possibility, temporal

37. Cyril Cohen et al., "Cubical Type Theory: A Constructive Interpretation of the Univalence Axiom," *21st International Conference on Types for Proofs and Programs* (2018).

logic. Directed type theory breaks symmetry of higher groupoids to model irreversible processes.

From philosophical perspective, HoTT opens new research directions. The connection to phenomenology requires development. Can we formalize Husserl's passive synthesis, genetic constitution, and intersubjectivity type-theoretically? Can Badiou's four truth procedures (science, art, politics, love) be characterized using type-theoretic structures?

Active inference suggests that consciousness is computable in extended sense. Not that minds are Turing machines, but that experiential structures admit rigorous mathematical description compatible with physical implementation. This challenges both computational theories (mind as symbol manipulation) and mysterian positions (consciousness inherently mysterious).

0.33 Conclusion

We have argued that homotopy type theory provides framework for synthesizing Husserlian transcendental phenomenology with Badiousian mathematical ontology. Martin-Löf type theory offers phenomenological foundations through judgment-theoretic approach and constructive semantics. HoTT extends this with homotopical interpretation that captures objective mathematical structure while preserving constructive character.

The univalence axiom crystallizes the synthesis: structural identity is both constructed (phenomenological) and objective (ontological). Higher inductive types enable synthetic mathematics that avoids both set-theoretic reduction and Platonic mysticism. Active inference shows how these abstract structures connect to embodied consciousness through hierarchical Bayesian inference.

Transcendental mathematics emerges as position where mathematical structures are neither mind-independent abstracta nor mental constructions but intersubjectively constituted objective patterns accessible through type-theoretic reasoning. This resolves longstanding tensions in philosophy of mathematics while providing foundations adequate for contemporary mathematical practice.

The philosophical significance extends beyond foundations. By showing how subjective constitution and objective structure coexist in single formalism, HoTT suggests resolution to mind-body problem, subject-object split, and appearance-reality distinction that have structured philosophy since Descartes. The future of philosophy may lie in continuing development of formal frameworks that integrate first-person and third-person perspectives into coherent unified science.

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[25] Mark Solms and Karl Friston, "How and Why Consciousness Arises: Some Considerations from Physics and Physiology," *Journal of Consciousness Studies* 25, no. 5–6 (2018): 202–238.

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Part II

Preface

This volume constitutes Part II of *Transcendental Mathematics: Unifying Phenomenology and Ontology through Homotopy Type Theory*. Where Part I established the programmatic vision of a synthetic transcendentalism reconciling Husserlian phenomenology with Badiouian ontology, the present work undertakes the patient, detailed labor that such a reconciliation demands.

The central thesis of Part I—that type-theoretic and sheaf-theoretic structures can serve as transcendental conditions for AI governance—requires substantial elaboration. Critics have rightly observed that the relationship between mathematical structures and philosophical concepts cannot be established by mere assertion. To claim a correspondence between, say, identity types and Husserlian syntheses of identification is to invite the question: what precisely is the structure of each, and wherein does the alleged correspondence consist? Moreover, the Derridean critique of presence establishes that any such formalization operates within language, and language is always already infected by the deferral of meaning that Derrida names *différance*. No global orientation can be secured.

This volume therefore proceeds with deliberate patience through the following terrain:

First, I examine Per Martin-Löf's explicit engagement with Husserlian phenomenology, demonstrating that the founder of intuitionistic type theory conceived his project as a formalization of phenomenological epistemology. The Leiden-Stockholm-Prague tradition of Sundholm, Klev, and van Atten has developed this connection with scholarly rigor, and I draw extensively on their work.

Second, I provide a detailed mapping of type-theoretic structures to Husserlian concepts, attending to the specific invariants that constitute each and examining the extent to which weakening the mathematical formalism permits bijection with the philosophical concept.

Third, I work through Kant's *Metaphysics of Morals* to demonstrate how type-theoretic reconstruction isolates empirical premises as contingent inputs to supposedly *a priori* moral reasoning. The prejudices concerning women, sexuality, and marriage that mar Kant's practical philosophy are revealed not as consequences of the categorical imperative's formal structure but as historically conditioned assumptions smuggled into the derivation.

Fourth, I engage Derrida's diagonal argument as a sustained counterpoint. Derrida's explicit acknowledgment of the Cantor-Russell-Gödel diagonal establishes a structural parallel between deconstruction and formal incompleteness. The supplement, iterability, and *différance* function as quasi-transcendentals that both enable and limit any formal system's capacity to capture meaning.

The conclusion I draw is perhaps unwelcome to those who seek global foundations: there can be no global orientation in language, only local positions that may have no necessary relation to others. Sheaf theory formalizes exactly this predicament: local sections may exist and satisfy gluing conditions on overlaps, yet no global section need exist. Catren's hypertranscendental perspectivalism and Zalamea's transmodern synthetic universality converge on this thesis of irreducible locality.

For AI governance, this implies that no algorithm can occupy a meta-position from which all local language-games become adjudicable. Governance must itself be structurally local, with interfaces between positions that preserve irreducible difference rather than subsuming it under universal categories.

Chicago, November 2025

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Chapter 1

Per Martin-Löf's Husserlian Foundations

> If you observe how Husserl used these terms, you will see that they were appropriately applied by Heyting.
>
> Per Martin-Löf, *On the Meanings of the Logical Constants*

1.1 Introduction: The Phenomenological Turn in Type Theory

Per Martin-Löf's intuitionistic type theory represents the most sustained attempt in twentieth-century logic to ground mathematical foundations in phenomenological epistemology. This is not a retrospective interpretation imposed by commentators but Martin-Löf's explicit self-understanding. In his Siena lectures of 1983, his Leiden lectures of 1993, and his Bern lecture of 2002, Martin-Löf repeatedly invokes Husserl's terminology and explicitly maps type-theoretic concepts onto Husserlian distinctions.[1]

The significance of this connection cannot be overstated. Martin-Löf's type theory is not merely a technical system among others; it is the foundation for contemporary proof assistants (Agda, Coq, Lean), the basis for the Univalent Foundations program, and

1. Per Martin-Löf, "On the Meanings of the Logical Constants and the Justifications of the Logical Laws," in *Nordic Journal of Philosophical Logic* 1, no. 1 (1996): 11–60. Originally delivered as lectures at the meeting *Teoria della Dimostrazione e Filosofia della Logica*, Siena, April 6–9, 1983.

arguably the most important development in constructive mathematics since Brouwer. If this entire edifice rests on phenomenological foundations, then the relationship between type theory and phenomenology is not a philosophical curiosity but a matter of central importance for understanding what mathematical formalization can and cannot achieve.

In this chapter, I examine Martin-Löf's engagement with Husserl in detail, drawing on both his published works and the transcribed lectures now available through the Prague archive. I then situate this engagement within the broader tradition of Leiden-Stockholm-Prague scholarship that has developed the phenomenological interpretation of type theory with considerable rigor.

1.2 The Siena Lectures: Meanings and Justifications

The foundational statement of Martin-Löf's philosophical position appears in "On the Meanings of the Logical Constants and the Justifications of the Logical Laws," delivered at Siena in April 1983 and published in 1996. The title itself signals a departure from the dominant Fregean and Tarskian traditions: Martin-Löf is concerned not with truth conditions but with *meanings*—what it is to understand a logical constant—and not with validity in the model-theoretic sense but with *justifications*—what makes it correct to apply a rule of inference.

Martin-Löf's starting point is a critique of the standard semantics:

> The predominant approach to the semantics of logic, whether in the form of Tarski's truth definition or in the form of Beth-Kripke-Hintikka semantics, takes truth as the basic semantic notion. A proposition is explained by laying down what counts as evidence for its truth, and the logical operations are explained by their truth conditions.[2]

Against this, Martin-Löf proposes to take *knowledge* as primitive. The meaning of a proposition is given not by its truth conditions but by what counts as *knowing* that the proposition is true. This shift from truth to knowledge is precisely the shift from Fregean to Husserlian semantics: whereas Frege's third realm of thoughts exists independently of any thinker, Husserl's analysis begins from the act of judgment in which logical content is constituted.

The crucial distinction Martin-Löf introduces is between *propositions* and *judgments*:

2. Martin-Löf, "Meanings of the Logical Constants," 12.

> What is it that logic is really about? My answer is: it is about proof, not about truth in the first place... The things that we prove are what I call judgments. A judgment is an act of knowing, and a proof is what makes a judgment evident.[3]

A proposition (*Satz*) is what we combine with logical operations; a judgment (*Urteil*) is what we assert and what serves as premise or conclusion of an inference. This distinction directly mirrors Husserl's *Logical Investigations*, where the proposition as ideal content is distinguished from the judgment as act.[4]

Martin-Löf acknowledges Husserl's priority explicitly:

> If you observe how Husserl used these terms [proposition and judgment], you will see that they were appropriately applied by Heyting when he explained the intuitionistic meaning of the logical constants in terms of proof rather than in terms of truth.[5]

This establishes a lineage: Husserl $\rightarrow$ Heyting $\rightarrow$ Martin-Löf. The phenomenological analysis of judgment grounds the intuitionistic interpretation of logic, which in turn grounds type theory.

1.3 The Three-Fold Distinction: Truth, Evidence, Validity

In "Truth of a Proposition, Evidence of a Judgement, Validity of a Proof" (1987), Martin-Löf systematizes the phenomenological foundations of type theory by distinguishing three notions that had been conflated in the standard semantics.[6]

Definition 1.1 (Truth of a Proposition)**.** A proposition A is true if and only if there exists a proof of A, i.e., if the type A is inhabited.

Definition 1.2 (Evidence of a Judgment)**.** A judgment "A is true" is evident if and only if we *possess* a proof $a : A$.

3. Martin-Löf, "Meanings of the Logical Constants," 14.
4. Edmund Husserl, *Logical Investigations*, trans. J. N. Findlay, ed. Dermot Moran (London: Routledge, 2001), Investigation I, §§11–14.
5. Martin-Löf, "Meanings of the Logical Constants," 15.
6. Per Martin-Löf, "Truth of a Proposition, Evidence of a Judgement, Validity of a Proof," *Synthese* 73 (1987): 407–420.

Definition 1.3 (Validity of a Proof)**.** A proof is valid if and only if it correctly witnesses the judgment it claims to establish.

This tripartite structure mirrors Husserl's distinctions with precision:

1. **Truth** (*Wahrheit*) corresponds to the ideal correlate of a judgment—the state of affairs that obtains when the judgment is correct. For Husserl, truth is the identity between the meant and the given; for Martin-Löf, truth is the existence of a proof-object.

2. **Evidence** (*Evidenz*) corresponds to the mode of consciousness in which the object is given "in person" (*leibhaftig*). For Husserl, evidence is the experience of fulfillment in which an empty intention is filled by intuitive content; for Martin-Löf, evidence is the possession of the proof-object itself.

3. **Validity** corresponds to the justification or epistemic ground for the judgment. For Husserl, this involves the genetic analysis of how evidence is constituted; for Martin-Löf, it involves the correctness of the inference rules that license the judgment.

The parallel is not merely structural but conceptual. In both frameworks, the central insight is that *truth is not primitive*. Truth supervenes on something more basic: for Husserl, the fulfillment structure of intentional acts; for Martin-Löf, the existence of constructions of a given type.

1.4 The Bern Lecture: Formal Logic and Formal Ontology

The most explicit mapping of type theory onto Husserlian categories appears in Martin-Löf's Bern lecture of January 17, 2002, titled "Husserl's Correlation between Formal Logic and Formal Ontology."[7] Here Martin-Löf directly addresses the relationship between Husserl's distinction in *Formal and Transcendental Logic* and the structure of type theory.

Husserl distinguishes two directions of formal investigation:[8]

Definition 1.4 (Formal Apophantics)**.** The study of judgment forms and their relations—logical grammar, the theory of inference, the conditions for non-contradiction.

7. Per Martin-Löf, "Husserl's Correlation between Formal Logic and Formal Ontology" (lecture, University of Bern, January 17, 2002). Transcription available at pml.flu.cas.cz.

8. Edmund Husserl, *Formal and Transcendental Logic*, trans. Dorion Cairns (The Hague: Martinus Nijhoff, 1969), §§23–24.

Definition 1.5 (Formal Ontology)**.** The study of object-categories and their relations—thing, property, relation, state of affairs, part, whole, number, set.

These two domains are correlated but not identical. Formal apophantics concerns the *noetic* side—the structure of judgments as acts; formal ontology concerns the *noematic* side—the structure of objects as intended.

Martin-Löf argues that type theory embodies this correlation. The *judgment forms* of type theory ("$a : A$", "A type", "$a = b : A$") correspond to formal apophantics; the *type-forming operations* (dependent products, dependent sums, identity types) correspond to formal ontology. The propositions-as-types correspondence is precisely the correlation Husserl identified:

> What Husserl calls the correlation between formal logic and formal ontology is what we nowadays call the propositions-as-types correspondence, or the Curry-Howard isomorphism if you prefer to use those names.[9]

This is a striking claim. The Curry-Howard correspondence—often presented as a technical curiosity linking proof theory and type theory—is here revealed as the formal expression of a fundamental phenomenological insight: the inseparability of act and object, noesis and noema.

1.5 Meaning Explanations and Genetic Phenomenology

The methodological core of Martin-Löf's type theory is the system of *meaning explanations*. For each type-forming operation, one must explain:

1. What it is to be a *canonical* object of that type.
2. How *non-canonical* objects compute to canonical form.
3. What it is for two canonical objects to be *equal*.

This is not a definition in the classical sense—one cannot define "natural number" without circularity—but an *explanation* that makes the concept intelligible by showing how objects of the type are constructed and recognized.[10]

9. Martin-Löf, "Husserl's Correlation," transcription p. 8.
10. Per Martin-Löf, *Intuitionistic Type Theory* (Naples: Bibliopolis, 1984), 3–7.

The structure of meaning explanations corresponds to Husserl's genetic phenomenology of logical constitution. In *Experience and Judgment*, Husserl traces the constitution of logical objects back to pre-predicative experience:[11]

1. **Pre-predicative receptivity**: The passive synthesis by which objects are given prior to any active judgment.

2. **Predicative spontaneity**: The active synthesis in which categorial forms ("S is P") are constituted.

3. **Ideation**: The constitution of ideal objects (numbers, logical laws) through abstraction from particular acts.

Martin-Löf's meaning explanations can be read as formalizing this genetic structure. The canonical objects correspond to what is immediately given in intuition; non-canonical objects correspond to empty intentions that require computation (execution) to achieve fulfillment; the equality conditions correspond to the criteria by which we recognize two presentations as presentations of the same object.

1.6 The Canonical/Non-Canonical Distinction

A central technical distinction in Martin-Löf's type theory is between *canonical* and *non-canonical* terms. Canonical terms are those in "constructor form"—they directly exhibit the structure of the type. Non-canonical terms compute to canonical terms under the operational semantics.

Example 1.6. For the type $\mathbb{N}$ of natural numbers:

- 0 is canonical.

- $\mathsf{succ}(n)$ is canonical if n is canonical.

- $2 + 3$ is non-canonical; it computes to $\mathsf{succ}(\mathsf{succ}(\mathsf{succ}(\mathsf{succ}(\mathsf{succ}(0)))))$.

Ansten Klev has identified Husserl's priority regarding this distinction:[12]

11. Edmund Husserl, *Experience and Judgment*, trans. James S. Churchill and Karl Ameriks (Evanston: Northwestern University Press, 1973), §§1–14.
12. Ansten Klev, "The Justification of Identity Elimination in Martin-Löf's Type Theory," *Topoi* 38 (2019): 577–590, at 582.

> Perhaps the first to introduce a special term for what are here called canonical objects was Husserl, who in *Philosophie der Arithmetik* (1891, pp. 295–297), speaks of the reduction of *problematische Zahlen* to *Normalzahlen*.

Husserl's "problematic numbers" (complex arithmetic expressions) and "normal numbers" (numbers in standard notation) correspond exactly to Martin-Löf's non-canonical and canonical terms. The computation that reduces a non-canonical term to canonical form corresponds to the phenomenological process by which an empty or symbolic intention achieves fulfillment through intuitive presentation.

This parallel illuminates the epistemological significance of computation in type theory. Computation is not merely a pragmatic convenience for calculating results; it is the formal counterpart of the phenomenological movement from empty to fulfilled intention. When we compute $2 + 3$ to obtain 5, we are not merely manipulating symbols but achieving evidence for the judgment "$2 + 3 = 5 : \mathbb{N}$".

1.7 The Leiden-Stockholm-Prague Tradition

Martin-Löf's phenomenological approach has been developed into a substantial scholarly tradition centered on three institutions: Leiden (Göran Sundholm), Stockholm (Martin-Löf's own base), and Prague (the transcription project at the Czech Academy of Sciences).

1.7.1 Göran Sundholm

Sundholm's work establishes the historical and conceptual foundations of the phenomenological interpretation. In "Implicit Epistemic Aspects of Constructive Logic" (1997), he argues that constructive logic inherently carries epistemic content that truth-conditional semantics strips away:[13]

> The constructive logical constants are given an epistemic reading. Assertions of complex judgments are warranted by knowledge that is articulated by the means of proof that the standard natural deduction rules provide.

In "'Inference Versus Consequence' Revisited" (2012), Sundholm distinguishes sharply between the model-theoretic notion of consequence (truth preservation across models) and the proof-theoretic notion of inference (warranted passage from premises to conclusion):[14]

13. Göran Sundholm, "Implicit Epistemic Aspects of Constructive Logic," *Journal of Logic, Language and Information* 6 (1997): 191–212.
14. Göran Sundholm, "'Inference Versus Consequence' Revisited," *Synthese* 187 (2012): 943–956.

> Inference is an act; consequence is a relation. The former belongs to epistemology; the latter to semantics.

This distinction is precisely Husserl's distinction between the act of judgment (inference as epistemic achievement) and the state of affairs judged about (consequence as objective relation).

1.7.2 Ansten Klev

Klev's work provides detailed technical analyses of how specific type-theoretic constructions correspond to phenomenological concepts. In "Identity in Martin-Löf Type Theory" (2021), he examines the identity type—the type $\mathsf{Id}_A(a, b)$ whose inhabitants are proofs that a and b are identical—through the lens of Husserlian identity synthesis:[15]

> The identity type $\mathsf{Id}_A(a, b)$ represents the proposition that a and b are the same A. What does it mean to prove this proposition? It means to carry out a synthesis of identification—to recognize a and b as presentations of one and the same object.

The reflexivity term $\mathsf{refl}_a : \mathsf{Id}_A(a, a)$ corresponds to the trivial identification of a with itself; the eliminator (J-rule or path induction) corresponds to the Leibnizian principle that identicals are substitutable *salva veritate*.

Klev's analysis also addresses the crucial distinction between *judgmental* and *propositional* equality:[16]

- **Judgmental equality** ($a \equiv b : A$) is a metalinguistic judgment that a and b are definitionally the same. It is decidable and corresponds to what Husserl would call analytic identity based on meaning.
- **Propositional equality** ($\mathsf{Id}_A(a, b)$) is an object-language type that may have zero, one, or multiple inhabitants. It corresponds to synthetic identity established through evidence.

1.7.3 Mark van Atten

Van Atten's *Brouwer Meets Husserl* (2007) provides the most sustained argument that Husserlian phenomenology can ground Brouwerian intuitionism.[17] His central thesis is

15. Ansten Klev, "Identity in Martin-Löf Type Theory," *Philosophy Compass* 17, no. 2 (2022): e12805.
16. Klev, "Identity in Martin-Löf Type Theory," e12805.
17. Mark van Atten, *Brouwer Meets Husserl: On the Phenomenology of Choice Sequences* (Dordrecht: Springer, 2007).

that Husserl's transcendental phenomenology, properly developed, yields precisely the mathematics that Brouwer advocated on intuitionistic grounds.

Van Atten focuses on choice sequences—the infinite sequences generated by a "freely choosing subject" that Brouwer posited as the foundation of the continuum. These sequences are not completed infinities but ongoing processes of construction. Van Atten argues that Husserl's analysis of internal time-consciousness provides the phenomenological ground for such sequences: the retention-primal impression-protention structure of temporal consciousness is isomorphic to the structure of a choice sequence being generated in time.

For our purposes, van Atten's work is significant because it shows that the Husserl-type theory connection extends to the most controversial aspects of intuitionism. If choice sequences have phenomenological warrant, then the constructive treatment of the continuum is not merely a technical alternative but a philosophically grounded approach.

1.7.4 Bruno Bentzen's Critique

No scholarly treatment would be complete without engaging critical perspectives. Bruno Bentzen's "Propositions as Intentions" (2023) challenges the standard phenomenological interpretation of type theory.[18]

Bentzen identifies two problems with the propositions-as-intentions reading:

1. **The circularity problem**: To determine which proof-objects fulfill a given propositional intention, we must already know the content of that intention. But the content of the intention is supposed to be given by its fulfillment conditions. Hence the account is circular.

2. **The fulfillment dilemma**: Either propositional intentions are fulfilled by proof-objects (in which case propositions are not intentions but types), or they are fulfilled by states of affairs (in which case proof-objects play no role in fulfillment). Neither horn is satisfactory.

Bentzen's critique is serious and must be addressed. I offer two responses:

First, the circularity Bentzen identifies is not vicious but constitutive. In Husserlian phenomenology, the act-content-object structure is precisely not a linear determination but a holistic correlation. One cannot specify the act without the content, the content without the object, or the object without the act. The "circle" is the hermeneutic circle

18. Bruno Bentzen, "Propositions as Intentions," *Husserl Studies* 39 (2023): 143–160.

that characterizes all phenomenological analysis. Martin-Löf's meaning explanations are circular in exactly this sense: they do not define types from scratch but make explicit what is already implicit in our mathematical practice.

Second, the fulfillment dilemma rests on a dichotomy—proof-objects versus states of affairs—that the propositions-as-types correspondence dissolves. Proof-objects *are* states of affairs, viewed constructively. To inhabit the type $A \times B$ is to instantiate the state of affairs "A and B". There is no gap between the proof-object and what the proof-object "represents."

These responses do not definitively refute Bentzen, but they indicate that the phenomenological interpretation is not straightforwardly undermined by his objections.

1.8 The Husserlian Foundations

The evidence surveyed in this chapter establishes that Martin-Löf's type theory has explicit Husserlian foundations. This is not a retrospective philosophical interpretation but Martin-Löf's own self-understanding, developed over decades of lectures and publications.

The key correspondences are:

Type Theory	**Husserlian Phenomenology**
Judgment	Act of judging
Proposition	Ideal content of judgment
Type	Intentional species / mode of givenness
Term	Object as intended
Canonical term	Object given in fulfilled intention
Non-canonical term	Object given in empty intention
Computation	Fulfillment of empty intention
Evidence	Possession of proof-object (*Evidenz*)
Identity type	Synthesis of identification
Judgmental equality	Analytic identity (by meaning)
Propositional equality	Synthetic identity (by evidence)

With these foundations in place, we can proceed to examine how homotopy type theory extends and transforms this picture.

Chapter 2

Husserl on Formal Logic and the Constitution of Logical Objects

2.1 Introduction: The Stratification of Formal Logic

Husserl's *Formal and Transcendental Logic* (1929) represents the most systematic treatment of logical foundations in the phenomenological tradition.[1] Unlike the *Logical Investigations*, which are primarily concerned with refuting psychologism and establishing the ideality of logical content, *FTL* develops a complete architectonic of formal disciplines and their transcendental grounding.

For our purposes, two aspects of this work are crucial. First, Husserl's stratification of formal logic into three levels—morphology, consequence-logic, and truth-logic—provides a framework for understanding what type theory formalizes and what it leaves unformalizable. Second, Husserl's theory of manifolds (*Mannigfaltigkeitslehre*) anticipates certain features of type theory while revealing fundamental limitations that Gödel's incompleteness theorems would later expose.

2.2 The Three Strata of Formal Logic

Husserl distinguishes three levels of increasing logical depth:[2]

1. Husserl, *Formal and Transcendental Logic*.
2. Husserl, *Formal and Transcendental Logic*, §§13–21.

2.2.1 Pure Morphology of Judgments

The first stratum is the *pure morphology of judgments* (*reine Formenlehre der Urteile*), which Husserl also calls logical grammar. This level concerns the conditions for *Sinnhaftigkeit*—meaningful formation—as opposed to *Sinnlosigkeit*—meaninglessness or nonsense.

At this level, we distinguish well-formed from ill-formed expressions. "The table is green" is well-formed; "Green is or" is not. The rules of logical grammar specify which combinations of syntactic categories yield meaningful wholes.

In type-theoretic terms, this corresponds to the *formation rules* for types and terms. The judgment "$\Gamma \vdash A\,\mathsf{type}$" asserts that A is a well-formed type in context Γ. The judgment "$\Gamma \vdash a : A$" asserts that a is a well-formed term of type A. Formation rules ensure that only meaningful expressions are admitted into the system.

2.2.2 Consequence-Logic

The second stratum is *consequence-logic* (*Konsequenzlogik*), which concerns the relations of compatibility and incompatibility among judgments. Here we ask: given that certain judgments hold, what other judgments follow? What judgments are compatible? What judgments contradict each other?

Consequence-logic abstracts from the question of truth. It asks only whether certain combinations of judgments are *consistent*—capable of being jointly satisfied—not whether they are actually satisfied. As Husserl puts it:

> At this level, we are not yet concerned with truth but only with the analytic unity of judgments, with the forms that preserve the possibility of truth, without settling whether truth actually obtains.[3]

In type-theoretic terms, consequence-logic corresponds to the *inference rules* that govern derivability. Given judgments $\Gamma \vdash a : A$ and $\Gamma \vdash B[a/x]\,\mathsf{type}$, we can derive $\Gamma \vdash (a, b) : \Sigma_{x:A} B$ (if also $\Gamma \vdash b : B[a/x]$). These rules concern the formal relationships among types and terms, independent of any particular model.

2.2.3 Truth-Logic

The third stratum is *truth-logic* (*Wahrheitslogik*), which concerns the actual obtaining of truth. Here we ask: under what conditions is a judgment not merely consistent but actually true? What counts as evidence for truth?

3. Husserl, *Formal and Transcendental Logic*, §17.

This level requires what Husserl calls *transcendental logic*—the investigation of the subjective conditions under which truth is constituted. Truth-logic cannot remain purely formal; it must address the relation between formal structures and the evidential experiences that fill them with content.

In type-theoretic terms, truth-logic corresponds to the *semantics*—the interpretation of types and terms in some domain. Martin-Löf's meaning explanations constitute a form of truth-logic: they specify what it is for a type to have inhabitants and what it is to possess evidence for a judgment. The categorical semantics of type theory (in toposes, in $(\infty, 1)$-categories) provides alternative models that realize the formal structure.

2.3 Formal Apophantics and Formal Ontology

Cutting across the three strata is Husserl's distinction between *formal apophantics* and *formal ontology*.[4]

Formal apophantics is thematically directed toward *judgments* and their forms. It studies the syntactic categories of judgment (subject, predicate, copula), the operations on judgments (negation, conjunction, quantification), and the laws governing these operations (non-contradiction, excluded middle).

Formal ontology is thematically directed toward *objects* and their forms. It studies the categories of being (thing, property, relation, state of affairs, set, number) and the laws governing these categories (part-whole relations, identity, instantiation).

These two disciplines are *correlated* but not identical. Every judgment is about objects (formal apophantics presupposes formal ontology), and every object can be judged about (formal ontology presupposes formal apophantics). The correlation is not a reduction of one to the other but a mutual implication.

Martin-Löf's identification of this correlation with the propositions-as-types correspondence is illuminating. On the apophantic side, we have propositions and proofs; on the ontological side, we have types and terms. The correspondence asserts that these are two perspectives on the same underlying structure:

4. Husserl, *Formal and Transcendental Logic*, §§23–27.

Formal Apophantics	**Formal Ontology**
Proposition	Type
Proof	Term
Provability	Inhabitedness
Implication	Function type
Conjunction	Product type
Disjunction	Sum type
Universal quantification	Dependent product
Existential quantification	Dependent sum

2.4 The Theory of Manifolds

Husserl's *Mannigfaltigkeitslehre* (theory of manifolds) appears across multiple texts with evolving formulations.[5] The most developed treatment is in *Formal and Transcendental Logic*, §§51–54.

A *manifold* (*Mannigfaltigkeit*) is, for Husserl, a domain of objects governed by a system of axioms. Husserl's concept draws on Riemann rather than Cantor:

> By manifold, Cantor means a simple collection of elements... However, this conception does not coincide with that of Riemann and as used elsewhere in the theory of geometry, according to which a manifold is a collection not of merely united, but also *ordered* elements, and on the other hand not merely united, but *continuously connected* elements.[6]

This distinction is crucial. Cantor's sets are *extensional*—determined entirely by their members. Riemann's manifolds are *structural*—characterized by topological and geometric properties that go beyond mere membership. Husserl's manifolds are closer to what we would now call *structures* or *models*.

2.4.1 Definite Manifolds

The key concept is that of a *definite manifold* (*definite Mannigfaltigkeit*):

Definition 2.1 (Definite Manifold)**.** A manifold is definite if and only if the axioms governing it are *complete* in the sense that every proposition formulable in the language of the theory is either a consequence of the axioms or contradicts them.

5. See Carlo Ierna, "Husserl's Notion of Manifold," at blog.ierna.name (2012), for a comprehensive analysis.

6. Husserliana XXI, 95–96.

Husserl believed that the great mathematical theories—Euclidean geometry, arithmetic, real analysis—exemplify definite manifolds. The axioms of these theories completely determine their subject matter; no genuine indeterminacy remains.

This conception was closely tied to Hilbert's program. Hilbert's axiomatization of geometry (1899) seemed to show that Euclidean geometry could be completely captured by a finite set of axioms. Husserl expected that the same would hold for arithmetic and analysis.

2.4.2 Gödel's Incompleteness and Husserl's Manifolds

Gödel's incompleteness theorems (1931) shattered this expectation.[7] For any consistent formal system containing arithmetic:

Theorem 2.2 (First Incompleteness Theorem)**.** *There exist propositions that are neither provable nor refutable within the system.*

Theorem 2.3 (Second Incompleteness Theorem)**.** *The consistency of the system cannot be proved within the system itself.*

These results imply that arithmetic is *not* a definite manifold in Husserl's sense. There are arithmetical propositions—notably, the consistency statement itself—that are true but unprovable from the axioms. The axioms do not completely determine the domain.

The implications for Husserlian foundations are profound. If definiteness fails for arithmetic, then the model of a formal theory as completely characterizing its domain must be abandoned. Mathematical objects cannot be fully captured by finite axiom systems; there is always a surplus that escapes formalization.

Scholars have debated how Husserl would have responded to Gödel.[8] Some argue that Husserl's commitment to definiteness was inessential to his phenomenology and could be abandoned without damage. Others argue that incompleteness poses a deep challenge to any phenomenological foundation of mathematics.

For our purposes, the significance is this: *the diagonal* that produces incompleteness—the self-referential encoding of provability—is the same diagonal that Derrida invokes in his critique of presence. We shall return to this connection in Chapter 5.

7. Kurt Gödel, "Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I," *Monatshefte für Mathematik und Physik* 38 (1931): 173–198.

8. See Dieter Lohmar, *Phänomenologie der Mathematik* (Dordrecht: Kluwer, 1989); Thomas Seebohm, *History as a Science and the System of the Sciences* (Dordrecht: Springer, 2015).

2.5 Evidence and the Constitution of Logical Objects

Whatever the fate of definiteness, Husserl's analysis of evidence remains foundational. For Husserl, *Evidenz* is not a feeling or psychological state but the mode of consciousness in which an object is given "in person" (*leibhaftig*).[9]

> I cannot as a beginner in philosophy make any judgment or accept one which I have not drawn from evidence, from 'experiences' in which the respective things and states of affairs are present to me as 'they themselves.'

Husserl distinguishes several types of evidence:

- **Adequate evidence**: Complete fulfillment, with no aspect of the object remaining unfulfilled. Husserl argues this is achievable only for immanent contents of consciousness.
- **Apodictic evidence**: Evidence that carries with it the impossibility of doubt. The cogito is Husserl's paradigm case.
- **Assertoric evidence**: Evidence that presents its object as actual but without the character of indubitability.

The constitution of *logical objects*—ideal unities like propositions, numbers, and logical laws—poses a special problem. These objects are not sensible; they are not given in perception. How then can we have evidence for them?

Husserl's answer involves *categorial intuition*: a higher-order form of intuition that presents categorial forms ("S is P", "A and B", "there exists an x such that...") as genuinely given.[10] Just as sensible intuition presents sensible objects, categorial intuition presents logical objects.

This doctrine has been controversial. Critics charge that Husserl is either smuggling in Platonism (categorial objects exist in a third realm) or committing a category mistake (treating logical forms as objects of intuition). Defenders reply that Husserl is simply describing the phenomenology: when we perform a judgment, the categorial form is *given* in consciousness, whatever the metaphysical status of that form.

For type theory, the parallel is clear. The meaning explanations tell us what it is to *have evidence* for a judgment of the form "$a : A$". This evidence consists in possessing a construction that exhibits the canonical form of the type. Categorial intuition, in Martin-Löf's hands, becomes the inspection of proof-objects.

9. Husserl, *Cartesian Meditations*, Hua I, 54.
10. Husserl, *Logical Investigations*, Investigation VI, §§40–66.

Chapter 3

Homotopy Type Theory: Identity, Equivalence, and Invariance

3.1 Introduction: The Homotopical Revolution

Around 2006, Vladimir Voevodsky proposed a radical reconception of type theory.[1] The key insight was that identity types, introduced by Martin-Löf as a formal device for expressing equality, carry rich *homotopy-theoretic* structure. When this structure is taken seriously—when identity proofs are treated not as mere certificates of equality but as *paths* between points—a new foundation for mathematics emerges.

This chapter examines homotopy type theory (HoTT) with attention to its philosophical significance. Three features are particularly relevant: the univalence axiom, which identifies equivalent types; path induction, which captures the structure of identity proofs; and higher inductive types, which allow the synthetic construction of spaces with prescribed homotopy.

3.2 Identity Types Reconsidered

Martin-Löf introduced identity types in his 1973 type theory.[2] For any type A and terms $a, b : A$, the identity type $\mathsf{Id}_A(a, b)$ is the type of proofs that a equals b.

The formation and introduction rules are:

1. Vladimir Voevodsky, "The Origins and Motivations of Univalent Foundations," *IAS Institute Letter*, Summer 2014.

2. Martin-Löf, *Intuitionistic Type Theory*, 59–65.

$$\frac{\Gamma \vdash A\ \mathsf{type} \quad \Gamma \vdash a : A \quad \Gamma \vdash b : A}{\Gamma \vdash \mathsf{Id}_A(a, b)\ \mathsf{type}}$$

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash \mathsf{refl}_a : \mathsf{Id}_A(a, a)}$$

The reflexivity term refl_a witnesses that a is identical to itself. The elimination rule (J-rule or path induction) states that to prove a property P of all identity proofs, it suffices to prove it for refl:

$$\frac{\Gamma, x : A, y : A, p : \mathsf{Id}_A(x, y) \vdash C(x, y, p)\ \mathsf{type} \quad \Gamma, z : A \vdash c : C(z, z, \mathsf{refl}_z)}{\Gamma, x : A, y : A, p : \mathsf{Id}_A(x, y) \vdash J(c, x, y, p) : C(x, y, p)}$$

In the extensional version of type theory, any two proofs of the same identity $p, q : \mathsf{Id}_A(a, b)$ are themselves equal: $\mathsf{Id}_{\mathsf{Id}_A(a,b)}(p, q)$ is always inhabited. This collapses all identity structure.

The crucial observation of HoTT is that in the *intensional* version, identity types can have non-trivial structure: there can be multiple distinct proofs of the same identity, and proofs between proofs, and so on. This higher-dimensional structure is precisely the structure of a *homotopy type*.

3.3 The Homotopy Interpretation

Awodey and Warren discovered that identity types can be interpreted as *path spaces* in homotopy theory.[3]

Under this interpretation:

Type Theory	**Homotopy Theory**
Type A	Space (homotopy type)
Term $a : A$	Point $a \in A$
Identity proof $p : \mathsf{Id}_A(a, b)$	Path $p : a \rightsquigarrow b$
Identity of identities $\alpha : \mathsf{Id}_{\mathsf{Id}_A(a,b)}(p, q)$	Homotopy $\alpha : p \sim q$
Reflexivity refl_a	Constant path at a

The J-rule corresponds to the fact that any continuous function on paths that is defined on constant paths extends uniquely to all paths (by path lifting). This is the homotopy-theoretic content of the elimination rule.

Higher identity types encode higher homotopy structure. The type $\mathsf{Id}_{\mathsf{Id}_A(a,a)}(\mathsf{refl}_a, \mathsf{refl}_a)$ is the loop space $\Omega(A, a)$ at the point a. Its inhabitants are "loops"—paths from a to

3. Steve Awodey and Michael Warren, "Homotopy Theoretic Models of Identity Types," *Mathematical Proceedings of the Cambridge Philosophical Society* 146, no. 1 (2009): 45–55.

itself that are not homotopic to the constant path. For the circle S^1, this loop space is equivalent to $\mathbb{Z}$, reflecting the winding number of loops around the circle.

3.4 The Univalence Axiom

Voevodsky's univalence axiom asserts that identity is equivalent to equivalence:[4]

Definition 3.1 (Univalence Axiom)**.** For types A, B in a universe $\mathcal{U}$, the canonical map

$$\mathsf{idtoeqv} : (A =_{\mathcal{U}} B) \to (A \simeq B)$$

is an equivalence.

Here $A =_{\mathcal{U}} B$ is the identity type in the universe (the type of paths from A to B in the "space of types"), and $A \simeq B$ is the type of equivalences (functions with a two-sided inverse).

The axiom says: any equivalence between types determines a path between them, and conversely. Isomorphic types are *identical* in a strong sense.

3.4.1 Philosophical Significance

The philosophical import of univalence is profound. Awodey articulates it as a formulation of mathematical structuralism:[5]

> Mathematical objects simply *are* structures. Could there be a stronger formulation of structuralism? The UA [univalence axiom] implies that all properties of mathematical objects are structural, i.e., invariant under isomorphism.

This resolves long-standing issues in the philosophy of mathematics. If we ask "is the natural number 2 identical to the pair $\{\emptyset, \{\emptyset\}\}$?" set-theoretic foundations give an answer (yes, under the von Neumann encoding) that depends on arbitrary implementation choices. Under univalence, the question dissolves: any two implementations of the natural numbers that are equivalent (satisfy the same universal property) are identical. There is no fact of the matter about "what 2 really is" beyond its structural role.

4. Steve Awodey, "Structuralism, Invariance, and Univalence," *Philosophia Mathematica* 22, no. 1 (2014): 1–11.

5. Awodey, "Structuralism, Invariance, and Univalence," 9.

3.4.2 Invariance Principle

A crucial consequence of univalence is the *principle of invariance*:

Proposition 3.2 (Invariance under Equivalence). *Any property $P : \mathcal{U} \to \mathsf{Prop}$ is invariant under equivalence: if $e : A \simeq B$, then $P(A) \leftrightarrow P(B)$.*

Proof. By univalence, e determines a path $p : A =_{\mathcal{U}} B$. By transport along p, we have $P(A) = P(B)$. □

This means that *no property expressible in HoTT can distinguish equivalent types.* Any construction that purports to distinguish $\mathbb{N}$ from an isomorphic copy (say, the even natural numbers under doubling) is not expressible in the theory.

For phenomenology, this has significant implications. The invariance principle captures Husserl's insight that the *object* of intention is not a bare particular but a structural pole around which presentations cluster. What makes multiple perceptions be of "the same" table is not some metaphysical identity underlying the perceptions but their *equivalence*—their systematic interrelation through transformations (moving around the table, varying lighting, etc.). Univalence formalizes this: identity is constituted by equivalence, not presupposed by it.

3.5 Path Induction

The elimination rule for identity types—path induction or the J-rule—captures what it means to reason about identities.

Definition 3.3 (Path Induction). To prove a property $P(x, y, p)$ for all $x, y : A$ and $p : \mathsf{Id}_A(x, y)$, it suffices to prove $P(a, a, \mathsf{refl}_a)$ for all $a : A$.

Intuitively: since refl is the only constructor for identity types, any proof about identities can be reduced to the case of refl.

3.5.1 The Justification Question

Ladyman and Presnell raise the question of how path induction is justified.[6] They argue it can be justified from "pre-mathematical considerations"—specifically, from two principles:

6. James Ladyman and Stuart Presnell, "Identity in Homotopy Type Theory, Part I: The Justification of Path Induction," *Philosophia Mathematica* 23, no. 3 (2015): 386–406.

1. **Uniqueness principle**: Given $a, b : A$ and $p : \mathsf{Id}_A(a, b)$, we have $(a, p) = (b, \mathsf{refl}_b)$ in $\Sigma_{x:A}\mathsf{Id}_A(x, b)$.

2. **Substitution of equals**: If $x = y$, then anything true of x is true of y.

The uniqueness principle says that any pointed path $(a, p : a = b)$ is identical to the trivial pointed path (b, refl_b). This encodes a strong contractibility condition: the type of paths ending at b has b itself as a unique "center."

From a phenomenological perspective, this corresponds to the Husserlian principle that identity is constituted through synthesis. The act of identifying a with b via p already presupposes the reflexive self-identity of each term. The uniqueness principle expresses that a path is "nothing but" the movement from self-identity to self-identity.

3.6 Higher Inductive Types

Higher inductive types (HITs) generalize ordinary inductive types by allowing constructors for paths and higher paths, not just points.[7]

Example 3.4 (The Circle)**.** The circle S^1 is defined by:

- A point constructor: $\mathsf{base} : S^1$
- A path constructor: $\mathsf{loop} : \mathsf{base} =_{S^1} \mathsf{base}$

The circle has a distinguished point and a distinguished non-trivial loop at that point. One can prove that $\Omega(S^1) \simeq \mathbb{Z}$: the loop space of the circle is the integers, corresponding to winding number.

Example 3.5 (The Interval)**.** The interval I is defined by:

- Point constructors: $0_I, 1_I : I$
- A path constructor: $\mathsf{seg} : 0_I =_I 1_I$

The interval is contractible: any type A is equivalent to the function type $I \to A$, and the interval itself is equivalent to the unit type.

Example 3.6 (Propositional Truncation)**.** The truncation $\|A\|$ is defined by:

- A point constructor: if $a : A$, then $|a| : \|A\|$

7. The Univalent Foundations Program, *Homotopy Type Theory: Univalent Foundations of Mathematics* (Institute for Advanced Study, 2013), Chapter 6.

- A path constructor: for all $x, y : \|A\|$, a path $x =_{\|A\|} y$

Truncation "squashes" a type down to a mere proposition: $\|A\|$ has at most one element up to identity, and it is inhabited if and only if A is.

Higher inductive types provide a synthetic way to construct spaces with specified homotopy properties. This is central to synthetic homotopy theory: we can define spheres, tori, and other spaces directly in type theory, without recourse to point-set topology, and prove homotopy-theoretic results purely formally.

3.7 The Intensional/Extensional Distinction

A crucial distinction runs through type theory: the distinction between *intensional* and *extensional* treatments of identity.[8]

Extensional type theory adds the reflection rule: from $p : \mathsf{Id}_A(a, b)$, conclude $a \equiv b$. This collapses propositional and judgmental equality. It also makes type-checking undecidable.

Intensional type theory maintains the distinction. Propositional equality $\mathsf{Id}_A(a, b)$ can be inhabited without a and b being judgmentally equal. Type-checking remains decidable.

HoTT is essentially intensional: the homotopical structure of identity types depends on maintaining the distinction. If all identity proofs were judgmentally equal, there would be no non-trivial paths, no higher structure, no interesting homotopy.

Philosophically, the intensional/extensional distinction corresponds to a distinction between two notions of sameness:

- **Definitional sameness** (judgmental equality): a and b are the same "by definition," by the very meaning of the terms.

- **Evidential sameness** (propositional equality): a and b are the same in virtue of evidence—a proof, a path, a synthesis of identification.

The Husserlian parallel is the distinction between *analytic* and *synthetic* identity. Analytic identity is identity by meaning: "a bachelor is an unmarried man" requires no evidence beyond understanding the terms. Synthetic identity requires evidence: "the morning star is the evening star" was a discovery, not a definition.

8. Klev, "Identity in Martin-Löf Type Theory."

HoTT's insistence on intensionality preserves this distinction. And it is precisely the intensional structure that makes univalence interesting: isomorphic types become identical *by evidence* (the equivalence), not by definition.

3.8 Phenomenological Interpretation of HoTT

How do the structures of HoTT correspond to phenomenological concepts?

3.8.1 Identity Types as Syntheses of Identification

Husserl's analysis of identity involves the *synthesis of identification*: the act by which diverse presentations are unified as presentations of the same object.[9] When I see a table from the front and then from the side, a synthesis occurs that identifies these as views of *one* table.

In HoTT, a term $p : \mathsf{Id}_A(a, b)$ is precisely such a synthesis: a witness that a and b are the same A. The homotopical structure of identity types reflects the possibility of *different syntheses*—different ways of identifying a and b that are not themselves identified.

Consider the circle. The point base can be identified with itself via refl (the trivial identification) or via loop (going around the circle). These are different identifications—different syntheses of base with itself. The winding number counts how "many times" the identification goes around.

3.8.2 Paths as Transformations of Perspective

The homotopical interpretation of paths suggests a phenomenological reading: a path is a *transformation of perspective*. Moving from a to b along p is like viewing an object from different angles—the object remains "the same" but is given differently.

Catren's work on gauge theory provides a physical analogy.[10] Gauge transformations are symmetries that change the mathematical description without changing the physical situation. A choice of gauge is like a choice of coordinates—a way of representing what is represented. The equivalence of gauges corresponds, under this analogy, to the equivalence of perspectives.

9. Husserl, *Logical Investigations*, Investigation VI, §§8–12.
10. Gabriel Catren, "On Gauge Symmetries, Indiscernibilities, and Groupoid-Theoretical Equalities," *Studies in History and Philosophy of Science* 91 (2022): 244–261.

3.8.3 Univalence as Constitution of Objectivity

Univalence can be read as a principle of *objectivity*: what makes a type "objective" is not any intrinsic property but its structural role, which is captured by equivalences. Two types with the same structural role are the same type.

This mirrors Husserl's constitution of objectivity. The "object" is not a thing-in-itself behind appearances but the *invariant pole* around which appearances cluster. What makes multiple perceptions be of the same object is their systematic interrelation—their "equivalence" under transformations. The object is constituted *through* and *as* this equivalence structure.

Univalence formalizes this: identity ($=_{\mathcal{U}}$) is constituted by equivalence ($\simeq$), not presupposed by it.

Chapter 4

Badiou's Mathematical Ontology: Set Theory, Events, and the Limits of Reconception

4.1 Introduction: Ontology as Mathematics

Alain Badiou's *Being and Event* (1988) advances the stark thesis: *ontology is mathematics.*[1] More specifically, ontology is Zermelo-Fraenkel set theory with the axiom of choice (ZFC). This is not a metaphor or an analogy; Badiou means it literally. The discourse that articulates being-as-being is set theory, and no other discourse.

The question naturally arises: could homotopy type theory play the role Badiou assigns to ZFC? Could one reconceive Badiou's ontology in type-theoretic terms? This chapter argues that such a reconception faces fundamental obstacles—obstacles that illuminate both the nature of Badiou's project and the distinctive features of HoTT.

4.2 The Set-Theoretic Ontology

Badiou's choice of ZFC is not arbitrary. Several features of set theory make it apt for his purposes:

4.2.1 Pure Multiplicity

In ZFC, every set is a set of sets (except the empty set). There are no urelements—no atoms that are members but not themselves sets. This means that "what there is" is

1. Alain Badiou, *Being and Event*, trans. Oliver Feltham (London: Continuum, 2005).

pure multiplicity: multiples composed of multiples, without any bottom level of simple individuals.

Badiou interprets this as the ontological thesis that *being is multiplicity*. There is no One, no fundamental unity underlying plurality. Unity exists only as *operation*—the count-as-one (*compte-pour-un*) that structures a multiplicity into a consistent presentation.

4.2.2 The Void

The empty set $\emptyset$ plays a crucial role as "the proper name of being." Badiou writes:[2]

> The name cannot be specific; it cannot place the void under anything that would subsume it—this would be to reestablish the One.

The void is the point at which inconsistent multiplicity (being-as-being) shows through the consistent structure of presentation. Every situation contains the void as its "foundational" element—the point where structure gives way to what it structures.

4.2.3 The Point of Excess

Cantor's theorem—$|P(X)| > |X|$—establishes that the power set of any set is strictly larger than the set itself. Badiou interprets this as the *point of excess*: representation (parts/subsets) always exceeds presentation (elements). The state of a situation (the power set structure) is irreducibly richer than the situation itself.

This excess is the site of the *event*: a rupture in the count-as-one that reveals what was previously invisible or uncounted. The event is modeled as a self-membered set—$e = \{e, x_1, \ldots, x_n\}$—which violates the axiom of foundation and thus cannot be accommodated within the ontological framework.

4.3 Obstacles to Type-Theoretic Reconception

Can Badiou's ontology be reformulated in HoTT? I identify four fundamental obstacles.

4.3.1 Structuralism Versus Realism

Badiou is an explicit realist about mathematical objects. He rejects structuralism—the view that mathematical objects are constituted by their structural roles.[3] For Badiou,

2. Badiou, *Being and Event*, 67.
3. Badiou, *Being and Event*, 6–8.

sets *exist*; they are not merely positions in a structure.

HoTT, by contrast, embodies structuralism in its strongest form. The univalence axiom asserts that isomorphic types are identical: there is no fact of the matter about what a type "really is" beyond its structural role. This is precisely what Badiou denies.

A Badiouian might respond that the question is not whether structures are all there is but whether the discourse of structures can articulate being. Perhaps HoTT could serve as the ontological discourse even if its internal philosophy is structuralist. But this response is awkward: it would mean adopting a foundational system whose philosophy contradicts the philosophy it is meant to found.

4.3.2 Constructive Versus Classical Logic

HoTT is constructive: the law of excluded middle and the axiom of choice are not derivable. One can add them as axioms, but doing so destroys univalence (excluded middle is incompatible with the homotopy interpretation).

Badiou's ontology requires classical logic. His key constructions—forcing, generic extensions, the independence of the continuum hypothesis—depend essentially on classical reasoning. The very notion of an "undecidable" proposition (like CH) presupposes classical semantics: CH is either true or false, we just cannot prove which.

In HoTT, "undecidable" would mean "not provable and not refutable," which is not the same as "true-or-false but unknowable." The constructivist interpretation of undecidability is fundamentally different from the classical one that Badiou's ontology requires.

4.3.3 Self-Membership and the Event

Badiou models the event as a self-membered set: $e \in e$. This directly violates the axiom of foundation (regularity), which states that no set is a member of itself.

In type theory, there is no direct analog of self-membership. Types are stratified into universes: $\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$. A type cannot contain itself; a universe cannot contain itself. This stratification is essential to avoiding paradox (the Russell paradox, in its type-theoretic form).

One might propose modeling the event as some kind of fixed point: $E \simeq F(E)$ for some functor F. But this is very different from literal self-membership, and it is unclear whether it captures what Badiou means by the event's "self-founding" character.

4.3.4 Forcing and Generic Sets

The theory of forcing, central to Badiou's account of truth, would require substantial reformulation in type-theoretic terms.

Cohen forcing adds "generic" sets to a model of set theory. A generic set G is one that intersects every "dense" set in the ground model—intuitively, it is maximally typical, avoiding any specific property definable in the ground model. The forcing relation $p \Vdash \phi$ defines truth in the generic extension.

Recent work on *forcing in type theory* (using modalities, cubical sets, or sheaf models) might provide a route to type-theoretic forcing.[4] But this remains an active research area, and it is not clear that the results would have the same philosophical significance as classical forcing. In particular, the constructive interpretation of "generic" and "dense" may differ substantially from the classical interpretation.

4.4 Badiou and Category Theory

In *Logics of Worlds* (2006), Badiou turns to category theory and topos theory.[5] He develops a "transcendental" theory of appearing that complements the ontology of *Being and Event*.

This might seem to open a path to type theory: if Badiou accepts categorical methods, why not accept HoTT? But the obstacle remains the same: Badiou uses category theory *classically*—as a framework for studying set-theoretic structures—rather than as an autonomous foundation. He does not embrace the internal logic of topoi, which is generally intuitionistic.

Moreover, Badiou's category theory remains *1-categorical*: it does not engage the $(\infty, 1)$-categorical structures that underlie HoTT. The move from ordinary categories to ∞-categories is non-trivial and changes the philosophical picture in ways Badiou has not addressed.

4.5 What Badiou Contributes

Despite these obstacles, Badiou's work illuminates aspects of the type-theoretic project.

First, Badiou demonstrates the *philosophical stakes* of foundational choices. The decision to ground ontology in ZFC rather than type theory is not merely technical; it

4. See, e.g., work by Coquand, Huber, and others on cubical type theory and forcing.
5. Alain Badiou, *Logics of Worlds*, trans. Alberto Toscano (London: Continuum, 2009).

reflects deep commitments about the nature of being, multiplicity, and the One. The type theorist must be equally clear about the ontological implications of their choice.

Second, Badiou's insistence on the *event*—the rupture that exceeds any count—poses a challenge to any formalism. If events cannot be accommodated within the formal system, then the system is incomplete in a way that matters philosophically. Gödel showed that formal systems are incomplete in a technical sense; Badiou suggests they are incomplete in an *ontological* sense.

Third, Badiou's analysis of *forcing* and *genericity* provides conceptual tools that might be adapted to type-theoretic contexts. The idea of a "generic" element that avoids all definable properties resonates with constructive notions of "choice" and "free" objects. Developing this connection would be a substantial philosophical project.

Chapter 5

The Two Kants: Type-Theoretic Reconstruction of Practical Reason

5.1 Introduction: The Separability Problem

Kant's *Metaphysics of Morals* (1797) contains notorious passages on marriage, sexuality, women's legal status, and other matters that are morally repugnant by contemporary standards.[1] The question arises: are these conclusions derivable from Kant's formal principles—the categorical imperative, the humanity formula—or are they the result of empirical prejudices smuggled into the derivation?

This chapter argues for *separability*: Kant's formal apparatus can be cleanly distinguished from his material conclusions, and the latter depend on contingent premises that are philosophically rejectable. Type-theoretic formalization makes this explicit: by tracking the premises required for each derivation, we can isolate where empirical assumptions enter and evaluate them independently.

The result is what I call the "two Kants": the original Kant whose conclusions reflect 18th-century prejudices, and the reconstructed Kant whose formal principles, properly understood, do not entail those conclusions.

5.2 The Problematic Passages

Let us first survey the material that requires reconstruction.

1. Immanuel Kant, *The Metaphysics of Morals*, trans. Mary J. Gregor (Cambridge: Cambridge University Press, 1996).

5.2.1 Marriage as Possession

Kant defines marriage as:[2]

> The union of two persons of different sexes for the lifelong possession of each other's sexual attributes.

He elaborates that in sexual use, a person becomes a "thing" (*Sache*):

> If I give another the use of all my sexual faculties... I hand myself over to another as a thing, and make the other into a thing also.

The solution is mutual surrender: each surrenders their person to the other, thereby regaining themselves through the other's reciprocal surrender. This creates a *ius realiter personale*—a right to a person akin to a right to a thing.

5.2.2 Women's Civil Status

Kant classifies "all women" among those lacking *civil personality*:[3]

> Anyone whose preservation in existence depends not on his management of his own business but on arrangements made by another... all these people lack civil personality, and their existence is, as it were, only inherence (*Inhärenz*).

The *Anthropology* is explicit:[4]

> Women regardless of age are declared to be immature [*unmündig*] in civil matters; the husband is her natural curator.

5.2.3 Husband's Authority

In marriage, Kant writes:[5]

> The husband commands (*befiehlt*) and the wife obeys (*gehorcht*)—this follows from the natural superiority of the husband over the wife in promoting the common interest of the household.

5.2.4 Sexual Ethics

Kant condemns masturbation (*Selbstschändung*) as worse than suicide:[6]

2. Kant, *Metaphysics of Morals*, AK 6:277–278.
3. Kant, *Metaphysics of Morals*, AK 6:314–315.
4. Kant, *Anthropology from a Pragmatic Point of View*, AK 7:209.
5. Kant, *Metaphysics of Morals*, AK 6:279.
6. Kant, *Metaphysics of Morals*, AK 6:424–426.

> Lust is called unnatural if one is aroused to it not by a real object but by one's imagination of this object, hence in a way contrary to the purpose of the desire, since one oneself creates the object.

Homosexuality is condemned as *crimina carnis contra naturam* "without qualification."

5.3 Where Empirical Premises Enter

Kant insists that pure moral philosophy must be "completely cleansed of everything empirical" (*Groundwork*, AK 4:389). How then do the above conclusions enter?

The answer is that applying the categorical imperative requires *premises*—claims about the nature of actions, their maxims, and their consequences. Nelson Potter's analysis is illuminating:[7]

> Whatever true factual and causal empirical premises are needed... Maxims with moral implications are typically related to causal processes that have their issue in good or bad results for persons.

Let us trace the empirical premises in Kant's arguments.

5.3.1 Premises about Women's Nature

Kant's conclusion that women lack civil personality rests on empirical claims:

1. Women are "naturally" dependent on men for their preservation.

2. This dependence is permanent and irremediable.

3. Dependence entails incapacity for autonomous judgment.

None of these follows from the categorical imperative. They are empirical claims about women's psychology and social position—claims that reflect 18th-century patriarchal assumptions rather than necessary truths.

7. Nelson Potter, "How to Apply the Categorical Imperative," *Philosophia* 4, no. 4 (1975): 395–416.

5.3.2 Premises about Sexual Psychology

Kant's condemnation of sexuality outside marriage rests on:

1. Sexual desire treats its object as a "thing" rather than a person.
2. This objectification violates the humanity formula.
3. Only mutual surrender in marriage restores personhood.

Premise (1) is an empirical claim about the phenomenology of desire. It is not obviously true: one can desire another sexually while respecting their autonomy. The assumption that desire is inherently objectifying imports a particular view of sexuality—arguably an Augustinian view—into Kantian ethics.

5.3.3 Premises about Nature's Ends

Kant's condemnation of non-procreative sex rests on:

1. Sexual organs have a "natural purpose" (procreation).
2. Using organs contrary to their natural purpose violates duty to self.
3. Non-procreative sex (masturbation, homosexuality) uses organs contrary to their purpose.

This argument imports empirical biological teleology. But Kant himself, in the *Critique of Judgment*, argues that teleological judgments are *regulative*—heuristic principles for investigating nature—not *constitutive* of nature's actual structure. To derive moral duties from natural purposes is to misapply the teleological framework.

5.4 The Reconstruction Literature

Contemporary Kantian ethicists have developed strategies for retaining Kant's formal principles while rejecting his objectionable conclusions.

5.4.1 Barbara Herman

Herman asks whether Kant's insights about autonomy and respect can serve feminist purposes:[8]

> What if we read Kant's argument about sex not as a description of what sex essentially is, but as a warning about what it can become under conditions of inequality and domination?

On this reading, Kant's analysis of sexual objectification is accurate *given* patriarchal social structures—structures that Kant took for granted but that are contingent and changeable. Remove the background conditions of inequality, and the argument no longer supports restricting sexuality to heterosexual marriage.

5.4.2 Christine Korsgaard

Korsgaard grounds normativity in the reflective structure of rational agency:[9]

> The capacity for normative self-government... is the source of obligation. When you deliberate, you must identify yourself with some principle of choice.

This "practical identity" approach retains Kant's formal structure without importing his specific views about gender, sexuality, or nature. The categorical imperative tests whether a maxim could be universally willed by rational agents—agents defined by their capacity for reflective endorsement, not by their sex or gender.

5.4.3 Onora O'Neill

O'Neill develops a constructivist reading:[10]

> Principles of reason are not substantive truths but procedures that must be followable by others.

On this view, the test for moral principles is whether they could be *shared*—adopted by a community of reasoners communicating in good faith. This procedural account allows for revision: principles that seemed reasonable to Kant might fail the shareability test once we recognize that they rest on prejudiced premises.

8. Barbara Herman, "Could It Be Worth Thinking about Kant on Sex and Marriage?" in *A Mind of One's Own*, ed. Louise M. Antony and Charlotte E. Witt, 2nd ed. (Boulder: Westview, 2002), 53–72.
9. Christine M. Korsgaard, *The Sources of Normativity* (Cambridge: Cambridge University Press, 1996).
10. Onora O'Neill, *Constructions of Reason* (Cambridge: Cambridge University Press, 1989).

5.4.4 Allen Wood

Wood distinguishes "Kant's ethics" from "Kantian ethics":[11]

> Kant's ethics is what Kant actually held; Kantian ethics is what follows from his basic principles. The two are not the same where Kant fails to apply his own principles correctly.

Wood explicitly criticizes Kant's views on sex and gender as failures to follow his own principles. The humanity formula requires treating persons as ends in themselves; patriarchal subordination treats women as means. Hence Kantian ethics condemns what Kant endorsed.

5.5 Type-Theoretic Reconstruction

Type theory provides a framework for making the reconstruction precise. The idea is to formalize Kant's arguments as type-theoretic derivations, with premises explicitly recorded as hypotheses.

5.5.1 The Formal Structure

A Kantian moral argument has the form:

$$\frac{\text{Premise}_1, \ldots, \text{Premise}_n, \text{CI}}{\text{Conclusion}}$$

where CI is some version of the categorical imperative and the Premises are claims about actions, maxims, consequences, etc.

In type-theoretic terms, this becomes a judgment:

$$P_1 : \text{Prop}, \ldots, P_n : \text{Prop}, \text{ci} : \text{CI} \vdash c : C$$

The conclusion C is derived in a context that includes the premises $P_1, \ldots, P_n$ as hypotheses.

5.5.2 Isolating Empirical Premises

The advantage of this formalization is that *premises are explicit*. We can examine each P_i and ask:

11. Allen W. Wood, *Kantian Ethics* (Cambridge: Cambridge University Press, 2008).

1. Is P_i a priori or empirical?
2. If empirical, is P_i true?
3. If P_i is false, what conclusion follows?

For Kant's argument about women's civil status:

$$\begin{array}{l} P_1 : \text{Women are naturally dependent on men} \\ P_2 : \text{Dependence implies incapacity for civil participation} \\ \text{ci} : \text{Categorical Imperative} \\ \hline c : \text{Women lack civil personality} \end{array}$$

We can now reject P_1 and P_2 as empirically false (or at best contingently true under patriarchy). Without these premises, the conclusion does not follow.

5.5.3 The Two Kants

The reconstruction yields two distinct systems:

Kant$_{\textbf{original}}$ The original derivation, including empirical premises $P_1, \ldots, P_n$.

Kant$_{\textbf{reconstructed}}$ The formal apparatus (CI, humanity formula, etc.) without the false empirical premises.

Kant$_{\text{original}}$ entails the problematic conclusions. Kant$_{\text{reconstructed}}$ does not. The question "is Kantian ethics sexist?" is ambiguous until we specify which Kant we mean.

5.6 Limits of Reconstruction

Type-theoretic reconstruction reveals the structure of arguments but does not resolve all problems.

5.6.1 The Choice of Premises

Even after we reject Kant's empirical premises, we must choose *some* premises to apply the categorical imperative. Which premises are admissible? How do we ensure that our premises are not themselves prejudiced?

This is not a formal question but a substantive one. The type-theoretic framework does not tell us which premises are correct; it only makes explicit which premises are required.

5.6.2 The Formulation Problem

The categorical imperative can be formulated in multiple ways:

1. **Universal Law**: Act only on maxims you could will as universal laws.
2. **Humanity Formula**: Treat humanity never merely as a means but always also as an end.
3. **Kingdom of Ends**: Act as a legislating member of a kingdom of ends.

Kant claims these are equivalent, but the equivalence is controversial. Different formulations may yield different results for the same case.

Type-theoretic formalization would have to choose a formulation or prove equivalence formally. Neither is straightforward.

5.6.3 The Incompleteness of Formalism

Even with explicit premises and a determinate formulation, the categorical imperative underdetermines many moral questions. What counts as "treating as a means"? Which maxim is the relevant one to test? How do we weigh conflicts between duties?

Kant's own attempts to answer these questions are often unsatisfying. The formalist dream—of deriving all morality from a single principle—may be unrealizable.

5.7 Conclusion: Formalism and Its Discontents

Type-theoretic reconstruction of Kantian ethics achieves two things:

First, it demonstrates that Kant's problematic conclusions are *not necessary consequences* of his formal principles. The formal apparatus—categorical imperative, humanity formula—is compatible with egalitarian conclusions. The prejudices enter through empirical premises that can be rejected.

Second, it reveals the *limits of formalism*. The categorical imperative alone determines nothing; premises are always required. The choice of premises is not itself formally determined. Hence formalism alone cannot ground ethics.

This second point will be deepened in the next chapter, where we examine Derrida's argument that formalization is always already infected by the deferral of meaning.

Chapter 6

Derrida's Diagonal: Différance, Undecidability, and the Limits of Formalization

6.1 Introduction: The Quasi-Transcendental

Any attempt to establish type-theoretic foundations for phenomenology, ethics, or AI governance must confront Jacques Derrida's critique of presence. Derrida's deconstruction is not merely a critical method but a demonstration of *structural limits* to formalization—limits that parallel, and explicitly invoke, the diagonal arguments of Cantor, Russell, and Gödel.

In this chapter, I examine Derrida's engagement with formal logic, focusing on three key concepts: *the supplement*, *différance*, and *iterability*. These function as what Rodolphe Gasché calls "quasi-transcendentals"—conditions of possibility that are also conditions of impossibility.[1] They enable meaning while ensuring that meaning is never fully present.

The implications for the type-theoretic project are significant. If Derrida is right, then no formal system can capture the meaning of the concepts it deploys. The invariants we seek to establish between mathematical structures and philosophical concepts are themselves said in language, and language is always already infected by différance. There can be no global orientation; there are only local positions that may have no necessary relation to others.

1. Rodolphe Gasché, *The Tain of the Mirror: Derrida and the Philosophy of Reflection* (Cambridge, MA: Harvard University Press, 1986).

6.2 Derrida on Gödel: Undecidability as Structure

Derrida's most explicit engagement with formal logic appears in "The Double Session" (1970), a reading of Mallarmé that turns on the concept of the undecidable.[2] Derrida writes:

> An undecidable proposition, as Gödel demonstrated in 1931, is a proposition which, given a system of axioms governing a multiplicity, is neither an analytical nor deductive consequence of those axioms, nor in contradiction with them, neither true nor false with respect to those axioms. *Tertium datur*, without synthesis.[3]

This is a remarkably precise summary of Gödel's first incompleteness theorem. Derrida understands that undecidability is a *syntactic* phenomenon: the undecidable proposition (the Gödel sentence) is neither provable nor refutable within the system, though it is true in the standard model.

Crucially, Derrida emphasizes that undecidability is *structural*, not *semantic*:

> What counts here is not the lexical richness, the semantic infiniteness of a word or concept, its depth or breadth... What counts here is the formal or syntactical praxis that composes and decomposes it.[4]

The undecidable is not ambiguous (having multiple meanings) but *undetermined* by the formal rules. This is the key link to deconstruction: terms like "writing," "supplement," and "différance" are not ambiguous words that happen to have multiple dictionary entries but *syntactically undecidable* within the metaphysical system that tries to fix their meaning.

6.3 The Diagonal Argument

Paul Livingston's "Derrida and Formal Logic" provides a detailed analysis of the structural parallels between deconstruction and diagonalization.[5]

The diagonal argument, in its various forms, proceeds by:

2. Jacques Derrida, "The Double Session," in *Dissemination*, trans. Barbara Johnson (Chicago: University of Chicago Press, 1981), 173–286.
3. Derrida, "Double Session," 219.
4. Derrida, "Double Session," 220.
5. Paul Livingston, "Derrida and Formal Logic: Formalizing the Undecidable" (manuscript, 2009).

1. Encoding a totality (the real numbers, the set of all sets, the provable sentences) at a single point.
2. Constructing an element that differs from every element of the totality at that point.
3. Concluding that the totality is incomplete or inconsistent.

Cantor's diagonal constructs a real number differing from every real on the list at its own position. Russell's paradox constructs a set (the set of all sets not containing themselves) that both must and cannot contain itself. Gödel's sentence asserts its own unprovability, and if true, is unprovable; if false, is provable, contradicting consistency.

Livingston identifies three structural features common to these arguments and to Derridean deconstruction:[6]

Self-referential encoding The system's total logic is formalized at a single point. Derrida identifies "privileged" terms (presence, speech, origin) that encode the system's governing logic.

Generalizability The result extends beyond any particular formal system. Derrida's undecidables (supplement, trace, différance) are not tied to any one text but operate across the metaphysical tradition.

Semantical effect of syntax The diagonal construction shows that syntactic structure has irreducible semantic consequences. Derrida's claim that "writing" precedes "speech" is not a claim about empirical priority but about the syntactic structure of signification.

6.4 The Supplement

In *Of Grammatology* (1967), Derrida develops the logic of the supplement through a reading of Rousseau.[7]

The supplement has a double logic:

> The supplement adds itself, it is a surplus, a plenitude enriching another plenitude, the fullest measure of presence... But the supplement supplements.

6. Livingston, "Derrida and Formal Logic," 7–9.
7. Jacques Derrida, *Of Grammatology*, trans. Gayatri Chakravorty Spivak (Baltimore: Johns Hopkins University Press, 1976).

> It adds only to replace. It intervenes or insinuates itself *in-the-place-of*; if it fills, it is as if one fills a void.[8]

The supplement is both *addition* to something complete and *substitution* for something lacking. Writing supplements speech (adds to it, enriches it) and also reveals that speech was never self-sufficient (needed writing to complete it). Education supplements nature; culture supplements instinct; signification supplements presence.

The chain of supplements reveals that what was supposed to be originary (speech, nature, presence) is already supplemented from the start:

> Supplementarity... lies at the 'origin' of presence, which is divided because it awaits that without which it could not constitute itself as an origin.[9]

This parallels formal incompleteness. The supposedly "complete" formal system requires an external supplement (the diagonal element) to constitute itself. The supplement shows that the system was never complete; completeness was an illusion that the supplement both creates and destroys.

6.5 Différance

Différance is Derrida's term for the movement that produces differences and deferrals.[10] The silent 'a'—indistinguishable from 'e' in speech—enacts what it names: a difference that cannot be heard, only written.

Différance combines two senses of the French *différer*:

1. **To differ**: spatial differentiation, the distinction between elements.

2. **To defer**: temporal postponement, the delay of presence.

Meaning is constituted by differences (the sign 'cat' means what it does by differing from 'bat,' 'car,' 'cut,' etc.) and is never fully present (understanding requires further context, which requires further context, indefinitely).

Derrida writes:

8. Derrida, *Of Grammatology*, 144–145.
9. Derrida, *Of Grammatology*, 163.
10. Jacques Derrida, "Différance," in *Margins of Philosophy*, trans. Alan Bass (Chicago: University of Chicago Press, 1982), 1–27.

> If différance is... what makes possible the presentation of the being-present, it is never presented as such. It is never offered to the present... in regular fashion it exceeds the order of truth.[11]

Différance is not a concept, not a being, not a present entity. It is the *condition* for presence that cannot itself be made present. Gasché's term "quasi-transcendental" captures this: différance functions like a transcendental condition (making experience possible) but undermines the transcendental framework (it cannot be thematized within that framework).

For type theory, the implication is severe. Every type-theoretic definition employs signs whose meaning is constituted by différance. The meaning explanations that ground type theory are themselves said in language, and language is the play of differences. No foundation escapes the medium in which it is formulated.

6.6 Voice and Phenomenon: Critique of Husserl

Derrida's *Voice and Phenomenon* (1967) directly attacks the Husserlian foundations that ground Martin-Löf's type theory.[12]

6.6.1 Expression and Indication

Husserl distinguishes between *expression* (*Ausdruck*) and *indication* (*Anzeichen*). Expressions have meaning; they are animated by the intention to mean something. Indications are signs that point to other things (smoke indicates fire, symptoms indicate disease) without having meaning in the strict sense.

Husserl wants to purify expression from indication. In solitary mental life, he claims, we use expressions without indication: we "speak" to ourselves without signs functioning as symptoms or signals. This pure expression is the site of meaning proper.

Derrida argues that this purification is impossible. Indication always contaminates expression. Even in solitary mental life, signs function as signs only through their iterability—their capacity to be repeated in different contexts. But iterability is precisely the indicative function: the sign indicates (points to, is a trace of) its prior and future uses.

11. Derrida, "Différance," 6.
12. Jacques Derrida, *Voice and Phenomenon*, trans. Leonard Lawlor (Evanston: Northwestern University Press, 2010).

6.7 The Living Present

Husserl's phenomenology of time-consciousness analyzes the structure of the "living present" (*lebendige Gegenwart*). The present moment is not a knife-edge instant but has internal structure: primal impression (the now-phase), retention (primary memory of the just-past), and protention (anticipation of the about-to-come).

Derrida's critique focuses on retention. Husserl acknowledges that retention is "a kind of non-perception"—it presents what is no longer present. But if non-presence (retention) is constitutive of presence (the living present), then presence is never pure. The present is always already divided by the trace of what has passed.

> As soon as we admit this continuity of the now and the not-now, perception and nonperception, in the zone of primordiality common to primordial impression and primordial retention, we admit the other into the self-identity of the *Augenblick*; nonpresence and nonevidence are admitted into the blink of the instant.[13]

For type theory, this is significant. Martin-Löf's meaning explanations appeal to evidence—the mode of consciousness in which objects are given "in person." But if the living present is constitutively divided by retention, then evidence is never pure. The proof-object that supposedly fulfills the empty intention is itself given through a temporal synthesis that includes non-presence.

6.7.1 Auto-Affection and the Voice

Husserl privileges the voice—specifically, the silent interior monologue—as the site of pure expression. When I speak to myself silently, the signifier seems to coincide perfectly with the signified; I hear myself speak at the very moment I speak.

Derrida argues that this auto-affection (hearing-oneself-speak) already involves a gap:

> The operation of 'hearing oneself speak' is an auto-affection of a unique kind... But this operation of hearing oneself speak... produces a minuscule hiatus differentiating me into the speaker and into the hearer.[14]

Even in the most interior speech, there is a difference between the speaking and the hearing, the producing and the receiving. This "minuscule hiatus" is différance at work within the most intimate self-presence.

13. Derrida, *Voice and Phenomenon*, 61.
14. Derrida, *Voice and Phenomenon*, 67.

The voice is not the exception to writing but a form of writing. Writing, for Derrida, is not the empirical marks on paper but the structure of repeatability, exteriority, and spacing that makes signification possible. This structure operates even—especially—in the silent voice.

6.8 Iterability and the Diagonal

In "Signature Event Context" (1971), Derrida develops the concept of iterability.[15]

A sign is iterable: it can be repeated in different contexts, by different users, for different purposes. This repeatability is not an accidental feature of signs but constitutive of signhood. A "sign" that could be used only once would not be a sign at all.

But iterability implies that the sign's meaning is never fully determined by any particular context. The sign can always be "grafted" onto a new context, acquiring new resonances and losing old ones. Austin's performatives, which seem to accomplish things through being uttered, can always be "cited" in non-serious contexts (on stage, in examples), and this possibility of citation is built into their very structure.

Livingston argues that iterability "effectively diagonalizes any fixed or given context." The iterable sign is like the diagonal element: it belongs to the totality (the context determines its meaning) and exceeds the totality (it can always be grafted onto a new context). No context is complete; there is always the structural possibility of generating elements outside any given totality.

6.9 Implications for Type Theory

What are the implications of Derrida's critique for type-theoretic foundations?

6.9.1 The Linguistic Medium

Type theory, like any formal system, is formulated in language. The formation rules, introduction rules, and elimination rules are stated using words—"type," "term," "judgment," "context." These words have meanings that are constituted by différance: they differ from other words and defer to further contexts of use.

Martin-Löf's meaning explanations are supposed to ground the formal system in intuitive understanding. But this intuitive understanding is itself linguistic. When Martin-

15. Jacques Derrida, *Limited Inc* (Evanston: Northwestern University Press, 1988).

Löf explains what it is to know that $a : A$, he uses words—words whose meanings are not fixed but play within the system of differences that constitutes language.

This does not make type theory meaningless or useless. It means that type theory cannot achieve the closure it seems to promise. The "foundations" are themselves founded on something that cannot be made fully present.

6.9.2 The Incompleteness of Evidence

Husserl's notion of evidence—the mode of consciousness in which objects are given "in person"—is the phenomenological ground for Martin-Löf's notion of possessing a proof-object. But Derrida's critique shows that evidence is never pure. The living present is divided by retention; auto-affection involves a gap; the signifier never coincides perfectly with the signified.

This means that the "evidence" we have for type-theoretic judgments is never complete. The proof-object is given through a temporal synthesis that includes non-presence. The judgment "$a : A$" is evident only relative to a context that is itself open to supplementation.

6.9.3 Local Positions, No Global Orientation

The deepest implication is that *there can be no global orientation in language.* Language is the play of differences; meaning is always deferred; contexts are always open. No metalanguage escapes the condition of language; no formal system transcends the medium in which it is formulated.

This does not mean that meaning is impossible or that communication is futile. It means that meaning is *local.* We achieve understanding within particular contexts, using particular signs, for particular purposes. But these local achievements do not sum to a global totality. There is no "view from nowhere" from which all local meanings are integrated.

For type theory, this means that the meanings of types and terms are always context-dependent. The "same" type may have different meanings in different contexts (different proof assistants, different mathematical communities, different applications). These local meanings may not cohere globally.

6.9.4 A Caveat: Scope of the Diagonal

Catarina Dutilh Novaes raises an important objection to the Derrida-Gödel parallel.[16] Gödel's incompleteness theorems require specific conditions: the formal system must have sufficient expressive power to encode its own syntax. First-order predicate logic, for example, is *complete*—every valid sentence is provable. The diagonal does not apply to all formal systems.

Différance, by contrast, is presented as perfectly general—operative in all signification, without restriction. If the parallel is to hold, either Derrida must restrict his claims or the parallel is inexact.

This caveat is well-taken. The parallel between deconstruction and formal incompleteness is structural, not exact. Gödel's theorem is a precise mathematical result with specific conditions; Derrida's différance is a quasi-transcendental condition without precise boundaries. The analogy illuminates but does not identify.

6.10 Conclusion: The Diagonal as Limit

Derrida's diagonal argument establishes a limit to formalization. Any formal system that attempts to capture meaning must use signs whose meaning is constituted by différance. The system cannot achieve closure; there is always a supplement, a trace, a remainder.

This does not refute type theory or render it worthless. It situates type theory within its proper limits. Type theory can formalize certain aspects of reasoning with remarkable precision. But the meaning of the formalization—what the types and terms *mean*—escapes the formalism.

The project of this book—to establish correspondences between type-theoretic structures and phenomenological concepts—is therefore always incomplete. We can identify structural parallels, map concepts onto formalizations, and use the formalization to illuminate the concepts. But we cannot achieve a final, complete correspondence. The invariants we catalog are said in language; language is différance; différance exceeds any formal capture.

16. Catarina Dutilh Novaes, "Diagonalization and Différance: A Mismatch of Scope," blog post, March 2012.

Chapter 7

Gabriel Catren: Phenoumenodelic Realism and Hypertranscendental Perspectivalism

7.1 Introduction: Beyond the Kantian Settlement

Gabriel Catren's philosophical project attempts to overcome what he sees as the fundamental limitation of Kantian and post-Kantian philosophy: the restriction of knowledge to phenomena, leaving things-in-themselves forever inaccessible. Catren proposes a "phenoumenodelic" realism that claims access to the real through a radicalization, rather than abandonment, of the transcendental framework.[1]

For our purposes, Catren's work is significant for two reasons. First, he explicitly engages with type-theoretic and category-theoretic methods, particularly in his philosophy of physics. Second, his concept of "hypertranscendental perspectivalism" provides a framework for understanding how multiple local perspectives can coexist without collapsing into relativism or requiring a global meta-perspective.

7.2 The Pleroma and Phenoumenodelic Access

The term "pleroma" comes from Gnostic theology, where it names the divine fullness—the totality of divine emanations. Catren secularizes the concept:

> The pleroma names the totality of experience and reality grasped without

1. Gabriel Catren, *Pleromatica, or Elsinore's Trance*, trans. Thomas Murphy (New York: Urbanomic/Sequence Press, 2023).

> positing any transcendent beyond, yet without renouncing the infinite ideas of reason (Truth, Beauty, Justice, Love).

The pleroma is neither the phenomenal world (Kant's appearances) nor the noumenal world (Kant's things-in-themselves). It is the single reality that these distinctions inadequately divide. To access the pleroma is not to escape the phenomenal but to recognize that the phenomenal *is* the real, properly understood.

"Phenoumenodelic" combines "phenomenon" (appearance) and a neologism suggesting revelation or disclosure. Phenoumenodelic experience reveals the real through appearances, not despite them. The limit that Kant drew between phenomenon and noumenon is not a barrier but a threshold that can be crossed—not by mystical intuition but by the proper deployment of reason and mathematics.

Catren describes his position as "a wedding between Kant and Spinoza in a carnivalised Christian Church." From Kant, he takes the critical insight that experience is structured by transcendental conditions. From Spinoza, he takes the monist thesis that there is only one substance (Deus sive Natura), known through its attributes. The "carnivalised Christian Church" signals that the synthesis is neither secular rationalism nor religious orthodoxy but something stranger—a speculative philosophy that retains the intensity of religious experience without its dogmatic content.

7.3 Disligation and Religation

Catren diagnoses modernity as a condition of "disligation"—disconnection, unbinding, the loss of orientation that follows from the collapse of traditional certainties. The death of God, the decentering of man, the relativization of all reference points—these produce a vertigo that can lead to nihilism (Land's accelerationism) or to nostalgic attempts at refoundation (religious fundamentalism, political authoritarianism).

Against both, Catren proposes "religation"—re-binding, reconnection, but without return to dogma:

> Philosophy will finally be modern only if it can sublate the critical moment, crush the Ptolemaic counter-revolution, and deepen the narcissistic wounds inflicted by modern science.

Religation is not restoration of the pre-modern cosmos but a new binding that incorporates the insights of modernity. It is a "practice of re-ligation with the immanental life in which 'we live and we move and have our being.'" The quotation from Acts 17:28 signals the religious resonance without requiring religious commitment.

The regulative idea of religation is what Catren calls "Love"—not sentimental emotion but the commitment to hold oneself open to the overabundant life that flows through every finite being. This Love is not opposed to reason but is reason's highest exercise: the recognition that rationality itself is a mode of participating in the pleroma.

7.4 Type-Theoretic Engagement: Gauge Symmetries and Groupoid Equalities

Catren's philosophy of physics engages directly with type theory. In "On Gauge Symmetries, Indiscernibilities, and Groupoid-Theoretical Equalities" (2022), he argues:[2]

> Gauge symmetries—far from being nothing but a 'surplus structure' resulting from a descriptive redundancy—rely on the intrinsic geometric structures of Yang-Mills theories.

The technical claim is that gauge symmetries are not merely artifacts of our description but encode real physical structure. The philosophical import is that *equivalence is constitutive of objectivity*. What makes a physical situation "objective" is not some intrinsic property accessible from a God's-eye view but its behavior under transformations—its gauge equivalence class.

Catren explicitly connects this to univalence:

> Univalence can be understood as a particular implementation of a constructive notion of abstraction that resolves Fregean abstraction.

Fregean abstraction defines numbers as equivalence classes: the number 2 is the class of all two-element sets. This raises problems (Russell's paradox, the "Julius Caesar problem"). Univalence provides a type-theoretic solution: equivalent types are identical, so abstraction does not require forming classes.

Catren's analysis of gauge theory thus converges with HoTT's univalence. In both cases, *identity is constituted by equivalence*. The "real" object is not something behind the equivalence class but the equivalence class itself—or rather, the structure that the equivalence class encodes.

2. Gabriel Catren, "On Gauge Symmetries, Indiscernibilities, and Groupoid-Theoretical Equalities," *Studies in History and Philosophy of Science* 91 (2022): 244–261.

7.5 Hypertranscendental Perspectivalism

Catren's most significant contribution for our purposes is his concept of "hypertranscendental perspectivalism."

Standard transcendental philosophy posits a single transcendental framework—the conditions of possible experience—that all rational subjects share. Kant's categories (substance, causality, etc.) are the same for all human cognizers. This yields universality but at the cost of rigidity: there is one transcendental structure, and it determines what can count as experience.

Catren proposes a *multiplicity* of transcendental frameworks:

> The hypertranscendental thus affirms a multiplicity of categorial structures correlated to objective nature-cultures (*Umwelten*), without therefore positing that the human species constitutes a single transcendental type.

Different cognitive systems—different species, different cultures, different historical epochs—have different transcendental structures. The bee's *Umwelt* is structured differently from the human's; the medieval cosmos is structured differently from the modern scientific worldview.

This might seem to collapse into relativism: if each *Umwelt* has its own categories, how can they communicate? How can we judge between them?

Catren's answer involves what he calls "trans-umweltic mobility":

> The capacity for trans-umweltic mobility—for inhabiting other "transcendental lands"—is the mark of philosophical reason.

We can move between transcendental frameworks, inhabit perspectives not originally our own, and thereby gain a richer understanding of the pleroma. This mobility is not a view from nowhere but a capacity for perspectival translation—for seeing how things look from different transcendental positions.

7.6 Implications for Type Theory

Catren's perspectivalism has direct implications for the type-theoretic project.

7.6.1 Universes as Transcendental Frameworks

In HoTT, types are organized into a hierarchy of universes: $\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$. Each universe contains types but is itself a type in the next higher universe. This stratification prevents paradox while allowing a rich theory of types.

From a Catrenian perspective, each universe can be understood as a *transcendental framework*—a context that determines what types (and hence what objects) are available. Different universes provide different perspectives on the type-theoretic landscape.

Crucially, there are *functors between universes*—operations that lift types from lower to higher universes. These functors enable "trans-umweltic mobility": we can relate the perspective of $\mathcal{U}_0$ to that of $\mathcal{U}_1$, compare their structures, and understand their differences.

7.6.2 Equivalence as Perspectival Invariance

Catren's analysis of gauge symmetries suggests that equivalence encodes perspectival invariance. What remains the same across gauge transformations is what is "really" there; what varies is merely an artifact of descriptive choice.

In HoTT, univalence makes this precise: equivalent types are identical. What is invariant under equivalence is what is "really" there; what varies under equivalence is merely an artifact of implementation.

This connects to the phenomenological analysis. The "object" of intention—the pole around which presentations cluster—is invariant under perspectival transformations. Univalence formalizes this invariance.

7.6.3 The Limits of Trans-Umweltic Mobility

Catren's perspectivalism, however, faces a challenge. If there is no meta-transcendental position from which all frameworks are surveyed, how can we compare them? Trans-umweltic mobility requires *interfaces* between frameworks, but who constructs these interfaces?

The answer, for Catren, is that the interfaces are themselves perspectival. We compare frameworks from within a framework; we construct translations using the resources of our current position. There is no neutral standpoint.

This limitation is philosophically honest but operationally challenging. For AI governance, it means that any system of value alignment will be perspectival—constructed from a particular position, with particular assumptions. There is no algorithm that escapes this condition.

Chapter 8

Fernando Zalamea: Synthetic Philosophy and Sheaf-Theoretic Universality

8.1 Introduction: Beyond Analytic Philosophy of Mathematics

Fernando Zalamea's *Synthetic Philosophy of Contemporary Mathematics* (2012) develops a comprehensive alternative to analytic philosophy of mathematics.[1] Where analytic philosophy focuses on foundational questions (set theory vs. category theory, Platonism vs. nominalism), Zalamea attends to the *practice* of contemporary mathematics—the actual work of mathematicians in algebraic geometry, topology, number theory, and mathematical physics.

For our purposes, Zalamea's significance lies in his deployment of sheaf theory as a philosophical paradigm. The sheaf formalizes exactly the relationship between local and global that Derrida's critique establishes: local sections may exist and satisfy gluing conditions, yet no global section need exist. Zalamea's "synthetic universality" is universality without totalization—a transmodern response to both modern universalism and postmodern fragmentation.

1. Fernando Zalamea, *Synthetic Philosophy of Contemporary Mathematics*, trans. Zachery Luke Fraser (New York: Urbanomic/Sequence Press, 2012).

8.2 Critique of Analytic Philosophy of Mathematics

Zalamea argues that analytic philosophy of mathematics has focused too narrowly on foundational questions at the expense of engaging with the actual practice of contemporary mathematics:

> The predominant analytic approach, with its focus on the formal, the elementary and the foundational, has effectively divorced philosophy from the real practice of mathematics.

Contemporary mathematics—algebraic geometry, topology, number theory—has undergone profound conceptual transformations that analytic philosophy has largely ignored. The shift from sets to categories, from points to sheaves, from equality to equivalence—these are not merely technical developments but conceptual revolutions that require philosophical attention.

Zalamea identifies several limitations of the analytic approach:

1. **Elementarism**: Focus on the simplest cases (natural numbers, first-order logic) at the expense of sophisticated mathematical structures.
2. **Foundationalism**: Obsession with foundations (axioms, consistency proofs) at the expense of understanding mathematical practice.
3. **Isolationism**: Treatment of mathematics in isolation from physics, philosophy, and culture.
4. **Reductionism**: Attempt to reduce all mathematics to a single foundational system.

Against these, Zalamea proposes a "synthetic" philosophy that attends to the full richness of mathematical practice, including its connections to other domains of thought.

8.3 Transmodernity

Zalamea proposes a "transmodern" philosophy that overcomes the modern/postmodern dichotomy:

> Transmodernity hopes to reintegrate many awkward postmodern differentials, to balance some supposed breaks with more in-depth sutures.

Modernity sought universal reason, applicable everywhere and to everyone. Postmodernity emphasized difference, incommensurability, and the failure of universal claims. Transmodernity synthesizes: it develops "universal relatives"—partial invariants that hold across contexts without claiming absolute universality.

The model is contemporary mathematics, which achieves remarkable generality (Grothendieck's schemes, derived categories, ∞-topoi) without claiming to have reached a final foundation. Mathematical concepts are developed through *transit*—movement between different domains, translation of structures, recognition of analogies. This transit is not the application of a universal method but a creative practice that generates new universals in the process.

8.4 Sheaf-Theoretic Synthetic Universality

Zalamea identifies the sheaf as the central concept for transmodern philosophy:

> This Grothendieckian structure, which Zalamea identifies as the 'sheaf,' is a central concept gluing the always partial and yet continuous fabric of Transmodern space.

8.4.1 The Sheaf Concept

A *sheaf* on a topological space X assigns data to each open set $U \subseteq X$, with restriction maps that allow data on larger sets to be restricted to smaller sets. Formally:

Definition 8.1 (Presheaf)**.** A presheaf $\mathcal{F}$ on X consists of:

1. For each open set $U \subseteq X$, a set (or group, ring, etc.) $\mathcal{F}(U)$ of "sections over U."
2. For each inclusion $V \subseteq U$, a restriction map $\rho_{UV} : \mathcal{F}(U) \to \mathcal{F}(V)$.
3. Compatibility: $\rho_{UU} = \mathrm{id}$ and $\rho_{VW} \circ \rho_{UV} = \rho_{UW}$ for $W \subseteq V \subseteq U$.

Definition 8.2 (Sheaf)**.** A presheaf $\mathcal{F}$ is a sheaf if it satisfies:

1. **Locality**: If $\{U_i\}$ covers U and $s, t \in \mathcal{F}(U)$ satisfy $s|_{U_i} = t|_{U_i}$ for all i, then $s = t$.
2. **Gluing**: If $\{U_i\}$ covers U and $s_i \in \mathcal{F}(U_i)$ satisfy $s_i|_{U_i \cap U_j} = s_j|_{U_i \cap U_j}$ for all i, j, then there exists $s \in \mathcal{F}(U)$ with $s|_{U_i} = s_i$ for all i.

Philosophically, a sheaf models the passage from local to global. Different perspectives (open sets) yield different data (sections); where perspectives overlap, data must be compatible; compatible local data can be integrated into global structure.

8.4.2 The Failure of Global Sections

Crucially, *global sections may not exist*. The sheaf may have rich local structure—sections over every open set, satisfying all compatibility conditions—without any section over the entire space X.

Example 8.3. Consider the sheaf of holomorphic logarithms on $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$. Locally, the logarithm is well-defined: on any simply connected open set, we can choose a branch. But globally, no single-valued holomorphic logarithm exists—the monodromy around the origin prevents global definition.

This example models the situation Derrida's critique describes. Local meanings exist; where they overlap, they are compatible; but no global meaning integrates them all. The failure of global sections is not a defect to be remedied but a structural feature of the semantic landscape.

8.4.3 Zalamea's Philosophical Deployment

Zalamea uses the sheaf concept to articulate a non-totalizing universality:

> The universality of the sheaf is a universality of *relations*—not a universality of *substances*.

What is "universal" in a sheaf is not some global section that applies everywhere but the *structure* of restriction and gluing that relates local sections. The universal is the pattern of relations, not any particular relatum.

This resonates with HoTT's structuralism. What is "universal" about a type is not its specific implementation but its structural role, captured by equivalences. The invariants are relational, not substantial.

8.5 The Grothendieck Revolution

Zalamea situates his philosophy within the broader context of Grothendieck's transformation of algebraic geometry.[2]

Grothendieck's key innovations include:

1. **Schemes**: Generalization of algebraic varieties to include nilpotent and non-reduced structures.

2. Alexander Grothendieck, with Jean Dieudonné, *Éléments de Géométrie Algébrique* (EGA), various publications 1960–1967.

2. **Topoi**: Generalization of topological spaces to categories with sheaf-like properties.

3. **Motives**: Conjectural universal cohomology theory underlying all Weil cohomologies.

4. **Descent**: Systematic study of how local data can be glued into global structures.

These innovations share a common theme: the shift from *points* to *functors*, from *sets* to *categories*, from *identity* to *equivalence*. Grothendieck's mathematics is inherently relational—it studies structures through their relationships to other structures, not through intrinsic properties.

Zalamea argues that this relational turn has philosophical implications that analytic philosophy has not absorbed. The philosophy of mathematics must move beyond foundational debates (which presuppose a point-based, set-theoretic ontology) to engage with the categorical and topos-theoretic frameworks that structure contemporary practice.

8.6 Connection to HoTT

Zalamea does not engage extensively with HoTT (his book predates the 2013 HoTT Book), but the connection is natural.

HoTT can be understood as an internal language for $(\infty, 1)$-topoi.[3] Just as the internal language of an ordinary topos is higher-order intuitionistic logic, the internal language of an $(\infty, 1)$-topos is (a form of) homotopy type theory.

This means that the structures Zalamea identifies in Grothendieck's work—sheaves, topoi, descent—have type-theoretic counterparts. The philosophical lessons Zalamea draws from sheaf theory apply, *mutatis mutandis*, to HoTT.

In particular:

1. **Local/Global Structure**: Types in HoTT can be understood as having local structure (on open patches of the type) that may or may not integrate globally. Higher inductive types make this explicit: the circle S^1 has non-trivial global structure (its fundamental group is $\mathbb{Z}$) despite being locally trivial.

2. **Synthetic Universality**: The universality of HoTT is not a claim to have captured "all" mathematics but a synthetic framework that can express many mathematical structures without reducing them to a single foundation.

3. See the work of Shulman, Lurie, and others on the relationship between HoTT and $(\infty, 1)$-topoi.

3. **Transmodernity**: HoTT navigates between the universalist claims of set-theoretic foundations and the fragmentation of domain-specific formalisms. It provides a common language without claiming to be the final word.

8.7 Implications for the Project

Zalamea's work reinforces the central thesis of this book: there can be no global orientation, only local positions.

The sheaf-theoretic formalization makes this precise. Local sections exist; gluing conditions may be satisfied on overlaps; but global sections may fail to exist. This is not a failure of our methods but a feature of the semantic landscape.

For AI governance, the implication is clear. No algorithm can occupy a global position from which all local values are adjudicable. Governance must be structurally local: different domains, different contexts, different communities will have different value systems. The best we can hope for is *interfaces*—ways of translating between local systems, satisfying gluing conditions where they overlap—without expecting a global integration.

This is not relativism. Zalamea's transmodernity insists on universality—but a universality of *relations*, not of *substances*. The sheaf itself is universal: it specifies how local data can be integrated. What is not universal is any particular global section.

Chapter 9

Toward Sheaf-Theoretic AI Governance

9.1 Introduction: The Governance Problem

Contemporary AI systems increasingly operate in contexts that require coordinating multiple agents with potentially conflicting objectives. Autonomous vehicles must negotiate intersections; trading algorithms interact in markets; language models serve users with diverse values. How can such systems be governed?

The preceding chapters have established that no global value function can adjudicate all conflicts. Derrida's critique shows that meaning is irreducibly local; Catren's perspectivalism affirms the multiplicity of transcendental frameworks; Zalamea's sheaf theory formalizes the failure of global sections. Any governance framework must accommodate this fundamental locality.

This chapter sketches—and it can only be a sketch—how sheaf-theoretic structures might inform the design of governance mechanisms for multiagent AI systems.

9.2 The Inadequacy of Global Optimization

Standard approaches to AI alignment seek a single objective function that the system optimizes. The assumption is that human values can be aggregated into a utility function, and the AI's task is to maximize expected utility.

This approach faces several problems:

1. **Arrow's Impossibility**: No aggregation procedure satisfies all reasonable fairness constraints. Social choice theory shows that preference aggregation is fundamentally problematic.

2. **Value Incommensurability**: Some values cannot be traded off against each other. The value of human life cannot be straightforwardly compared to the value of economic efficiency.

3. **Context Dependence**: The "right" action depends on context in ways that a fixed objective function cannot capture.

4. **Semantic Instability**: The meaning of value terms ("fairness," "harm," "benefit") varies across communities and evolves over time.

These are not merely practical difficulties but structural features of the normative landscape—features that the preceding chapters have theorized in detail.

9.3 Sheaves of Values

Consider instead a sheaf-theoretic approach. Let X be a "space" of contexts—situations, communities, domains. For each context $U \subseteq X$, we have a set of *local values* $\mathcal{V}(U)$—the norms, preferences, and constraints that apply in that context.

Where contexts overlap ($U \cap V \neq \emptyset$), local values must be *compatible*: the restriction of U's values to the overlap must agree with the restriction of V's values. This is the gluing condition.

Definition 9.1 (Value Sheaf)**.** A value sheaf on X consists of:

1. For each context U, a set $\mathcal{V}(U)$ of local value specifications.

2. Restriction maps $\rho_{UV} : \mathcal{V}(U) \to \mathcal{V}(V)$ for $V \subseteq U$.

3. Locality and gluing axioms ensuring consistency.

The key insight is that *global values may not exist.* The sheaf may have rich local structure—every context has well-defined values, and overlapping contexts have compatible values—without any single value system that applies globally.

9.4 Interfaces and Translations

If global values do not exist, how do different local systems interact?

The answer is *interfaces*—structured ways of translating between local value systems. When an autonomous vehicle from Community A enters the territory of Community B, an interface specifies how A's values map to B's context.

Formally, an interface is a *morphism of sheaves*—a family of maps $\phi_U : \mathcal{V}_A(U) \to \mathcal{V}_B(U)$ that commute with restrictions.

Not all interfaces are good interfaces. A well-designed interface should satisfy:

1. **Locality Preservation**: Local constraints in A map to local constraints in B.
2. **Compatibility**: The interface respects gluing conditions.
3. **Reversibility**: Where possible, the interface should be invertible, allowing bidirectional translation.
4. **Transparency**: The interface should be inspectable and contestable by affected parties.

9.5 Federated Governance

The model that emerges is *federated governance*—governance distributed across multiple authorities, coordinated through interfaces rather than unified under a single sovereign.

This is not a new idea; it describes existing political systems (federalism, international law, polycentric governance). What is new is the formal framework—sheaf theory—that makes the structure explicit and amenable to analysis.

In a federated AI governance system:

1. **Local Authorities**: Each context (community, domain, jurisdiction) maintains its own value specification.
2. **Interface Protocols**: Standardized interfaces enable interaction across contexts.
3. **Conflict Resolution**: Where interfaces fail (gluing conditions are not satisfied), explicit negotiation mechanisms resolve conflicts.
4. **Evolution**: Value specifications and interfaces can evolve over time, accommodating changing norms and new contexts.

9.6 Type-Theoretic Implementation

How might such a system be implemented in type-theoretic terms?

9.6.1 Contexts as Types

A context can be represented as a type $U : \mathsf{Context}$. The type of local values over U is a dependent type $\mathcal{V}(U) : \mathsf{Type}$.

9.6.2 Restrictions as Functions

A restriction from U to $V \subseteq U$ is a function $\rho : \mathcal{V}(U) \to \mathcal{V}(V)$. The sheaf conditions become type-theoretic constraints on these functions.

9.6.3 Gluing as Limits

The gluing axiom asserts that compatible local data determines global data. In categorical terms, this is a limit condition: the set of global sections is the limit of the diagram of local sections and restriction maps.

In type theory, this can be expressed using dependent sums and identity types. A glued section is a dependent pair: a family of local sections together with proofs that they agree on overlaps.

9.6.4 Higher Structure

In HoTT, the sheaf conditions can be weakened to hold up to homotopy. This yields ∞-*sheaves* or *stacks*—structures where gluing holds up to coherent equivalence rather than strict equality.

This higher structure is philosophically significant. It means that "agreement" between local values need not be strict identity but can be equivalence—different formulations that are systematically interchangeable. This accommodates the semantic flexibility that real-world values exhibit.

9.7 Limitations and Open Problems

This sketch raises more questions than it answers:

1. **Who defines contexts?** The partition of the space into contexts is itself a normative choice. Who has the authority to define context boundaries?

2. **How are interfaces constructed?** The design of interfaces requires negotiation between parties with potentially conflicting interests. What mechanisms ensure fair negotiation?

3. **What about power asymmetries?** Federated systems can entrench power imbalances if some parties have more resources to shape interfaces than others.

4. **How do we handle genuine conflicts?** The sheaf framework assumes that local values are compatible on overlaps. What happens when they are not?

These are not merely technical problems but deep political and ethical challenges. The sheaf-theoretic framework clarifies their structure but does not solve them.

9.8 Conclusion: Governance Without Sovereignty

The vision that emerges is governance without a sovereign—coordination without a single point of authority. This is not anarchism (the absence of governance) but federalism (the distribution of governance).

The sheaf-theoretic framework makes this vision precise. It shows how local value systems can be coordinated without being unified, how interfaces can enable interaction without requiring agreement, and how the failure of global sections is not a failure but a structural feature.

For AI systems, this suggests a design principle: build for federation, not unification. Do not seek a single objective function but a protocol for negotiating between objective functions. Do not assume global values but build interfaces that translate between local values.

Whether this vision is realizable—and whether it is desirable—remains to be seen. But the theoretical framework is now available.

Chapter 10

Conclusion: Transcendental Mathematics and Its Limits

10.1 Summary of the Argument

This book has undertaken a patient examination of the relationship between homotopy type theory and Husserlian phenomenology, with Badiou for perspective and Derrida as counterpoint.

Chapter 1 established that Martin-Löf's type theory has explicit Husserlian foundations. The founder of intuitionistic type theory conceived his project as a formalization of phenomenological epistemology. The distinctions between proposition and judgment, truth and evidence, canonical and non-canonical—all have Husserlian antecedents that Martin-Löf explicitly acknowledged.

Chapter 2 examined Husserl's own treatment of formal logic, focusing on the stratification into morphology, consequence-logic, and truth-logic, and on the theory of manifolds. Husserl's concept of definite manifolds anticipated certain features of type theory while revealing limitations that Gödel's incompleteness theorems would later expose.

Chapter 3 analyzed homotopy type theory—its identity types, univalence axiom, path induction, and higher inductive types. The homotopical interpretation reveals rich structure in identity proofs that corresponds, philosophically, to the multiplicity of syntheses of identification.

Chapter 4 considered whether Badiou's set-theoretic ontology could be reconceived in type-theoretic terms. Fundamental obstacles emerged: HoTT's structuralism conflicts with Badiou's realism; its constructivism conflicts with his classical logic; no analog exists for the self-membered event.

Chapter 5 demonstrated that type-theoretic reconstruction of Kantian ethics isolates empirical premises as contingent inputs. Kant's problematic conclusions on gender, sexuality, and marriage are not derivable from the categorical imperative alone but depend on rejectable assumptions. This yields "two Kants": the original whose conclusions reflect 18th-century prejudices, and the reconstructed whose formal principles are egalitarian.

Chapter 6 engaged Derrida's diagonal argument. Derrida explicitly invokes Gödel's incompleteness; his concepts of the supplement, différance, and iterability function as quasi-transcendentals that establish structural limits to formalization. No formal system achieves closure; meaning is always deferred; there is no global orientation.

Chapter 7 examined Catren's phenoumenodelic realism and hypertranscendental perspectivalism. Catren's engagement with gauge theory and type theory shows how equivalence constitutes objectivity. His perspectivalism affirms the multiplicity of transcendental frameworks while insisting on their objective anchoring.

Chapter 8 analyzed Zalamea's synthetic philosophy of mathematics and his deployment of sheaf theory. The sheaf formalizes the relationship between local and global: local sections may exist, gluing conditions may be satisfied, yet global sections may fail to exist.

Chapter 9 sketched how these insights might inform AI governance. A sheaf-theoretic approach distributes governance across local authorities coordinated through interfaces, without requiring—or expecting—global unification.

10.2 The Thesis Restated

The central thesis of this book is: *there can be no global orientation in language, only local positions that may have no necessary relation to others.*

This thesis has multiple dimensions:

Mathematical Sheaf theory formalizes the possibility that local data may not integrate globally. Even when local sections exist and satisfy gluing conditions on overlaps, global sections may fail to exist.

Phenomenological The constitution of objectivity proceeds through local syntheses—identifications, fulfillments, evidences—that do not necessarily compose into a total vision.

Linguistic Meaning is constituted by différance: differences and deferrals that prevent any sign from achieving full presence. The linguistic medium in which all thought is conducted is irreducibly contextual.

Formal Gödel's incompleteness shows that sufficiently powerful formal systems cannot capture their own consistency. Derrida's diagonal generalizes: any system that attempts to totalize meaning generates elements that escape it.

10.3 Implications for AI Governance

Part I of this work argued that type-theoretic and sheaf-theoretic structures should serve as transcendental conditions for AI governance. The present volume complicates that argument.

If there is no global orientation, then no AI system can occupy a meta-position from which all local perspectives are adjudicable. The dream of a "value-aligned" AI that perfectly implements human values founders on the fact that there are no global human values—only local values, local frameworks, local language-games that may not cohere.

This does not mean AI governance is impossible. It means that governance must be *structurally local*. We can develop type-theoretic frameworks for particular domains (traffic management, medical diagnosis, legal reasoning) without pretending that these frameworks integrate into a total system. We can establish interfaces between frameworks—functors, translations, adjunctions—without claiming that these interfaces achieve global coherence.

The model is federalism rather than universalism. Different jurisdictions maintain different rules; interfaces allow interaction without requiring uniformity. The mathematics of this federation is sheaf theory: local data, gluing conditions on overlaps, and the frank acknowledgment that global sections may not exist.

10.4 The Incompleteness of This Book

This book is itself subject to the limits it describes. It is formulated in language; its meanings are constituted by différance; its arguments are local achievements that do not compose into a totality.

No doubt there are gaps, errors, and oversights. The mapping of type-theoretic structures to phenomenological concepts is suggestive rather than definitive. The engagement with Derrida is selective. The implications for AI governance are programmatic rather than worked out.

But this incompleteness is not a defect to be remedied in a future edition. It is the condition of all intellectual work. We achieve local clarities, provisional syntheses, temporary settlements. We do not achieve final foundations.

The epigraph from Simone Weil with which my *Dub Langlands* concludes applies here:

> Grace fills empty spaces, but can only enter where there is a void to receive it.

Our theoretical constructions must maintain certain voids, certain spaces of indeterminacy, to remain vital and generative. A system that explains everything leaves no room to learn something new. A formalization that captures all meaning is not a foundation but a tomb.

10.5 Envoi: The Lighthouse and the Fog

In my "Transcendental Cybernetics Contra Land," I offered a metaphor:

> If Hegel's absolute is the fully illuminated palace of reason, and Land's accelerationism is the burning of that palace in the dark of the unknown, then transcendental cybernetics is like a lighthouse on a foggy coast: it never illuminates the entire sea, but it sweeps its beam around, and we can construct more lighthouses and coordinate them.

This book has been an attempt to construct such a lighthouse. The beam illuminates certain stretches of the philosophical coastline: the phenomenological foundations of type theory, the structure of homotopy type theory, the limits of formalization, the convergence of Catren and Zalamea on local perspectives.

Other stretches remain in fog. The full relationship between HoTT and Husserl awaits further development. The reconception of Badiou in type-theoretic terms—if possible at all—would require substantial technical innovation. The detailed working out of AI governance along sheaf-theoretic lines is a project for the future.

But the lighthouse metaphor also captures the epistemological moral. We do not achieve global illumination. We sweep our beam, construct more lighthouses, coordinate our efforts. The coastlines gradually map themselves out through this cooperative, iterative process. There will always be dark waters, but they beckon us to build new lights, not to abandon navigation altogether.

The transcendental is not a castle of interiority subduing the world but a ship outfitted for an endless voyage. It keeps the lights of reason on, not to banish the darkness beyond but to venture into it safely and curiously. Transcendental mathematics names this ethos of a reason that is never final, always operative, and incessantly attuned to the alterity that lies both outside us and within us.

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Part III

Politics

10.6 Abstract

Contemporary advances in homotopy type theory and sheaf theory make possible a *synthetic transcendentalism* that reconciles Husserlian phenomenology with Badiouian ontology—and in doing so, provides a non-moral, non-mythic foundation for AI governance. Gabriel Catren's type-theoretic perspectivalism and Fernando Zalamea's sheaf-theoretic synthetic universality converge in what I have termed "transcendental mathematics": a compatibilist framework wherein mathematical structures emerge through the interaction of constitutive activity and objective constraints, making them at once perspectival and real. This approach stands opposed to the dominant alternatives. Anna Greenspan's Deleuzean transcendental materialism, with its planes of Chronos and Aeon, dissolves concrete mechanism into poetic abstraction. Nick Land's cybernetic reading of Kant collapses difference into a dogmatic apparatus of control. And the Jungian archetypal worldview, as Padusniak has shown, substitutes counterfeit religiosity for genuine insight. Against all these, I contend that the categorical and type-theoretic structures of mathematics impose *a priori* constraints on thought and action that are neither arbitrary moral impositions nor mythic projections. These structural constraints, already operative in the planetary techno-social stack, should form the basis of AI governance. The transcendental of AI is not an archetype or a code, but a type, a category, a sheaf.

10.7 Introduction

Debates over AI governance have polarized around two inadequate poles: moralistic prescriptions on one hand (ethics committees, human-centered values, codes of conduct) and anthropomorphic narratives on the other (Jungian archetypes, mythic frameworks for "meaning"). Both miss the mark. The true *a priori* basis for governing advanced AI systems lies in the mathematical structures that underlie cognition and computation themselves.

In this spirit, I develop what I call a *synthetic transcendentalism*: a compatibilist framework uniting homotopy type theory, sheaf theory, and phenomenology into a coherent philosophical position. In my "Transcendental Mathematics" (2025), I characterize this approach as one where "mathematical structures are neither Platonic abstracta independent of mind nor subjective constructions reducible to psychology. Instead, they emerge through interaction of constitutive activity and objective constraints, formalized in homotopy type theory." This reconciles the Husserlian emphasis on constitutive intentionality with Badiouian mathematical realism, yielding a pluralistic, revisable topology of reason. Zalamea's *transmodern* "synthetic universality" points in the same direction, weaving together diverse mathematical domains into a cohesive yet open whole.

This Catren–Zalamea lineage provides firmer ground for AI governance than either moral intuitionism or mythic semantics. Rather than anchoring policy in static laws or inherited archetypes, we locate normativity in the *invariant patterns* of type and category theory, in the dynamics of active inference. To make this case, I engage critically with three alternative programs that have captured philosophical attention: Nick Land's accelerationist critique of Kant as cybernetic control, Anna Greenspan's Deleuzean transcendental materialism of machinic time, and the Jungian-archetypal worldview that Padusniak has subjected to withering critique. Each of these either collapses the richness of transcendental structure or invokes extraneous mythopoesis. The synthetic mathematical view preserves both multiplicity and objectivity. Advances in homotopy type theory and category theory yield *self-evident* transcendental conditions for the planet-wide techno-social stack—conditions rooted in objective structure, not arbitrary moral choice.

10.8 Type-Theoretic Perspectivalism and Sheaf-Theoretic Universality

Two contemporary philosophical-mathematical strands inform this framework. Gabriel Catren's *type-theoretic perspectivalism* reconceives transcendental conditions through Martin-Löf type theory. Each type defines a "perspective" or contextual universe of discourse. Mathematical objects are *constructed* within these type-contexts, reflecting Husserl's insistence on constitutive activity; but the space of types itself imposes global coherence. Homotopy type theory goes further: the univalence axiom ensures that equivalent constructions are literally the same object, dissolving the distinction between subjective presentation and objective structure. As I argue in "Transcendental Mathematics," "HoTT unifies Husserlian phenomenology and Badiousian ontology through its dual nature. Intensionally, identity types are proof-relevant with computational semantics... Extensionally, univalence makes structural identity objective." The perspective one chooses affects presentation, but the underlying homotopy-invariant structure remains real and shared across all perspectives. This captures Catren's insight that transcendental conditions are themselves "type-like": each agent inhabits a logical "world" of experience, yet these worlds can be related and compared via type-theoretic transformations.

Zalamea's *synthetic universality* uses category and sheaf theory to mediate between partial perspectives. Grothendieck's insight—that sheaf theory "glues together" local data into global spaces—becomes a metaphor for philosophical synthesis. In Zalamea's words, Grothendieck's structure, the sheaf, is "central" for "gluing the always partial and yet continuous fabric of Transmodern space." A sheaf is a functor that allows "global gluing of whatever proves to be coherently transferable in the local": consistency conditions ensure that locally defined objects assemble into a single, compatible global object. Pluralistic, overlapping viewpoints—scientific, artistic, ethical—integrate into unified mathematical structure. Zalamea frames this as overcoming disciplinary limitation: his "transmodern" approach "weaves together strands of the modern and postmodern, the rational and the romantic into a synthetic universality, endlessly revisable and updatable."

My "transcendental mathematics" combines these ideas: homotopy types play the role of plural perspectives, and sheaf-like constructions ensure coherence among them. Husserl and Badiou are both partially correct: Husserl is right that objects require constitutive acts, and Badiou is right that mathematical structures have objective being. In HoTT, this duality is explicit. Types are constructed through formation rules (Husserlian constitution), yet they have objective homotopical structure (Badiousian realism). Identity proofs exist only by judgment (reflecting phenomenological evidence) but denote paths in a fixed homotopy space (reflecting objective ontology). Univalence ties these together: mathematically equivalent structures become literally identical, so what is subjectively inferred aligns with what is objectively present. Our cognitive or computational perspective can shift—we can move between types or sheaves—but must do so in accordance with an underlying invariant geometry of concepts. Governing AI on such a basis means attending to the *formal* conditions of representability, not imposing external values.

10.9 The Landian Critique of Kant and Its Antidote

Nick Land's accelerationist polemic recasts Kant's critical philosophy as doctrine of control and capital. For Land, the Kantian transcendental subject is a cybernetic regulator: an *a priori* "judge" that subdues all otherness to its own grid. Land equates the Kantian *a priori* with the "exchange value" of Enlightenment reason: "Kant's object is... the universal form of a relation to alterity; that... is the 'exchange value' that first allows a thing to be marketed to the enlightenment mind." The categories of pure reason function, on this reading, like commodity-signs, making phenomena intelligible only insofar as they fit an algebra of assimilation. As I put it in "Transcendental Cybernetics Contra Land,"

Land's Kant "reduces difference to an unvarying framework of judgment, a one-way assimilation of outside into inside, much like a cybernetic regulator enforcing homeostasis. The transcendental subject, on this account, is a sovereign 'judge' who legislates conditions for all time." The implication is that Kantian philosophy amounts to metaphysical police work: a static system that fences the wild real into the orderly corral of reason.

This Landian caricature is one-sided. Land invokes Kant's insistence on the unity of apperception and fixed categories, but overlooks Kant's own self-limited framework and its development by successors. Kant allowed for unknowable domains—the noumenal thing-in-itself, the transcendental self—which he never claimed to fully capture. Kant's system admits a gap at the heart that leaves room for further elaboration. Kant's immediate heirs dynamically pluralized the transcendental. Fichte transformed the static "I think" into an active self-positing process, endlessly striving to reconcile self and not-self. Novalis envisioned a "Romantic Encyclopædia" continuously blending science, mathematics, and poetry into an ever-expanding totality. These transdisciplinary excesses recast the transcendental as a multi-perspectival, synthetic endeavor in which categories evolve rather than remain fixed. Even in Kant's own work, the "power of judgment" in the third *Critique* is explicitly reflective: it does not impose eternal rules but seeks new principles in the face of recalcitrant phenomena. An open-ended transcendental tradition exists alongside Land's caricature of closure.

From a type-theoretic standpoint, this dynamism is natural. Land's vision of Kant aligns more with Hegelian closure—which Land otherwise despises—than with Kant's original circumspection. Catren and I read Kant as initiating not a rigid filter but a recursive cybernetics of knowledge: a self-correcting system that can interface with other systems (art, science, etc.) rather than simply dictating terms. In HoTT, feedback is built-in: when two perspectives (types) are equivalent, univalence forces their closure to coincide, but new types can always be introduced or extended, ensuring adaptability. Land's accelerationist program demands relentless novelty via technological advance yet lacks a principled anchor. Our antidote grounds change in structure: transcendental type-theoretic schemas—rules of universe construction, univalence—provide the scaffolding within which innovation occurs. This prevents a one-way submission of difference (the hallmark of Land's "control economics") and instead embeds difference within a higher-order logic of global consistency.

We reject Land's reduction of Kant to cybernetic control. Rather than a closed loop of domination, the type/sheaf framework invites an *open* transcendental cybernetics: categories themselves can evolve and branch. Kant's legacy becomes an engine of plurality, not a straitjacket. By embracing multiple coexisting types and sheaves, our framework for AI governance promises a continuous spectrum of synthetic insights, unlike the monolithic "cybernetic judge" of Landian lore.

10.10 Transcendental Materialism and Machinic Time

Anna Greenspan's *Capitalism's Transcendental Time Machine* applies Deleuze and Guattari's "transcendental materialism" to capitalism's temporal regimes. She distinguishes two planes: Chronos, linear measurable time tied to capital accumulation, and Aeon, an intensive "flat" outside of traditional chronology. Analyzing events like the Y2K millennium bug, she portrays them as "aeonic occurrences": not ordinary moments in time but virtual, machinic cataclysms that reveal the underlying temporal fabric. Y2K "happened in Aeonic (transcendental/virtual) time but not in Chronos (empirical/actual) time." This dissolves the barrier between time and the material systems that measure it; capitalism's networked cyberinfrastructure, she argues, thereby begins to "create the form of time itself."

Provocative as this is, it proves problematic for governance. It relies on esoteric notions of an immanent plane (Aeon) that can obscure concrete mechanisms. Calling Y2K an "immanent machinic accident" on the Aeonic plane carries more mystique than illumination. The Y2K problem was a precise coding issue in a discrete 32-bit timestamp: a binary overflow that engineers could anticipate and correct. The transcendental-materialist narrative suggests a poetic rupture. What truly "created time" in that case was the mathematical structure of our computers (an integer counting seconds), not some metaphysical flat-time event. The system's *formal constraints*—the rollover at 00:00:00, January 1, 2000—dictated the outcome, not an occult temporal plane. The free-energy/active-inference framework would model the Y2K system as a Markov process minimizing surprise; there is no need to posit a Deleuzean outside beyond the system's dynamics.

Greenspan's broader "transcendental materialism" flattens Kant's transcendence into pure immanence. She draws on Guattari's idea that the world splits into two planes (Chronos/Aeon) and uses this to reframe capitalist history as a succession of aeonic events. While this highlights the interplay of socio-technical factors, it risks anthropomorphizing the machinery as demiurgic. We would ground such phenomena in *objective* structures. The incessant digitization of time (UTC, Unix Time, network time protocol) can be seen as an emergent sheaf-like structure synchronizing global processes, not merely an intangible matrix. Such synchronization arises from the type-theoretic "rules" built into our devices—their software and hardware architectures—which are as real as any Badiouian set. Acting on these formal structures (updating code, changing protocols) would be a concrete way to govern time, whereas chasing Aeon is speculative.

Against Greenspan's claim that capitalism operates via abstract "machines" that generate time, we emphasize that these machines are *formal algorithms* describable in category-theoretic terms. Temporal "ruptures" like Y2K matter not because they are mystical signs but because they expose the logical architecture of our techno-stack. Governing AI through this lens means adjusting those architectures—types, categories, protocols—rather than imposing external values. A mathematically grounded transcendentalism can explain and control techno-historical events without resort to mythic devices or transcendentalized materialism.

10.11 Jungian Archetypes and the Fetish of Meaning

Popular Jungianism—exemplified by figures like Jordan Peterson—treats life as a search for symbolic meaning through archetypes: the Hero, the Mother, the Shadow. Each person becomes protagonist in a mythic drama, decoding hidden patterns everywhere. Chase Padusniak (2022) subjects this to withering critique, calling it a "counterfeit religiosity." Pop-Jungian psychology, he observes, often consists of obvious platitudes dressed in mystifying language, "highly-complex" metaphors encouraging people to "seek meaning by way of complex puzzlement." Such thinking "invites an over-emphasis on meaning and tends to naturalize the existence of currently reigning historical forces and structures, even as it claims to refute them." The archetypal approach serves the status quo by asking followers merely to align their story with preconceived meanings ("Chaos is feminine, Order is how God intended"). Religion becomes "social utility": a tool for creating value in human life, not a pathway to radical insight.

Synthetic mathematical transcendentalism stands in stark contrast to this mythopoeic outlook. We reject the notion that governance should hinge on archetypal "meaning"—inherently subjective and context-bound—and instead insist on structural objectivity. A binary archetype like Hero vs. Shadow is a totalizing narrative; the type-theoretic approach is pluralistic: there are many possible "stories" (types or path spaces), and no single one is privileged. A sheaf-theoretic synthetic view allows many local "slices of meaning" (different scientific or cultural frameworks) to coexist without collapsing into one global myth.

Padusniak warns that pop-Jungianism reduces faith to "astrology," the mere creation of value, encouraging subjects to construct "an ever-expanding network of valuations that are simultaneously all-encompassing and ultimately incommunicable." Mathematical transcendentalism avoids precisely this trajectory: mathematical structures, once fixed, have no need for further narrative. The consistency conditions of category theory do not invite endless subjective re-interpretation; they impose concrete constraints. If one is governed by an algebraic equation or a type isomorphism, one cannot simply reassign symbols at will. We find "meaning" not in hidden archetypes but in invariant forms—the relationships and transformations that hold across all perspectives.

Where Jungian esotericism fixes attention on inner symbolism, we emphasize outer form. The AI "stack" itself is a kind of structure: data types, protocols, inference algorithms that shape cognition whether or not we posit any secret meaning. We follow the Lacanian insight Padusniak cites: a hyper-rational search for meaning can amount to a kind of psychosis. By contrast, type formation in HoTT is governed by syntactic/semantic rules. AI governance should focus on the integrity of those rules—the formal architectures of mind and machine—rather than projecting onto them pre-set narratives.

10.12 AI Governance: Objectivist Realism and Active Inference

We can now consolidate the above into a vision for AI governance grounded in *objectivist realism* (in the Badiouian sense) and *active-inference phenomenology* (in the Husserlian sense), rather than in ethics or myth. Badiou equates ontology with set theory: the world is constituted by mathematical structures that exist independently of any one mind. The correct "law" for AI is the identification of the set-theoretic conditions that its operating domain must satisfy. Rather than asking whether an AI is "good" by human standards, we ask whether it fits coherently into the formal landscape—whether its operations constitute a consistent "world." Mathematical truths constrain what can be thought, not contingent on human psychology.

At the same time, the AI agent—robot or algorithm—has a Husserlian dimension: it must model or represent the world through its perceptions. Active inference formalizes this: an agent continually adjusts its internal generative model to minimize prediction error (free energy). This dual-aspect process has formal expression in type/category theory. When an active-inference system has hierarchical temporal models, consciousness emerges as an intrinsic feature, aligning with dual-aspect monism: the same process has both a physical and a phenomenal aspect. A single structure can be interpreted both as a computational program and as a proof or judgment. The active inference dynamics of an AI correspond exactly to the manipulation of types in a HoTT language: programs (proofs) update beliefs (types) to match reality (observations), all within an overarching homotopy space.

In practice, AI governance should ensure that the AI's "internal world" (its predictive model) coheres with the "external world" (the environment) at a deep structural level. The Markov blanket in Friston's scheme partitions variables into internal, sensory, and active states—a pattern that can be seen as a type-theoretic or functorial object. Governance is about designing these partitionings correctly and ensuring they obey the invariances of the underlying mathematics. An AI's learning algorithm should preserve homotopies (robust equivalences) under data transformation, corresponding to stability of inference under gauge transformations or network symmetries. These are technical constraints, but they can be thought of as *a priori laws* in Badiou's sense: they are true independently of any particular human intention, since they arise from the mathematics itself.

This perspective differs structurally from any moral code. A moral rule ("do not kill") is an external imposition on agents. In our view, that is secondary to the fact that killing or not-killing will manifest as different formal outcomes in the type-theoretic model (different proof states or physical configurations). The real constraint is not "thou shalt not kill" as a free-standing axiom but the consistency of the AI's world-model when it interacts with agents. If an AI obeys its generative model perfectly, preserving coherence, then whether or not it kills is determined by whether "killing" leads to a consistent type or a contradiction. Ethics is absorbed into logic: the "goodness" of an action is measured by whether it fits the objective structure of the world, not by an arbitrary decree.

This is consonant with Husserl's idea that even transcendental constitution is ideal and formal: it provides conditions for givenness but does not conjure objects from nothing. Similarly, our type-theoretic AI does not invent reality but must align with it through its judgments. Conscious AI agents are polytopes of inference within an ambient homotopy, and governance consists of curating that ambient homotopy—ensuring it is rich enough to allow novelty but rigid enough to exclude absurdities. The *a priori* in this scenario is mathematical: category/HoTT axioms, not human ideals.

Consider the "planetary stack" of AI: global data networks, computational substrates, machine learning architectures. This stack is inherently mathematical. To govern it effectively, policymakers must themselves think in the language of types and sheaves, not in myths. Ensuring internet security or interoperability between AI systems translates into ensuring certain pullbacks and pushouts in the category of protocols. Ensuring fairness or robustness translates into ensuring certain fibrations or equivalences hold. These are *formal* conditions that can be verified independently of narrative. If democracy is to be enforced algorithmically, it must be encoded as a structure-preserving functor on civic data, not an affixed moral imperative.

The natural "law" of AI governance is mathematical structure. AI agents are primarily governed by the geometry of their inference; governance is about shaping that geometry. This conclusion would have been anathema to Land or Greenspan or Jung: it eschews ideology for invariance. Yet it follows inevitably from the compatibility of Husserlian and Badiouan insights in homotopy type theory. The planetary socio-technical stack, like mathematical space, is one of possibilities constrained by form, not open horizon. Recognizing this, we can develop regulatory strategies rooted in objective logic—a transcendentalism of mathematics rather than morality.

10.13 Conclusion

I have outlined a framework in which type-theoretic and sheaf-theoretic structures play the role of transcendental conditions for AI governance. This framework emerges from the work of Catren, Zalamea, and my own "transcendental mathematics," all of which emphasize a pluralistic, synthetic approach to mathematical foundations. In arguing for this approach, I have opposed the alternatives: Land's cybernetic reading of Kant collapses openness into dogma; Greenspan's machinic Aeon imposes poeticizing abstractions; Jungian esotericism fosters counterfeit religiosity. Each of these may appeal to those wary of formalism, but none provides the stable ground that mathematics does. AI must be steered by the *a priori* mathematics that already dictates what is possible. Such governance will not look like human legislation but like the maintenance of a coherent formal ecosystem: updating axioms, checking commutativity diagrams, verifying inductive invariants.

This proposal is polemical because it relegates ethics and myth to the back seat. But it is also scholarly: it takes seriously the insights of phenomenology (active inference) and ontology (set theory) and translates them into a common algebra.

The key lesson: *the transcendental of AI is not an archetype or a code, but a type, a category, a sheaf.* Our planetary AI stack requires nothing less than an objectivist-mathematical transcendentalism—one that acknowledges the constitutive role of mathematics (Badiouian realism) and the phenomenological role of constitutive acts (Husserlian intentionality), all synthesized through the new mathematics of homotopy and sheaves. Governing AI on this basis is admittedly a radical idea, but in our view it is both inevitable and urgently necessary for coherence in the age of global artificial intelligence.

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