Critique of Linguistic Reason

From the Analytic Tradition to
a Homotopic Metatheory of Language

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Dedicated to the memory of the ocean of metalanguages,
and to those who build interfaces where no global section exists.

> "Syntax, under the right conditions, is indeed sufficient for semantics, and meaning can be conferred upon syntactic expression if such conditions are satisfied."
>
> — Reza Negarestani

> "It is my belief that in the next decade and in the next century the technical advances forged by category theorists will be of value to dialectical philosophy, lending precise form with disputable mathematical models to ancient philosophical distinctions."
>
> — F. W. Lawvere

> "Grace fills empty spaces, but can only enter where there is a void to receive it."
>
> — Simone Weil

Preface

This book completes an argument begun in three earlier texts and, in completing it, transforms it. The *Prospectus to a Homotopic Metatheory of Language* (2023) announced a program: to argue for a Kantian–Hegelian idealism whose historical epochs are divided according to Michael Friedman's dynamics of reason, and whose metatheory of scientific and linguistic progress is expressed in the language of Homotopy Type Theory. *Transcendental Mathematics* (2025) established the phenomenological credentials of that language—the Husserlian foundations of Martin-Löf's judgments, the constitution-theoretic reading of identity types—and arrived at a negative thesis: **there can be no global orientation in language, only local positions that may have no necessary relation to others**. *The Mathematics of Neo-Rationalism* (2026) supplied the technical substrate in full: categorical logic from Lawvere's functorial semantics through the doctrinal hierarchy, and topos theory from the elementary axioms through Kripke–Joyal semantics and geometric morphisms.

What remained undone was the critique itself. Kant did not write a *Theory* of pure reason; he wrote a *Critique*—a tribunal before which reason's claims are examined, its legitimate employment secured, and its transcendent pretensions dismissed. The philosophy of language has never received such a tribunal. It has received foundations (Frege), therapies (Wittgenstein), tolerances (Carnap), hierarchies (Tarski), verdicts of indeterminacy (Quine), and programs of inferential explication (Brandom). But it has not received a systematic delimitation of the conditions under which meaning is possible at all, together with a principled demonstration of where the demand for meaning must fail.

This book is that tribunal. Its organon is the mathematics of the twenty-first century's foundational revolution: dependent type theory, homotopy type theory, the univalence axiom, fibered categories, descent, stacks, and the adjoint modalities in which Lawvere discerned the formal skeleton of Hegel's logic. Its verdict, stated in advance: the synthetic a priori, expelled from philosophy by Russell, the positivists, Quine, and Putnam, must be readmitted—but relativized, historicized, and fibered. Meaning is real, objective, and normatively binding; and it is irreducibly local, constituted in interaction, transported along context-shifts, amalgamated rather than abstracted, and never totalized in a global section.

The reader I imagine holds three books open at once: Brandom's *Making It Explicit*, the HoTT Book, and Grothendieck's SGA 1. Such readers are few. For everyone else, Part III and the appendices develop all the category theory and type theory the argument requires, following the self-contained treatment of my *Mathematics of Neo-Rationalism*; the philosophical chapters presuppose no more mathematics than they themselves introduce.

I thank David Corfield, whose correspondence—quoted in the Prospectus and put to work throughout this book—first pointed me toward linear homotopy type theory as the setting in which Brandom's singular terms and substitutional inferences find their formal home; and Steve Awodey, whose interpretation of identity types as topological path spaces forms the basis of the homotopical reading of language defended here.

Chicago, 2026

Note on Sources

This book is the fourth panel of a single project, and it borrows freely, with revision, from the other three. The premise and the verbatim program of Parts I–II and V derive from my *Prospectus to a Homotopic Metatheory of Language* (November 2023), whose sections from "Reclaiming the Kantian synthetic a priori" onward are here expanded into chapters, and whose earlier sections—the multi-agent circuit, the computational trinity, the Corfield and Awodey correspondence, the Zalamea and McLarty reviews, the Lawvere–Hegel diagrams—supply the recurring machinery. The mathematical exposition of categorical logic and topos theory in Part III and the appendices condenses my *Mathematics of Neo-Rationalism: Notes on Categorical Logic and Topos Theory* (2026), which should be consulted for full proofs, the complete doctrinal hierarchy, the gallery of toposes, and the guide to the literature. The fibrational semantics of Part IV follows the reconstruction of Brice Halimi's descent-theoretic account of context that I set out in "Gnostic Futurism: Notes Toward a Movement" (2026), where the same mathematics is read politically and curatorially; readers of that essay will recognize Part VI's final chapter as its systematic twin. The negative thesis that governs the whole—no global orientation in language, only local positions—is argued at book length, on phenomenological and deconstructive grounds, in *Transcendental Mathematics* (2025); the present work assumes its conclusions and undertakes to derive them structurally.

Contents

Introduction: The Idea of a Critique of Linguistic Reason

The tribunal

A critique, in the Kantian sense, is neither a refutation nor a foundation. It is a juridical proceeding: reason called before its own tribunal, required to exhibit the sources, the extent, and the limits of its claims. The first *Critique* asked how synthetic a priori judgments are possible, answered that they are possible as the forms of experience itself, and concluded that beyond experience they are impossible—the dialectic of pure reason being the record of what happens when the demand for the unconditioned outruns the conditions of givenness.

Philosophy of language in the twentieth century staged the entire Kantian drama again without noticing. The analytic tradition began, at the height of neo-Kantianism, as a revolt against the "metaphysical bloatware"—Negarestani's phrase—of post-Hegelian speculation. Its instrument was the concept of analyticity: truth in virtue of meaning alone, deduction as tautology, mathematics as an immense system of dominos falling from definitions. Wittgenstein's *Tractatus* declared that all deductions are made a priori (5.133) and that where we cannot speak we must remain silent. Carnap radicalized the program: meaning itself was deracinated, replaced by syntax, and the choice among logical frameworks was declared a matter of tolerance rather than truth. Then the internal collapse: Tarski's undefinability theorem showed that every object language requires a metalanguage to define its truth predicate, and that metalanguage another, generating what Steve Awodey has called the *ocean of metalanguages*; Gödel showed that no sufficiently strong system totalizes its own consistency; Quine dissolved the analytic–synthetic distinction on which the whole edifice rested; and the synthetic a priori, three times expelled—by Russell, by the positivists, by Quine and Putnam—left behind a philosophy of language with no account of how meaning could be both objective and constituted.

The present book contends that this history is not a sequence of accidents but a transcendental dialectic: the necessary fate of any philosophy of language that demands a *global* theory of meaning—a single semantic value function, a universal metalanguage, a view from nowhere above the fibres of use. The critique's positive task is to exhibit the conditions under which meaning *is* possible: inferential, interactional, type-theoretic, and fibered. Its negative task is to prove, with the resources of descent theory, that these conditions never yield a global section, and that the demand for one is the paralogism of semantic reason.

The three questions

Kant's three questions—What can I know? What ought I to do? What may I hope?—have linguistic counterparts, and they organize this book.

What can be said? The Analytic of Inference (Parts I–II) answers: whatever can be given a role in the game of giving and asking for reasons. Meaning is use; use is inference; inference is normatively instituted between agents. Following Brandom, semantics is answerable to pragmatics; following Negarestani, syntax coupled with interaction suffices for semantics; following Girard, interaction itself is computationally encodable. The formal explication of this answer is the computational trinity: propositions are types, proofs are programs, and the space of reasons is an $(\infty, 1)$-topos in which types record directed material inference and higher morphisms record the reversible substitution of singular terms.

How is what is said possible? The Transcendental Machinery and the Fibrational Semantics (Parts III–IV) answer: through structure that is synthetic a priori in Martin-Löf's sense—pure geometry, arithmetic, and logic as modes of construction, not analysis—yet relativized in Friedman's sense, holding universally within an epoch and revisable across revolutions. The machinery is categorical logic (theories as structured categories, logical operations as adjoints), topos theory (the internal language, Kripke–Joyal forcing), univalence (identity as equivalence, Awodey's path-space interpretation), and, decisively, Halimi's fibrational semantics: context-dependence is context-shift-dependence; a content in a context is not the value of a meaning-function but the Cartesian transposition of a content along a shift; and the general meaning of an expression is not an abstracted invariant but an amalgamation of local contents glued by descent.

What may language hope for? The Dialectic and the Doctrine of Consequences (Parts V–VI) answer: not a universal language—Gödel, Tarski, and the failure of global sections foreclose it—but the patient construction of interfaces. The teleology of the whole process is Hegelian in Brandom's sense: a history of histories in which each relativized a priori is recollected, its contingency converted into the expressive rationality of the tradition. Lawvere's formalization of Hegel via adjoint modalities gives this teleology a mathematical body; the absence of global sections gives it its ethics. Against the libidinal nihilism whose slogan is *time will tell*, the critique concludes: **the other will speak**, and the task of a critical philosophy of language is to build the interfaces—reindexing functors, gluing data, Grothendieck coverings—through which that speech can be heard, translated, contested, and revised.

The central thesis

Thesis 0.1. The metatheory of linguistic and scientific progress can be expressed in the language of Homotopy Type Theory / Univalent Foundations. Language is co-extensive with sociality; general intelligence is realizable if and only if language is understood inferentially, in terms of semantic social norms defined interactionally between agents through a minimal syntax generating the whole system of rule-based roles. The semantics of such a language is fibrational: contents live in fibres over contexts, are transported along context-shifts by Cartesian lifts, and amalgamate by descent. And the fibration admits, in general, no global section: there is no view from nowhere in language—not as a matter of pessimism, but as a mathematical theorem about the structure of meaning.

Every clause of this thesis will be earned. Part I earns the historical clause by re-litigating the fate of the synthetic a priori. Part II earns the inferentialist clause. Part III earns the type-theoretic clause. Part IV earns the fibrational clause. Part V earns the historical-dialectical clause. Part VI earns the final, negative clause—and shows why it is not a defeat.

Part I

The Critical Problem: The Fate of the Synthetic A Priori

Chapter 1

The Deracination of Meaning: Wittgenstein, Carnap, and the Birth of Analyticity

1.1 Analytic philosophy at the height of neo-Kantianism

Analytic philosophy was created at the height of neo-Kantianism, in the immediate aftermath of German Idealism, and it defined itself against that aftermath. The obscure terms and concepts of the idealist inheritance—Hegel's philosophy above all—were, by the turn of the twentieth century, much overused; there was a flood of what Reza Negarestani calls *metaphysical bloatware*, inflationary metaphysics whose concepts had ceased to discharge any inferential labor. The founders of the analytic tradition responded not by repairing the concepts but by changing the currency in which conceptual debts are paid: from intuition to logic, from the synthetic to the analytic, from the system to the proposition.

Frege established the analytic demarcation of a priori reasoning that would inform Russell's logicism and Wittgenstein's construction of fact and tautology. Arithmetic, on Frege's account, is analytic: derivable from logic plus definitions, requiring no Kantian intuition of time. Geometry alone retained its synthetic status—a concession that the logical positivists would soon revoke by splitting geometry into pure (analytic, uninterpreted) and applied (empirical, a posteriori) components, filling every box of their fourfold table except the one Kant had built the entire critical philosophy to occupy. The Logical Positivists' boxes, as displayed in Neil Tennant's *The Taming of the True*, are exact: logic, arithmetic, and pure geometry in the analytic a priori; applied geometry in the synthetic a posteriori; and the synthetic a priori crossed out. The Quine–Putnam boxes go further: everything migrates to the synthetic a posteriori, and analyticity itself is crossed out. Martin-Löf's boxes—to which we return in Chapter 4—restore what both had cancelled: logic, arithmetic, and pure geometry as *synthetic* a priori, with only applied geometry a posteriori.

1.2 The Tractatus: deduction as tautology

Russell's pupil Wittgenstein gave the analytic program its most austere formulation. In the *Tractatus*, the propositions of logic are tautologies; they say nothing; their truth is recognizable from the symbol alone. "5.133 All deductions are made a priori." Mathematics is a logical method; its propositions are equations, hence pseudo-propositions; the world divides into facts, and where the facts give out—ethics, aesthetics, the meaning of life, the mystical—we must remain in silence.

The question this book presses against the Tractarian settlement is the one the young Carnap already pressed: is analyticity only meaningful when it is devoid of all content—when the only admissible propositions are positivistic logico-philosophical ones, all deductions implied from themselves deductively like a succession of dominos? Is all mathematics tautology? The Tractatus purchases the a priori at the price of its emptiness. Kant's question—how are synthetic judgments a priori possible?—is not answered but dissolved: there are none. The cost of the dissolution would only become visible when mathematics itself refused to behave analytically: when Gödel showed that the consequences of a formal system outrun any finite domino-chain of its definitions, and when the actual practice of twentieth-century mathematics—sheaves, schemes, toposes, homotopy types—exhibited exactly the ampliative, constructive, intuition-refashioning character that Kant had called synthesis.

1.3 Carnap: the deracination of meaning and the principle of tolerance

Carnap's critique of Wittgenstein is the hinge of the story. Where the Tractatus retained a residual semantics—the picture theory, the showing that cannot be said—Carnap's *Logical Syntax of Language* deracinated meaning altogether and substituted syntax. Philosophy becomes the logic of science; the logic of science becomes the syntax of the language of science; and questions of meaning are replaced by questions of derivability, consequence, and translatability, formulated in a metalanguage.

Two features of Carnap's program matter for the critique. First, the **principle of tolerance**: in logic there are no morals; everyone is free to build his own logic, his own form of language, as he wishes. We should be tolerant of a multitude of metalanguages for instituting the object language of scientific theory—the language in which one has propositions, or the arithmetic, say Peano arithmetic—through the supporting ocean of metalanguages (Gödel's encoding of Peano Arithmetic through prime numbers being the paradigm of such institution). Tolerance converts the Kantian question of validity into an engineering question of framework choice; external questions, asked outside any framework, are cognitively meaningless.

Second, the **minimal structure of communicability**. Carnap crystallized the bare minimum of structure required for an alien civilization to understand our language: propositions, and variables defining such propositions. This thought experiment—language reduced to its skeletal combinatorics, meaning to be reconstituted from structure alone—anticipates, in a way Carnap could not have foreseen, both the inferentialist thesis that syntax coupled with interaction suffices for semantics (Chapter 6) and the type-theoretic thesis that grammatical categories are types and communicability is the preservation of meaningfulness under typed substitution (Chapter 8).

The critique's verdict on Carnap is therefore double. His tolerance was correct as a rejection of the view from nowhere: there is no framework-independent standpoint from which the one true logic could be certified, and this book's fibrational semantics is, in one sense, Carnapian tolerance made mathematically exact—frameworks as fibres, translations as reindexing functors. But his deracination of meaning was a transcendental error: it mistook the impossibility of a *global* semantics for the impossibility of semantics *tout court*. The task is not to eliminate meaning in favor of syntax but to exhibit meaning as instituted *through* syntax-in-interaction—which requires precisely the machinery Carnap lacked: adjoint functors, dependent types, and descent.

1.4 The continual casting away of the synthetic a priori

There has been a continual casting away of the synthetic a priori: by Russell, for whom logic sufficed; by the positivists, for whom the analytic/synthetic and a priori/a posteriori distinctions crossed only in the two safe boxes; and later by Quine and Putnam, for whom even those boxes collapsed into a web of belief revisable at every node. The central question of this book can accordingly be posed in the classical idiom before it is posed in the categorical one: whether to admit, ontologically, metaphysical realism or anti-realism; and then, epistemologically, semantic realism or anti-realism. The wager of the chapters that follow is that the second question is prior to the first, that it is decided on the terrain of the philosophy of mathematics—either one is a Platonist or one is an intuitionist, as Dummett insisted—and that the intuitionist option, codified in Martin-Löf type theory and geometrized in HoTT, reopens the case for the synthetic a priori that the tradition thrice dismissed.

Chapter 2

Tarski contra Carnap: The Ocean of Metalanguages

2.1 The undefinability of truth

Tarski posed a great challenge to Carnap with the undefinability theorem: no consistent language sufficient for arithmetic can define its own truth predicate. Wherein every metalanguage necessitates a further metalanguage to encode the prior object language, the model-theoretic paradigm begins. Truth for the object language is definable only in an essentially richer metalanguage; truth for that metalanguage, in a meta-metalanguage; and so on up the hierarchy. Semantics, which Carnap had hoped to eliminate or at least to discipline syntactically, returns as an infinite tower.

The standard reading of this episode makes Tarski the victor: semantics is unavoidable, and it is stratified. The standard reading of Gödel makes the stratification vicious: the dream of a universally quantified language—a single language in which all languages, including itself, are interpreted—is doomed by incompleteness. What are the possibilities of defining a universally-quantified language, or is such an endeavor doomed to fail because of Gödel's theorem regarding incompleteness? The critique accepts the letter of both theorems and contests the moral usually drawn from them.

2.2 An alternative reading of the split

I am interested in an alternative reading of the split between Tarski and Carnap over metalogic regarding semantics. The alternative begins from intuitionistic logic, where propositions are taken as if the witness matters: to assert is to be prepared to produce evidence, and the meaning of a proposition is the type of its verifications. This line would later culminate in type theories such as Martin-Löf's and in substructural logics such as Girard's linear logic. On this reading, the semantic anti-realist position—the opposition between transcendent truth and constructive truth, situated between Platonist and intuitionist philosophy of mathematics—harkens back to Carnap's original principle of verification, now relocated:

verification becomes the witness of intuitionistic logic, and the witness is situated in an inferential network.

The Tarskian hierarchy then appears in a new light. It is not a regress to be lamented but a structure to be internalized. In the Univalent Foundations, the tower of metalanguages is reflected inside the theory itself as the cumulative hierarchy of universes: a type of types, a type of types of types, and so on. In Agda—the proof-checker for Univalent Foundations—the type system includes an infinite hierarchy of universes $\mathsf{Set}_i : \mathsf{Set}_{i+1}$; this hierarchy enables quantification over arbitrary types without the inconsistency that follows from $\mathsf{Set} : \mathsf{Set}$. The ascent that Tarski located between languages, type theory locates between universes—and, crucially, makes *internal*: the passage up and down is itself expressed by terms and types, morphisms and objects.

Theorem 2.1 (Transit)**.** *Apart from the original morphisms and objects directed at the lowest level, there exists a transit up and down—and vice versa—between object language and metalanguage through* $(\infty, 1)$*-topoi: the universe hierarchy internalizes semantic ascent, and the reversible substitutability of terms at higher dimensions internalizes semantic descent.*

Chapter 12 gives this theorem its precise content and its proof-sketch via object classifiers in $(\infty, 1)$-topoi. Here it functions as the critique's answer to Tarski: the ocean of metalanguages is real, but it is not outside language; it is language's own higher-dimensional structure. The undefinability of truth becomes the positive discovery that truth is *indexed*—by universe level, by context, by fibre—and the model-theoretic paradigm becomes a special, truncated case of the fibrational semantics developed in Part IV.

2.3 Findlay, Brandom, and the direction of ascent

The hierarchy admits two directions of travel, and the choice between them is philosophically loaded. As opposed to J. N. Findlay, who argues for semantic ascent in Hegel from the object-language to the meta-language—a move championed by Tarski—in order to define a judgment, Brandom argues for semantic *descent*: down to the bottom level, to the materially good inferences in which the use of expressions is actually instituted. The dispute is not verbal. Ascent treats the metalanguage as the locus of semantic authority: meaning is what the theorist says it is. Descent treats the object-level practice as the locus: meaning is what the practitioners do, and metalinguistic locutions—Sellars's "broadly metalinguistic locutions," which Brandom spends his career making explicit—are expressive instruments for bringing that doing into the game of reasons, not a tribunal above it.

The homotopic metatheory dissolves the exclusivity of the choice. In an $(\infty, 1)$-topos both directions are available and adjoint: ascent is the passage to a name in a universe, descent is instantiation; the '1' of the $(\infty, 1)$ corresponds to unidirected inference between types, the '∞' to the reversible substitutability of terms at all higher dimensions. Findlay's Hegel and Brandom's Hegel are the two adjoints of a single structure—an anticipation, at the level of metalogic, of the adjoint cycles that Chapter 17 will read, with Lawvere, as the mathematical form of the dialectic.

Chapter 3

Semantic Anti-Realism: Dummett, Martin-Löf, and Logic as if People Matter

3.1 Bivalence and the theory of meaning

One of the main readings of Frege's logicism—the school of philosophy of mathematics which believes in the fundamentality of atomic propositions—runs through Michael Dummett's work on the bivalence of truth statements and the justificatory power of demonstrating a proof to a witness via assertion. Dummett was the key proponent of the semantic anti-realist school, and he argued that all language can be reduced to its logical basis in the philosophy of mathematics: either one is a Platonist or one is an intuitionist. The realist holds that every proposition is determinately true or false independently of our capacity to know which; the anti-realist replies that a theory of meaning must be a theory of *understanding*, that understanding must be manifestable in use, and that a truth transcending all possible verification could play no role in any use we could manifest. Dummett was, in this, a proponent of verificationism—the doctrine Carnap and the logical positivists had held central to establishing the veracity of empirical phenomena—but purged of its phenomenalist accidents and relocated in the philosophy of mathematics, where the contrast between transcendent and constructive truth is sharpest.

Frege's own work already contained the seeds. The puzzle of the morning star and the evening star—how does one identify the same sense across different modes of presentation of one denotation?—forced the sense/reference distinction; and Frege's treatment of incomplete arithmetic expressions, functions awaiting saturation, provides a hint of the functional paradigm that Church's lambda calculus would make total: meaning as function, understanding as computation, the concept horse as a value of type $\iota \to o$.

3.2 Martin-Löf: proof as epistemology

The key disagreement between Martin-Löf and Dummett—small on the page, decisive in consequence—is whether the proof of a statement is an ontological or an epistemological claim, with Martin-Löf arguing for the latter and Dummett for the former. For Dummett, the proof-condition replaces the truth-condition as the content of the statement: what a proposition means *is* what would prove it. For Martin-Löf, the proposition's meaning is laid down by its introduction rules—what counts as a canonical verification—but the act of judgment, the demonstration, is an act of knowledge: evidence is the evidence *of a judging subject*, and the distinction between the truth of a proposition and the evidence of a judgment is the distinction between content and act. This is what Robert Harper calls "logic as if people matter": the witness is not a metaphor but a term; the assertion is not a shadow of the proposition but the primary phenomenon.

Transcendental Mathematics established the Husserlian genealogy of this position: Martin-Löf's meaning explanations, his distinctions between proposition and judgment, truth and evidence, canonical and non-canonical, all have explicit phenomenological antecedents. The present critique takes the genealogy as read and extracts the systematic point: the intuitionist option is not a restriction of classical logic but a *re-founding* of logic on the acts in which content is constituted. Per Martin-Löf, in his tracts on the philosophy of mathematics, accepts the synthetic a priori which Kant postulated as existent with geometry and arithmetic: pure geometry, arithmetic, and logic are modes of synthetic a priori knowledge, epistemologically—synthetic because their evidence is constructive, generated by acts of synthesis rather than extracted by analysis of concepts; a priori because the constructions are prior to and presupposed by every empirical employment.

3.3 From verification to inference

Yet verificationism, even in its refined Martin-Löf form, is not the last word. Through inferentialism, truths are related not to atomic verification but to *meaningfulness in context*—meaning-as-use in the full Brandomian sense, where the use that confers content is the use in the social game of giving and asking for reasons. The semantic anti-realist position, situated in an inferential network, sheds its residual atomism: the witness matters, but the witness is a move in a game whose rules are instituted between agents. David Corfield argues that the inferentialist school of Brandom is the most faithful way to read the philosophy of Homotopy Type Theory; the argument of Part II is that the converse also holds—HoTT is the most faithful way to formalize Brandom—and that the two readings jointly discharge the obligations that Dummett's challenge laid on any theory of meaning: manifestability (terms are programs; use is execution), molecularity (types are defined by their rules), and the rejection of holism-without-structure (the inferential network is a category, not a soup).

Chapter 4

Reclaiming the Kantian Synthetic A Priori

4.1 Martin-Löf's boxes

Set the three fourfold tables side by side. The Logical Positivists' boxes: analytic a priori = logic, arithmetic, pure geometry; synthetic a posteriori = applied geometry; the other two boxes crossed. The Quine–Putnam boxes: logic, arithmetic, geometry all synthetic a posteriori; analyticity itself under erasure. Martin-Löf's boxes: logic, arithmetic, and *pure geometry* in the **synthetic a priori**; applied geometry synthetic a posteriori; the analytic a priori reduced to the merely definitional. The history of twentieth-century philosophy of language is the passage from the first table to the second; the argument of this book is the passage to the third.

I will be arguing for the synthetic a priori in terms of *synthetic geometry*—and specifically Homotopy Type Theory, which sits at the nexus between theoretical computer science and algebraic topology. The word "synthetic" here carries both its Kantian and its mathematical senses, and the coincidence is not a pun but the thesis. Kantian synthesis: judgment that amplifies, that constructs its object in pure intuition rather than extracting predicates from concepts. Mathematical synthesis: axiomatic development from within, as Euclid develops geometry from postulates about construction, opposed to the analytic reduction of geometry to coordinates. HoTT is synthetic in both senses at once. Its types are not defined as sets of points with topological structure bolted on; they *behave* as spaces natively, and the homotopy theory unfolds from the inference rules themselves. The computational types can be transported directly into geometric terms through Steve Awodey's interpretation of the Univalence Axiom.

4.2 Awodey's interpretation

The interpretation of identity types as topological path spaces—which, as Awodey himself has noted, was his contribution, forming the basis of HoTT, while the Univalence Axiom itself was the work of Voevodsky—is the technical heart of the reclamation. The identity type $\mathrm{Id}_A(a, b)$ is interpreted as the space of paths from a to b in the space A; iterated identity types are homotopies, homotopies between homotopies, and so on into the ∞-groupoid structure of the type. The Univalence Axiom then asserts, for types A, B in a universe $\mathcal{U}$:

$$(A =_{\mathcal{U}} B) \;\simeq\; (A \simeq B),$$

identity is equivalent to equivalence. Identity itself becomes a structured, path-like, constructible relation—synthetic through and through. What Kant located in the pure intuition of space, HoTT locates in the formation, introduction, elimination, and computation rules of its types: an a priori that is generative, not merely regulative; constructive, not merely formal; and geometric without being spatial.

The Kantian question—how are synthetic a priori judgments possible?—thus receives a twenty-first-century answer: they are possible as the judgments of a dependent type theory with univalence, whose evidence is construction, whose objects are homotopy types, and whose identity relation is itself synthetic. The identity of types with topology extends the synthetic lineage of the a priori through a mathematical codification of intuitionism, which would otherwise remain mere mental constructions.

4.3 The relativization to come

But the reclamation cannot stop at Martin-Löf's boxes, for a reason Kant himself could not have foreseen and Michael Friedman has made central: the a priori is historically relativized. The a priori that made Newtonian mechanics possible—Euclidean geometry, the calculus, absolute simultaneity—was universally valid *during its epoch* and was relativized by the revolution that produced general relativity, whose own a priori (Riemannian geometry, the tensor calculus) had first to be philosophically refashioned from existent mathematics. If we are to follow Friedman's schema, as Corfield observes, then the period we are currently in is his *metascientific* one, where thinkers refashion mathematics and reformulate physical principles in a philosophically-minded way—think of Helmholtz, Mach, Clifford, Klein, Poincaré, . . . , Einstein; think now of the univalent foundations program itself. The synthetic a priori reclaimed in this chapter is therefore not Kant's fixed table of categories but a *relativized a priori*: universally valid within its period, constitutive of the very framework in which empirical questions can be posed, and yet revisable in revolution. Part V returns to the dynamics; Part IV supplies what Friedman's schema lacks, the mathematics of how one a priori is transported into another—reindexing along a context-shift, descent along a covering. The synthetic a priori is real; it is also fibered.

Remark 4.1. If time permits nothing else, let this chapter's claim be compared with the rationalist-nominalist reading of Jean Cavaillès, for whom mathematical experience is a determined concept and the development of theories proceeds by a necessity internal to the concepts themselves, indifferent to the constituting subject. The question the comparison poses—and Chapter 15 answers—is whether universality can be founded, as in Cavaillès, on the *generality* of ∞-groupoids in Homotopy Type Theory, i.e. globally; or whether it is necessary that metalinguistic locutions be situated in the *inferentialism* of Homotopy Type Theory, i.e. locally. Through the mathematical Absolute we arrive at both the contingent accidents and the mathematical necessity that allow for the development of new theorems; but the Absolute, we shall see, is a colimit never attained, a global section that need not exist.

Part II

The Analytic of Inference

Chapter 5

Inferentialism: Meaning as Use, Semantics as Ethics

5.1 Material inference and the game of reasons

The plan of this Part is to argue for the inferentialist account of meaning: that meaning is understood in terms of use, and that semantics is inherently an *ethical* question, tied to commitments to discursive norms. The claim is Brandom's, and its architecture bears rehearsing before it is formalized.

To grasp a concept is to master its inferential role: what follows from its application, what its application follows from, what is incompatible with it. The primary inferences are *material*: "Chicago is west of Pittsburgh, therefore Pittsburgh is east of Chicago" is good in virtue of the contents of "east" and "west," not in virtue of a suppressed conditional premise plus modus ponens. Formal validity is a derivative phenomenon: an inference is formally valid when it is materially good and remains good under all substitutions of its non-logical vocabulary. Brandom argues that most statements in a language need to be materially good and are *not* substitutable—not formally inferred. The logical vocabulary itself is expressive rather than foundational: conditionals make inferential commitments explicit as claimables; negation makes incompatibilities explicit; and, decisively for this book, the broadly metalinguistic locutions that Sellars identified—where Wittgenstein saw an unsystematic assortment of language games, Sellars saw systematic metalinguistic functions—are what Brandom's entire program undertakes to make explicit.

Semantics is answerable to pragmatics. Broadly speaking, semantics (content) is concerned with what is said, while pragmatics (use) is concerned with what one is *doing* in saying it. More precisely, in Negarestani's formulation of the Brandomian point: semantics asks what it is that one believes—or knows, claims—when one believes that p; pragmatics asks what it is that one must know *how to do* in order to count as producing a performance that expresses that content. Meaning is, at its most basic level, the justified use of mere expressions in social discursive linguistic practices; what counts as the justification of an expression is what counts as its meaning. And because justification is a matter of commitment and entitlement—of what one is responsible for, licensed to, and sanctioned over—the theory of meaning is a chapter of deontology. Semantics is ethics pursued by other means.

5.2 Singular terms and the two directions of inference

Two structural features of Brandom's account cry out for mathematical explication, and their explication is the bridge to homotopy type theory.

First, the distinction between **singular terms** and **predicates**. In *Making It Explicit* (Chapter 6), Brandom argues that any language capable of the full game of reasons must contain substitutable singular terms and directed inferences between predicates: singular terms are those expressions whose substitution-inferences are *symmetric* (if "Hesperus" can be substituted for "Phosphorus" salva veritate somewhere, it can be re-substituted back), while predicates stand in *asymmetric*, directed inferential relations ("is crimson" entails "is red" and not conversely). Brandom presents the argument with minimal formal treatment; and yet, as Corfield observes, this phenomenon makes very good sense in the context of HoTT, understood as arising from the different properties of *terms* and *types*. Indeed, the kind of category whose internal structure our type theory describes—an $(\infty, 1)$-topos—presents both aspects: the '1' corresponding to unidirected inference between types (morphisms between objects), and the '∞' referring to the reversible substitutability of terms (2-morphisms and higher between morphisms, all invertible).

Second, **semantic descent**. Against the Tarskian ascent to a metalanguage as the locus of judgment, Brandom descends to the bottom level, to the materially good inferences. Following Corfield's book on the philosophy of HoTT, it is agreeable that Homotopy Type Theory can correct the apparent formlessness of material inference through the terms and types of the $(\infty, 1)$-topos, where 2-morphisms, 3-morphisms, 4-morphisms, and so on are reversibly substitutable and therefore *formally* inferred. What Brandom could only gesture at—a formal treatment of material substitution—the higher dimensions supply: substitution salva veritate is a path; the network of substitutions is an ∞-groupoid; and the asymmetric backbone of material consequence is the 1-categorical direction that the groupoid dimensions leave invariant.

5.3 Corfield's proposal

The textual warrant for this bridge is worth quoting at length, for it fixed the program of this book. Corfield writes:

> I believe that there are grounds to hope that large portions of Brandom's program can be illuminated by type theory. Since Brandom's inferentialism derives in part from the constructive, proof-theoretic outlook of Gentzen, Prawitz, and Dummett, this might not be thought to be such a bold claim. However, his emphasis is generally on material inference, only some aspects of which he takes to be treatable as formal inference. For instance, the argument he gives in *Making*

> *It Explicit* for why we have substitutable singular terms and directed inferences between predicates is presented with a minimal formal treatment, and yet this phenomenon makes very good sense in the context of HoTT... On the other hand, his frequent use of formalisms... tells us that he recognizes the organizing power of logical languages... These additional functions of language pertain to the very framework that allows for our descriptive practices, and are what Brandom looks to make explicit in much of his work. My broader suggestion... is that we look to locate these locutions in features of our type theory, especially those modalities we will add.

And, in a private message that supplied this book its method:

> I think there's great potential in redoing Brandom with dependent type theory. Before we come to ask for reasons from someone, we must hold them to be meaningful, and part of the act of doing this is to parse what someone says. I think this can be understood in type-theoretic terms. We have the grammar correct if replacement of terms by others in the same type preserves *meaningfulness* (not meaning)... Vast tracts of Brandom's *Making It Explicit* could be given the type-theoretic treatment in terms of Intro and Elimination rules. There's also the structural inferentialism that HoTT provides.

Parsing precedes reasoning: before the game of giving and asking for reasons can be played with an utterance, the utterance must be held meaningful, and holding-meaningful is *typing*. Grammar is correct when replacement of terms by others of the same type preserves meaningfulness, not meaning: 'cup' and 'dog' can replace 'cat' in "I see a cat," but only 'dog' in "The cat is walking away"—the limited range of replacements is explained by a hierarchy of types and supertypes ($\mathsf{cat}, \mathsf{dog} : \mathsf{Animal}$; $\mathsf{cup} : \mathsf{Utensil}$; all three : Object), where the vector-space semantics of distributional NLP would instead embed everything in one metric space and read distinctness off distance. Chapter 19 returns to the comparison and, following Corfield's suggestion, takes *linear* HoTT as the best of both worlds: types for the grammar of meaningfulness, linear structure for the metric of similarity.

5.4 Intro and elimination rules as the form of explicitation

The type-theoretic treatment of *Making It Explicit* promised in Corfield's message can now be stated as a schema. Every Brandomian explicitating locution is a type former, and its paired introduction and elimination rules are the formal image of the two-sided pragmatics of commitment and entitlement:

- The **conditional** $A \to B$: introduction (abstraction) codifies entitlement-preserving inference as a claimable content; elimination (application, modus ponens) restores the inference to use. That the two are harmonious—that β-reduction normalizes—is the proof-theoretic form of the requirement that making an inference explicit adds no new commitment.

- **Conjunction** $A \times B$ and **disjunction** $A + B$: the additive bookkeeping of joint and alternative commitment.
- The **dependent product** $\prod_{x:A} B(x)$: universally binding commitment—what one is committed to for *every* instance a substitutable term may present; its introduction rule is the eigenvariable discipline of generality.
- The **dependent sum** $\sum_{x:A} B(x)$: existential commitment with witness—the anaphoric "there is something such that…" whose elimination rule is precisely the anaphoric discipline Brandom analyzes in his treatment of *tonk*-free existential idiom.
- The **identity type** $\mathrm{Id}_A(a, b)$: the substitution license itself, made claimable. Its elimination rule—path induction—says exactly that whatever is established of a transports along the license to b: substitution salva veritate as a first-class content.

The last item is the crux. In Brandom's hands, the substitution-inferential role of singular terms is a pragmatic status attributed by scorekeepers. In HoTT it is a type, with terms, with structure, with higher structure. The scorekeeper's bookkeeping is internalized as the ∞-groupoid of identifications. This is what it means to say that HoTT provides a *structural* inferentialism: the structure of the space of reasons is not described from outside but generated from within, by the rules.

Chapter 6

Interaction: The Bridge Between Syntax and Semantics

6.1 Negarestani's condition

The inferentialist account, left at Chapter 5, has a gap: it says that meaning is instituted by use in social practice, but not *how* mere syntax, mere parrotable noise, could come to bear normative statuses at all. Negarestani's formulation closes the gap by naming the missing ingredient:

> Syntax, under the right conditions, is indeed sufficient for semantics, and meaning can be conferred upon syntactic expression if such conditions are satisfied. These conditions are what the inferentialist theory of meaning, as a species of social-pragmatics or the use-theory of meaning, attempts to capture... While syntax by itself does not yield semantics, it does so when coupled with **interaction**. In this sense, pragmatics—at least in the sense defined by Brandom's inferentialist pragmatism—can be understood as a bridge between syntax and semantics.

The bridge between syntax and semantics is interaction. The formula compresses the whole analytic of this Part: semantics is not added to syntax from outside (by a model, a world, a designation relation) but precipitated within syntax by the discipline of exchange—assertion and challenge, commitment and assessment, move and countermove. What must be shown is that this precipitation can be characterized exactly; that "interaction" is not a metaphor but a mathematical structure.

6.2 Girard's ludics: interaction computationally encoded

Negarestani states that, through the logician Jean-Yves Girard's *ludics*, interactions between agents can be computationally encoded. Ludics arises from linear logic by taking the dialogical reading of proofs with full seriousness. Its primitive objects are *designs*: strategies specifying how a proponent responds, locus by locus, to the moves of an opponent. Two designs interact by playing against one another; the interaction either diverges or converges on the special action known as the *daimon*, the act of giving up. A *behaviour*—the ludic counterpart of a proposition—is a set of designs closed under biorthogonality: $\mathbf{G} = \mathbf{G}^{\perp\perp}$, where $\mathbf{G}^{\perp}$ is the set of all counter-designs whose interaction with every design in $\mathbf{G}$ converges. Meaning is thus defined *purely interactively*: a proposition is the equivalence class of all the ways of defending it, individuated by all the ways of attacking it. There is no external model; the semantics is generated by the orthogonality of conducts. The classical metalogical properties reappear as theorems about interaction—separation (designs are distinguished by the counter-designs that defeat them: an interactive Leibniz principle), and full completeness for the behaviours arising from logical formulas.

Ludics is therefore the existence proof for Negarestani's condition: syntax (designs are pure syntax, trees of loci and actions) coupled with interaction (orthogonality) yields semantics (behaviours) with no semantic primitives. What ludics does not supply is the *social* articulation: the differentiation of agential roles, the institution of the normative statuses that make an interaction a giving of reasons rather than a collision of automata. That articulation is the work of the next chapter's multi-agent circuit, and its formal home—this is the claim the Prospectus staked and this book redeems—is not bare linear logic but linear *dependent* homotopy type theory, which has the expressive power to encode predicates in the object language and, moreover, substitutable terms in the higher dimensions of the ∞-topos. The inferential network which institutes agential roles is expressed through singular and formal terms à la Brandom, vis-à-vis dlHoTT, thanks to David Corfield's suggestion.

6.3 The claim, globally and locally

The analytic can now be stated as a biconditional, readable in both directions. The global view: the tripartite historical process—metascience, revolution, philosophy, unfolding as a relativized a priori through the becoming of the philosophy of science (Part V)—leads to the following local view, and *vice versa*: language is co-extensive with sociality; general intelligence is realizable if and only if language is understood inferentially, in terms of semantic social norms defined interactionally between agents through a **minimal syntax** generating the whole system of rule-based roles. The vice versa matters: the local institution of meaning in interaction is not merely an application of the global metatheory but its generator—each scientific revolution is, locally, a renegotiation of inferential roles; each conversation is, globally, a miniature dynamics of reason. The fibrational semantics of Part IV will make this reciprocity exact: the global is what (sometimes) descends from compatible local data; the local is what the global (always) restricts to; and the asymmetry between the two directions—restriction always exists, gluing only sometimes—is the deepest structural fact about language that this critique has to report.

Chapter 7

Apperception, Judgment, Abduction: The Multi-Agent Circuit

7.1 The base case

The engine of the homotopic metatheory is a circuit of acts distributed over a plurality of agents. In the base case:

- **Agent 1 apperceives.** Apperception here is Kantian: the synthetic unity in which a manifold—for the scientific case, the physical world as pre-articulated by *existing mathematics*—is taken up into one consciousness of it. The apperception is contingent upon a *prior language* to encode it: a prior mathematical theory which is refashioned into physics, or, in the local linguistic case, a minimal syntax.

- **Agent 1 reasons, issuing a judgment.** Ratiocination upon the apperceived content forms a novel, *exact* judgment—in the scientific case, a physical theory; in the local case, an assertion with determinate inferential role.

- **Agent 2 understands and abducts.** The second agent does not merely receive; it infers backward to the best explication, purportedly rendering a past, unrigorous episteme from yesteryear into exactitude. But this explication is only partially ratiocination, because of the wellspring of yet-to-be-intelligible mathematical discoveries which will relativize philosophical thinking once again—call it Geist, the mathematical Absolute, or the unknown, whichever you prefer.

This process defines a **relativized a priori**. And note the loophole in the abductive mode of thinking: because abduction is ampliative and defeasible, the circuit does not close into a static equilibrium but unfolds *historically*, through the becoming of philosophy of science—through dialectics, or, if you prefer, generational research programmes. If one accepts the hypostasis of Ideas over matter, then ultimately the process approaches Absolute mind, the mathematical Absolute, world-historical spirit operating in the background processes. The critique neither asserts nor requires the hypostasis; it requires only the structure of the approach—and Part IV will show that the limit, like a global section, is regulative: it orients the gluing without being guaranteed to exist.

7.2 Generalization to n and m

Generalize now to cases n and m concurrently. In the background is the *inferential network* which institutes agential roles yet is itself defined through interaction between multi-agents: a generalization of the base case of agent 1 and agent 2 to agents n and m, all interrelated to each other. The point deserves emphasis because it inverts the usual order of explanation. The roles (apperceiver, judger, abducer; asserter, challenger, scorekeeper) are not prior to the network, as though fixed offices awaited their occupants; nor is the network prior to the roles, as though a structure could exist with nothing structured. Roles and network are co-instituted in interaction—exactly the situation category theory is built to describe, where objects are individuated by their morphisms and the morphisms presuppose their objects. Formally: the multi-agent system is a diagram of interactions; the agential roles are its limits and colimits; and the institution of roles-through-interaction is the statement that the diagram *generates* the category of roles as its completion.

The circuit also fixes the book's tripartite historical template, following Michael Friedman's *Dynamics of Reason* aside from Thomas Kuhn: (1) **metascience**, followed by (2) **revolution**, followed by post-hoc (3) **philosophy**. The first temporal period, metascience, is explicitly Kantian in terms of the (mathematical) presuppositions behind formulating physical theories—Kant had said geometry was a priori. This period is structured by a transcendental logic which necessitates a synthetic a priori of existent mathematical knowledge in order to philosophically refashion it into scientific theory. While universally valid during this period, this a priori is relativized as new scientific revolutions occur. The second period is scientific revolution itself. The final period is deliberation on the aftermath—Friedman's period of "philosophy." I claim that the overarching temporal teleology through the entirety of the three periods is explicitly Brandomian–Hegelian, while the first period is Kantian: apperception and judgment typify a formal Kantianism à la pure type theory, the bind between conceptual inference and the apperception of mathematical intuition; the recollective rationality that converts each relativized a priori into tradition is Hegel's, in Brandom's semantic reading. But instead of a neo-Kantian argument, the teleology here is Hegelian.

7.3 The diagrams after Lawvere

Diagrams of this dialectic are only possible after the logician William Lawvere's formalization of Hegel via categories (Chapter 17 supplies the mathematics; here we record the correspondences the Prospectus drew as a single table of triads, each resolving its opposition in a third term):

$$\begin{tikzcd} \textit{Metascience} \arrow[rr, dash] \arrow[dr, dash] & & \textit{Revolution} \arrow[dl, dash] \\ & \textit{Philosophy} & \end{tikzcd}$$

$$\begin{tikzcd} \textit{Type Theory} \arrow[rr, dash] \arrow[dr, dash] & & \textit{Category Theory} \arrow[dl, dash] \\ & \textit{Proof Theory} & \end{tikzcd}$$

$$\begin{tikzcd} \textit{Synthetic a priori} \arrow[rr, dash] \arrow[dr, dash] & & \textit{Pragmatic a priori} \arrow[dl, dash] \\ & \textit{Inferential roles} & \end{tikzcd}$$

$$\begin{tikzcd} \textit{linear HoTT} \arrow[rr, dash] \arrow[dr, dash] & & \textit{Quantum NLP} \arrow[dl, dash] \\ & \textit{AGI} & \end{tikzcd}$$

$$\begin{tikzcd} \textit{Kant} \arrow[rr, dash] \arrow[dr, dash] & & \textit{Conceptual Pragmatism (Lewis)} \arrow[dl, dash] \\ & \textit{Analytic Pragmatism (Hegel/Brandom)} & \end{tikzcd}$$

$$\begin{tikzcd} \textit{Affirmation} \arrow[rr, dash] \arrow[dr, dash] & & \textit{Negation} \arrow[dl, dash] \\ & \textit{Double Negation} & \end{tikzcd}$$

Each triangle is, formally, a unity of opposites resolved: the computational trinity (type theory, category theory, proof theory) at its center. That trinity is the subject of the next chapter.

Chapter 8

The Computational Trinity and the Functional Mind

8.1 The correspondence

The Curry–Howard correspondence states the identities on which the whole formal apparatus turns:

- Types correspond to logical formulas (propositions).
- Programs correspond to logical proofs.
- Evaluation corresponds to simplification of proofs.

Extended through Lambek to category theory, the correspondence becomes a trinity: **proofs = programs = morphisms**; propositions = types = objects; and the simply typed λ-calculus is the internal language of cartesian closed categories, so that $\beta\eta$-equivalence classes of terms just *are* morphisms. In the dependent setting the dictionary thickens into the table the HoTT Book displays and the Prospectus reproduced: dependent function types—Π-types—are equivalent to intuitionistic universal quantifiers of predicates and also to spaces of sections; dependent pair types—Σ-types—are equivalent to existential quantification and to total spaces above the base space; families of types are equivalent to predicates and to fibrations. A type becomes a proposition which is also a space. Terms $b(x)$ of families $B(x)$ become conditional proofs and also sections. A term of a type becomes a proof which is a point. Functions become intuitionistic implication which becomes a function space. Identity becomes equality becomes the path space A^I.

Types	**Logic**	**Sets**	**Homotopy**
A	proposition	set	space
$a : A$	proof	element	point
$B(x)$	predicate	family of sets	fibration
$b(x) : B(x)$	conditional proof	family of elements	section
$A + B$	$A \vee B$	disjoint union	coproduct
$A \times B$	$A \wedge B$	set of pairs	product space
$A \to B$	$A \Rightarrow B$	set of functions	function space
$\sum_{x:A} B(x)$	$\exists_{x:A} B(x)$	disjoint sum	total space
$\prod_{x:A} B(x)$	$\forall_{x:A} B(x)$	product	space of sections
Id_A	equality $=$	$\{(x,x) \mid x \in A\}$	path space A^I

8.2 Mind as what it does

The philosophical yield of the trinity, following Negarestani's program in *Intelligence and Spirit*: intelligence needs to be made intelligible; the mind must be approached *functionally*, in terms of what it does—behaviors and functions—and this functional description must exist within a "history of histories," which is to say, within spirit. The trinity supplies the functionalism with teeth. Reason is defined functionally as what it does by what it *computes*: proofs are programs. If mind is understood computational-functionally, then self-consciousness is co-extensive with public language through the social interactions of multi-agents, where sociality inherently realizes general intelligence, since natural semantics are generated by syntax coupled with interaction. Mind becomes only what it does functionally and therefore what is computable; spirit becomes the context-dependent codifications of public language in an inferential network of ∞-topoi.

Negarestani's own Hegel-inspired formalism—adopting Lawvere's functorial semantics—renders the formal order of self-consciousness, the logical $I = I$, as an identity of maps: treating I and I^* as objects with their respective identity maps, $I = I^*$ is really a map $f : I \to I^*$. Here I stands for formal self-consciousness and I^* for concrete self-consciousness: the claim that mind does not exhaust the real—that there is, in Negarestani's phrase, "reality in excess of the self or mind"—and that the freedom of self-consciousness is therefore a freedom exercised entirely *toward other self-consciousnesses*, never in view of some placeless, timeless standpoint. From the identity-map structure it then follows that $If = f = fI^*$, which Negarestani glosses as the realization of the formal condition of intelligence (I) *as* intelligence (I^*) in the work of rendering reality intelligible—an expansion of the intelligible that is at once an expansion of the real. Behind the formalism stands Hegel's dictum, which Negarestani quotes: "Self-consciousness attains its satisfaction only in another self-consciousness" (*Phenomenology of Spirit*, §175). The categorical rendering is not decoration; it is the precise statement that selfhood is a morphism, not an object—an insight the fibrational semantics will generalize: every "I" is indexed by a context, and its identity across contexts is a transport, not a substance.

8.3 Semantic ascent through the object language

One consequence for the theory of language should be fixed before the machinery of Part III is built. The semantic ascent from a language to a metalanguage—expressing, say, *redness*—exhibits a double character: substitutable, because in order to define red linguistically one needs ever more concepts to express it, black-boxed functionally, such as ROYGBIV; yet *singular* in term, because of the base-language understanding of the datum of red *qua* red. This double character can be instituted through quantum natural language processing, which describes sentence structure categorically with string diagrams defining a network of words, and this categorical approach can moreover be made equivalent, as Corfield suggests, to linear dependent Homotopy Type Theory. The traditional metalogical encodings of Tarski are relatively comparable through universes—the type of types, the type of types of types—found in the inherent inferentialism of UF/HoTT via ∞-groupoids. Both the definitional ascent (red analyzed through ROYGBIV) and the indexical descent (red as this very datum) are internal movements of one structure. The Tarskian tower, the Brandomian descent, the Findlayan ascent: Chapter 12 assembles them into a single theorem.

Part III

The Transcendental Machinery

Chapter 9

Categorical Logic: Theories as Categories, Operations as Adjoints

9.1 Two slogans

The mathematics on which the critique runs was developed at length in my *Mathematics of Neo-Rationalism*; this chapter and the next compress that development into what the argument strictly requires, so that the book stands on its own. Everything descends from two slogans, both Lawvere's.

Slogan 9.1. Theories are structured categories; models are structure-preserving functors; homomorphisms of models are natural transformations.

Slogan 9.2. The logical operations are adjoints.

The first slogan is *functorial semantics*. Instead of treating a theory as a bundle of uninterpreted syntax and a model as a set-based structure satisfying axioms, one distills the theory into a category $\mathcal{C}_{\mathbb{T}}$ carrying exactly the categorical structure needed to interpret the theory's logic—finite products for equational logic, finite limits for cartesian logic, and so on—and recovers models as functors $\mathcal{C}_{\mathbb{T}} \to \mathcal{C}$ preserving that structure. Syntax and semantics become objects of the same kind, related by functors; the classical metatheorems—soundness, completeness, the existence of free models—become statements about universal properties. The syntactic category $\mathcal{C}_{\mathbb{T}}$ has contexts $[x_1, \ldots, x_n]$ as objects and provable-equality classes of term-tuples as morphisms; it carries a tautological universal model U whose satisfaction relation *is* provability:

$$U \models s = t \quad \Longleftrightarrow \quad \mathbb{T} \vdash s = t.$$

One may think of $\mathcal{C}_{\mathbb{T}}$ as the Lindenbaum–Tarski category of $\mathbb{T}$: the syntax-invariant gadget presented by the theory, remembering all derived operations and all provable equations while forgetting which were basic. For any finite-product category $\mathcal{C}$, evaluation at U gives an equivalence

$$\mathrm{Hom}_{\mathrm{FP}}(\mathcal{C}_{\mathbb{T}}, \mathcal{C}) \;\simeq\; \mathrm{Mod}(\mathbb{T}, \mathcal{C}),$$

natural in $\mathcal{C}$. Classical **Set**-valued semantics almost never provides a single logically generic model; allowing models in arbitrary structured categories does.

Behind the equivalence stands Lawvere's duality, $\mathbf{Syntax} \simeq \mathbf{Semantics}^{\mathrm{op}}$: for any algebraic theory, the syntactic category is equivalent to the opposite of the category of finitely generated free models. An invariant copy of the syntax of $\mathbb{T}$ sits inside the opposite of its category of models. Philosophically, this is the first exact form of the book's anti-dualism about language: the sign-system and its interpretations are not two realms needing a bridge but one structure read in two directions. It is also the template for Halimi's fibrational semantics—syntax over semantics, base and fibre—to which Part IV turns.

9.2 Predicates as subobjects; substitution as pullback

For predicate logic, the semantic home of a predicate on an object A is the poset $\mathrm{Sub}(A)$ of subobjects of A: equivalence classes of monomorphisms into A, ordered by factorization. When $\mathcal{C}$ has pullbacks, Sub becomes a functor $\mathrm{Sub} : \mathcal{C}^{\mathrm{op}} \to \mathbf{Pos}$, sending $f : A \to B$ to the monotone map $f^* : \mathrm{Sub}(B) \to \mathrm{Sub}(A)$ given by pullback of monos along f. Formulas-in-context $\Gamma \mid \varphi$ are interpreted as subobjects $[\![\Gamma \mid \varphi]\!] \in \mathrm{Sub}([\![\Gamma]\!])$; sequents $\Gamma \mid \varphi \vdash \psi$ as containments $[\![\varphi]\!] \leq [\![\psi]\!]$; and—the structural key—*substitution of a term into a formula is pullback*:

$$[\![\Gamma \mid \varphi[t/y]]\!] \;=\; [\![t]\!]^* \, [\![y{:}B \mid \varphi]\!].$$

Every operation used to interpret a connective must be stable under pullback for the semantics to be well defined; stability is the categorical shadow of the syntactic fact that substitution commutes with the connectives.

9.3 Quantifiers as adjoints

Lawvere's second great observation: for $f : A \to B$, quantification along f is adjoint to substitution along f,

$$\exists_f \;\dashv\; f^* \;\dashv\; \forall_f,$$

with the adjunction laws

$$\exists_f U \leq V \iff U \leq f^* V, \qquad f^* V \leq U \iff V \leq \forall_f U$$

being precisely the two-way natural-deduction rules for the quantifiers, applied to the projection $\pi : [\![\Gamma]\!] \times [\![A]\!] \to [\![\Gamma]\!]$. The eigenvariable side condition of traditional proof theory is absorbed into the statement that one is transposing across an adjunction. Universal and existential quantifiers are adjoints to substitution—the epiphany, as the Prospectus recorded, that paved the way for a nuanced understanding of the comprehension axiom as an adjunction, and that Lawvere reached from an interest in the foundations of physics.

The same pattern delivers the whole doctrinal hierarchy. Reading downward, each fragment adds connectives and each class of categories adds the corresponding structure: algebraic logic ($=$ only) in finite-product categories; cartesian logic ($=, \top, \wedge$) in finite-limit categories; regular logic (adding $\exists$) in regular categories; coherent logic (adding $\bot, \vee$) in coherent categories; full first-order intuitionistic logic (adding $\Rightarrow, \neg, \forall$) in Heyting categories; higher-order logic (adding powersets) in elementary toposes. Each fragment is separated from the last by a preservation theorem—model classes of cartesian theories are closed under submodels, so the monoid-with-inverses theory ($\exists$ essential) is not cartesian; regular model classes are closed under products, so fields ($\vee$ essential) are not regular—and each two-way rule of the calculus is the syntactic trace of an adjunction. Conjunction and implication are related by the Heyting adjunction $(-) \wedge \varphi \dashv \varphi \Rightarrow (-)$; equality arises from the adjunction generated by the diagonal; comprehension, and even the powerset, arise from adjunctions. The second slogan is thus not a mnemonic but a metaphysical result: **the logical constants are not conventions (Carnap) nor tautology-generators (Wittgenstein) but universal structures**—adjoints, hence unique up to canonical isomorphism, hence precisely as objective as the mathematics of structure itself. This is the critique's answer to the conventionalist reading of logic: tolerance about frameworks, necessity within them; the necessity being the uniqueness of adjoints.

9.4 Curry–Howard–Lambek, proof-relevantly

Part I's model-theoretic reading records provability in hom-*posets*; the proof-theoretic reading records the proofs themselves in hom-*sets*. Lambek's theorem: the simply typed λ-calculus is the internal language of cartesian closed categories—every typed λ-theory presents a CCC whose objects are types and whose morphisms $A \to B$ are terms $x{:}A \vdash t : B$ modulo provable $\beta\eta$-equality, and this is half of an equivalence between λ-theories and CCCs. The doctrinal tower has a parallel proof-relevant tower: monoidal categories interpret linear and ordered calculi; CCCs the simply typed λ-calculus; locally cartesian closed categories dependent type theory; elementary toposes full intuitionistic higher-order logic; and—the rung this book adds to the ladder—$(\infty, 1)$-toposes interpret homotopy type theory with univalent universes. The two towers meet in the higher reaches: an elementary topos is simultaneously the model-theoretic environment for higher-order theories and, via its internal type theory, a proof-theoretic object.

For the philosophy of language the passage from poset to set to ∞-groupoid enrichment is the formal registration of three successive theses about reasons: that there is a fact of consequence (posets); that *how* one establishes a consequence matters—proofs are contentful, distinct, composable (sets); and that identifications between establishings themselves carry structure, all the way up (∞-groupoids). Brandom's material inference lives at the first level; his substitution-inferential analysis of singular terms demands the third.

Chapter 10

The Triumvirate of Topos Theory: Grothendieck, Mac Lane, and Lawvere

10.1 Uses and abuses of the history

Before the machinery of toposes is deployed, its provenance must be secured against a persistent misreading—for the misreading, if allowed to stand, would undermine the critique's use of the machinery. In his essay "The Uses and Abuses of the History of Topos Theory," Colin McLarty critically examines the historical development and understanding of topos theory and category theory, and his corrective is programmatic for this book. Topos theory has often been misunderstood as a mere generalization of set theory; McLarty is blunt: the view that toposes originated as generalized set theory is "a figment of set theoretically educated common sense" (McLarty 1990). The misconception hinders the proper understanding of category theory and of the categorical foundations of mathematics, for it was problems in geometry, topology, and related algebra that were the actual driving forces behind the development of categories and toposes. Category theory arose from a complicated array of practical problems in topology; it did not emerge from a contemplation of the timeless nature of mathematics but from the core of mathematical practice, offering foundational insights from within that practice. Topos theory arose from Grothendieck's work in geometry, Tierney's interest in topology, and Lawvere's interest in the foundations of physics.

The main point of categorical thinking, McLarty insists, is to "let arrows reveal structure"—an approach that defines objects only up to isomorphism (McLarty 1990). Set theory as practiced today is unique in *not* generally defining its objects up to isomorphism, which is precisely what makes it a challenging, and misleading, example for category theory. Students influenced by set-theoretic thinking misunderstand the essence of topos theory: they see objects as "structured sets" and arrows as "structure-preserving functions"—a misunderstanding that stems from a false history obscuring the real novelty of category theory. For the critique, the stakes of this historiography are transcendental rather than antiquarian. If toposes were generalized set theory, the fibrational semantics of Part IV would inherit set theory's constant, membership-based picture of content, and the entire apparatus would be one more model theory. Because toposes are instead the crystallization of *local-to-global* mathematical practice—cohomology, covering, continuous variation—the apparatus carries exactly the conceptual grain that a theory of contextual meaning requires. The tools for a theory of local meaning were built by people solving local problems; the false history conceals precisely this fitness.

10.2 Grothendieck: the geometric visionary

Grothendieck's introduction of toposes as generalized spaces was revolutionary in the strict sense: it changed what "space" means. Grothendieck's toposes generalize topological spaces so that algebraic and topological structure fuse in a single object—categories of sheaves on sites, where the site's covering families replace the open sets of classical topology, and where a "space" may have no points at all yet a rich cohomology. This was not generalization for its own sake. The étale topos of a scheme delivers the cohomology that the Zariski topology is too coarse to see; the Weil conjectures fell to it. And Grothendieck found—not by design, but in the course of the work—that many properties of the category of sets *lift* to toposes: exponentials, a subobject classifier, an internal logic. The discovery is the historical root of the transit theorem of Chapter 12: the set-like behavior of toposes is a *theorem about geometry*, not a definition imported from foundations. That logic emerges from geometry, rather than being imposed on it, is the deepest historical warrant for this book's claim that the logic of language emerges from the geometry of its contexts.

10.3 Mac Lane: laying the cornerstones

Saunders Mac Lane, collaborating with Eilenberg, pioneered the introduction of categories, functors, and natural transformations—famously inventing categories in order to define functors, and functors in order to define natural transformations, the concept the topological applications actually required. Mac Lane's foundational preoccupations led him into topos theory, and his Chicago lectures carried its core to a generation of students; his insistence on exactness clarified how toposes bind logic to geometry. In the economy of the present book, Mac Lane stands for a discipline the critique means to honor: the refusal to let profundity excuse imprecision. The dialectical readings of Part V—Lawvere's Hegel, Aufhebung as modal containment—are admissible in this book only because they pass through definitions of Mac Lane's standard: adjunctions with their triangle identities, not analogies with their atmospherics.

10.4 Lawvere: foundations as physics

F. W. Lawvere's foray into topos theory was underpinned by his interest in the foundations of physics—continuum mechanics above all—and by the ambition to establish *functorial* foundations for mathematics: not isolated categories but the traffic of functors between them, an emphasis that led him to probe the category of categories as a foundation. Three of his insights organize everything this book takes from him. First, the discovery already made central in Chapter 9: that the universal and existential quantifiers are adjoints to substitution—an epiphany that paved the way for understanding the comprehension axiom, too, as an adjunction, and ultimately for the doctrine that all logical constants are universal structures. Second, with Tierney, the elementary axiomatization of toposes—finite limits, exponentials, subobject classifier—which cut the concept loose from Grothendieck's sites and revealed the topos as a self-standing universe of variable sets; their work on partial map classifiers belongs to the same reconstruction. Third, the perspective that objects in a topos are *continuously variable sets*, juxtaposed against classical set theory's constant sets: a fresh lens on the theory of variable structure which is, in embryo, the entire fibrational semantics of Part IV—content as variation over a base, constancy as the degenerate case.

The synergy of the three is the historical instance of the book's systematic claim. Grothendieck's geometric insight laid the groundwork; Mac Lane's foundational prowess disciplined it; Lawvere's logical perspective disclosed what it meant. No one of the three positions subsumes the others; their collective legacy is an amalgamation, not an abstraction—three fibres of practice, glued on their overlaps into a subject none of them contained. The history of topos theory is itself a descent datum, and its lesson generalizes: brilliant minds converging on a singular idea do not converge *in* a master perspective, but along interfaces.

Chapter 11

Toposes and the Internal Language

11.1 The elementary axioms

Definition 11.1. An **elementary topos** is a category $\mathcal{E}$ with finite limits, which is cartesian closed, and which has a subobject classifier: an object Ω with a point true : $1 \to \Omega$ such that for every mono $m : S \rightarrowtail A$ there is a unique characteristic map $\chi_m : A \to \Omega$ making S the pullback of true along χ_m.

The definition is short; its consequences are an entire universe. Every topos has finite colimits, power objects Ω^A, image factorizations; its subobject posets are Heyting algebras, with the quantifier adjoints $\exists_f \dashv f^* \dashv \forall_f$ existing for every f; it interprets full intuitionistic higher-order logic. **Set** is a topos ($\Omega = \{0, 1\}$); G-sets form a topos; presheaves $\mathbf{Set}^{\mathcal{C}^{\mathrm{op}}}$ form a topos for any small $\mathcal{C}$, with $\Omega(C)$ the set of sieves on C; sheaves on any site form a topos—the Grothendieck toposes, which are the generalized spaces from which the subject arose.

The historiographical warrant for reading this definition geometrically rather than set-theoretically was established in the preceding chapter: toposes are the crystallization of local-to-global practice, and their internal set-like behavior is a discovered theorem, not an imported foundation.

11.2 Mitchell–Bénabou and Kripke–Joyal

Every topos speaks. The **Mitchell–Bénabou language** of $\mathcal{E}$ has a type for each object, terms for morphisms, and formulas as terms of type Ω; the connectives are the Heyting operations on Ω, the quantifiers the adjoints. Its semantics is **Kripke–Joyal forcing**: for a formula $\varphi(x)$ with x of type A and a generalized element $\alpha : C \to A$ (a *stage* C of definition), one defines $C \Vdash \varphi(\alpha)$ by structural recursion—

- $C \Vdash \varphi \wedge \psi$ iff $C \Vdash \varphi$ and $C \Vdash \psi$;
- $C \Vdash \varphi \vee \psi$ iff there is a covering (epimorphic) family $\{c_i : C_i \to C\}$ with each C_i forcing a disjunct;

- $C \Vdash \varphi \Rightarrow \psi$ iff for every $c : D \to C$, $D \Vdash \varphi(\alpha c)$ implies $D \Vdash \psi(\alpha c)$;
- $C \Vdash \exists y\, \varphi$ iff some cover of C supplies witnesses locally;
- $C \Vdash \forall y\, \varphi$ iff at every later stage, every element satisfies φ.

Truth in a topos is thus *local truth*: stage-relative, cover-mediated, monotone under restriction. Existence is local existence—a witness need only exist on a cover; disjunction is local decision. The generalized-element rules of ordinary categorical logic are the degenerate, global fragment of this semantics.

The philosophical cash value is immediate and large. Kripke–Joyal is the mathematically mature form of the "meaning is use in context" doctrine: the content of a claim is the system of its forcings across stages, and the logic this induces is intuitionistic not by fiat but by the geometry of covering. And it is the first appearance, inside the machinery itself, of the critique's negative thesis: a statement can be forced on every element of a cover—locally true everywhere—without being forced globally, exactly as local sections can fail to glue. The internal language of a topos is a language whose semantics is *already* fibrational.

11.3 Geometric morphisms: translations between universes

A **geometric morphism** $f : \mathcal{F} \to \mathcal{E}$ is an adjoint pair $f^* \dashv f_*$ with f^* left exact: the topos-theoretic image of a continuous map, and the correct notion of translation between mathematical universes. Toposes are custom-tailored universes for mathematics—a topos for the smooth world (synthetic differential geometry), for the computable world (the effective topos), for each forcing extension of set theory—and geometric morphisms are the interfaces between them. When Part VI argues that the response to semantic locality is the construction of interfaces, this is the mathematics being invoked: not vague "dialogue between frameworks" but adjoint pairs, with inverse image preserving the geometric fragment $(\top, \wedge, \exists, \vee)$—precisely the fragment whose truth is preserved under translation—while the non-geometric connectives $(\Rightarrow, \forall, \neg)$ may shift. The critique of pure language here acquires its table of categories: what is invariant under interface (geometric logic) and what is framework-relative (the rest). That the classically contentious operators are exactly the non-geometric ones is, to my knowledge, the deepest formal fact ever discovered about the limits of translation.

Chapter 12

Univalence and the Identity of Identity

12.1 Identity types and path induction

Dependent type theory's treatment of equality is its philosophical signature. For $a, b : A$ the identity type $\mathrm{Id}_A(a, b)$ is a type; its terms are identifications; and its elimination rule—path induction—says that to prove anything about all identifications it suffices to prove it for reflexivity. Nothing in the rules forces identifications to be unique: $\mathrm{Id}_A(a, b)$ may have many terms, and identifications between identifications form $\mathrm{Id}_{\mathrm{Id}_A(a,b)}(p, q)$, and so on. Every type is thereby an ∞-groupoid; every function is automatically a functor respecting all this structure; transport carries terms of $B(a)$ to terms of $B(b)$ along any $p : \mathrm{Id}_A(a, b)$.

Under Awodey's interpretation—identity types as topological path spaces—the tower of identity types is the homotopy structure of a space, and type theory becomes synthetic homotopy theory. *Transcendental Mathematics* read the multiplicity of identifications phenomenologically, as the multiplicity of syntheses of identification; the present critique reads it *linguistically*: an identification is a substitution license (Chapter 5), and the higher tower is the structure of the licenses' own inter-substitutability. Brandom's symmetric substitution-inferences commit him, whether he says so or not, to a groupoid of substitutions; HoTT says: all the way up.

12.2 Univalence

Definition 12.1. For types $A, B : \mathcal{U}$, univalence asserts that the canonical map $(A =_{\mathcal{U}} B) \to (A \simeq B)$ is an equivalence:

$$(A =_{\mathcal{U}} B) \;\simeq\; (A \simeq B).$$

Identity of types is equivalence of types. Isomorphic structures are identical—not by abuse of notation but by axiom; the structuralist practice of mathematics is made law. Voevodsky's axiom (on Awodey's basis) has three consequences the critique needs.

First, it completes the reclamation of the synthetic a priori (Chapter 4): even the logical relation of identity is constructive, path-like, synthetic.

Second, it grounds the *invariance of meaning under equivalent expression*. If linguistic contents are typed, and types are identified exactly up to equivalence, then content is individuated at precisely the grain that inferential role requires: no finer (notational variants are identified) and no coarser (inequivalent types remain distinct). Univalence is the formal principle behind the intuition that a translation preserving all inferential structure preserves meaning *simpliciter*—there is no further fact.

Third, it yields structural inferentialism in Corfield's sense. In an $(\infty, 1)$-topos, the universe is an object classifier: a map $A \to \mathcal{U}$ *is* a family, a predicate, a fibration; univalence says the classifier classifies up to equivalence. The Fregean apparatus of concept and object, the Tarskian apparatus of satisfaction, are absorbed into the geometry of classifying maps. Semantics does not sit above the language; it is a region of the language—which is exactly the situation Theorem 2.1 requires.

Chapter 13

Universes: The Transit Between Object Language and Metalanguage

13.1 The theorem

We can now redeem the promissory note of Chapter 2.

Theorem 13.1 (Transit, precise form)**.** *Let $\mathcal{E}$ be an $(\infty, 1)$-topos with its hierarchy of univalent object classifiers $\mathcal{U}_0 : \mathcal{U}_1 : \mathcal{U}_2 : \cdots$. Then:*

1. *(**Ascent**) Every family of types over a context—every predicate, every "metalinguistic" classification of expressions—is represented by a map into some $\mathcal{U}_i$: semantic ascent is naming in a universe, and it is available uniformly at every level, since each $\mathcal{U}_i$ is itself a type named in $\mathcal{U}_{i+1}$.*
2. *(**Descent**) Every map into $\mathcal{U}_i$ instantiates: pulling back the universal family converts the name back into the family named. Ascent followed by descent is the identity up to equivalence; by univalence, descent followed by ascent is too.*
3. *(**No totalization**) There is no universe of all universes: $\mathcal{U} : \mathcal{U}$ is inconsistent (Girard's paradox), so the transit never closes into a single all-encompassing language.*

Apart from the original morphisms and objects directed at the lowest level, there exists a transit up and down, and vice versa, between object language and metalanguage through $(\infty, 1)$-topoi.

In Agda—the proof assistant for Univalent Foundations—the hierarchy is concrete: the type system includes an infinite hierarchy of universes $\mathsf{Set}_i : \mathsf{Set}_{i+1}$, enabling quantification over arbitrary types without the inconsistency that follows from $\mathsf{Set} : \mathsf{Set}$. A type of types, or a type of types of types, corresponds to varying universe levels.

13.2 What the theorem settles

The theorem adjudicates the three disputes of Part I at a stroke.

Tarski. The undefinability of truth becomes clause (3): no single language totalizes its own semantics. But the hierarchy it generates is internal (clauses 1–2), not an external regress: the ocean of metalanguages is the universe hierarchy, navigable in both directions by terms of the theory itself. Model theory is what the transit looks like when truncated to sets.

Findlay versus Brandom. Semantic ascent (Findlay, Tarski) and semantic descent (Brandom) are the two directions of clause (2), and they are inverse up to equivalence. Neither has priority; the metalanguage has no authority the object language lacks, because the metalanguage is an object of the object language one level up. Brandom's descent to materially good inferences is vindicated as the direction of *instantiation*; Findlay's ascent is vindicated as the direction of *explicitation*; the dialectic between them is the univalent round-trip.

Carnap. The principle of tolerance survives as the plurality of universes and, more radically, of toposes; but the "external questions" Carnap dismissed as meaningless are re-admitted as internal questions one level up—and the questions with no level to be asked at (the universe of all universes, the truth of all truths) are diagnosed, not dismissed: they are the paralogisms of semantic reason, demands for the unconditioned that the structure of conditioning itself rules out. Kant's dialectic, replayed in type theory: the unconditioned is not meaningless but *regulative*—each $\mathcal{U}_i$ points to $\mathcal{U}_{i+1}$, the ascent never completes, and the totality of the ascent is an Idea in the Kantian sense, orienting inquiry without being an object of it.

This is the hinge of the book. Everything before it argued that meaning is inferential, interactional, and type-theoretic; everything after it argues that meaning so understood is *fibered*—indexed, transported, amalgamated—and that the regulative Idea of its totality is precisely a global section that need not exist.

Part IV

Fibrational Semantics

Chapter 14

Contexts as Fibres: Context-Dependence as Context-Shift-Dependence

14.1 Against the functional scheme

The dominant Anglophone tradition in the philosophy of meaning—running from Frege through Carnap, Montague, and Kaplan—treats linguistic meaning as a *function*: an expression's content in a given context is the value of its meaning evaluated at that context. Kaplan's character, Montague's intension, the possible-worlds apparatus generally: all instances of one scheme, content $= f(\text{context})$. Brice Halimi's reconstruction of context dependence through Grothendieck's theory of fibered categories and descent—the mathematical backbone this Part adopts and radicalizes—argues that the functional scheme cannot accommodate the actual phenomenology of context-dependent expression. Meaning is not an invariant obtained by abstraction; it does not sit behind its contextual instances as their common core, waiting to be evaluated. The alternative is a *functorial* scheme, and its watchword is the slogan that governs everything in this Part:

Slogan 14.1. Context-dependence = context-shift-dependence.

The content of an expression in a given context is not computed from scratch by evaluating a function at that context. It is the **transposition** of the content of that expression in some prior context of reference, transported along the context-shift that relates the two contexts. Contents do not answer to contexts one at a time; they answer to the *comparisons* between contexts. The semantic content of an utterance is determined not context by context, but from context to context.

14.2 The fibred category of contents

Construction 14.2. Let $\mathcal{C}$ be a category whose objects are **contexts** of utterance and whose morphisms $u : X' \to X$ are **context-shifts**: comparisons in which the current context X' is referred to a prior context of reference X. Let $\mathcal{S}$ be a category whose objects are **semantic contents in context**—each living above the particular context whose content it is—and whose morphisms are content-morphisms compatible with context-shifts. A functor

$$p : \mathcal{S} \longrightarrow \mathcal{C}$$

projects each content onto the context in which it lives. The **fibre** $\mathcal{S}_X := p^{-1}(X)$ above a context X is the category of all contents living in that context.

The crucial structural condition on p—what makes it a *fibration*—is the existence of **Cartesian lifts**. Given a context-shift $u : X' \to X$ and a content e above X, a Cartesian lift of u to e is a morphism $\hat{u} : u^*(e) \to e$ in $\mathcal{S}$, projecting onto u, with the universal property: whenever $\varphi : f \to e$ in $\mathcal{S}$ projects to a composite $p(\varphi) = u \circ v$, there is a unique $\psi : f \to u^*(e)$ above v with $\varphi = \hat{u} \circ \psi$.

$$\begin{tikzcd}
f \arrow[rr, "\varphi"] \arrow[dr, dashed, "\exists!\,\psi"'] & & e \\
& u^*(e) \arrow[ur, "\hat{u}"'] &
\end{tikzcd}$$

$$\begin{tikzcd}
p(f) \arrow[rr, "p(\varphi)"] \arrow[dr, "v"'] & & X \\
& X' \arrow[ur, "u"'] &
\end{tikzcd}$$

The upper triangle lives in $\mathcal{S}$; the lower in $\mathcal{C}$; p carries the one to the other; and the universal property asserts that the upper triangle is determined by the lower plus the choice of e. The object $u^*(e)$ is the transposition of e along u: its content as it appears once the context has been shifted. A choice of Cartesian lift for every pair (u, e) yields a **reindexing functor** $u^* : \mathcal{S}_X \to \mathcal{S}_{X'}$ for each shift, with coherence isomorphisms $(\mathrm{id}_X)^* \cong \mathrm{id}_{\mathcal{S}_X}$ and $(u \circ v)^* \cong v^* \circ u^*$, organizing the whole as an indexed category $\mathcal{C}^{\mathrm{op}} \to \mathbf{Cat}$.

A context-dependent semantic content thus does not have the form $f(x)$, as the functional scheme would have it, but the form $u^*(e)$.

14.3 "Red," and the unity of the machinery

Halimi's illustration. In a context X_1 of passive observation, the predicate "red" describes an object whose exterior surface is mostly red—a red bird. In a context X_2 of active production, the same predicate describes an object capable of *producing* a red surface—a red pen, whose casing may be black. The shift in usage is not the application of a single function $[\![\text{red}]\!]$ to two arguments; it is the transposition of the observational content e_1 along a context-shift $u : X_2 \to X_1$:

$$e_2 = u^*(e_1).$$

The two uses are not arbitrary; they are linked by a definite functorial transit.

Now observe what the reader of Part III already knows: **this machinery is not new to the theory; it is the theory**. Substitution in categorical logic is pullback—reindexing along a morphism of contexts (Chapter 9). Dependent types are families—fibrations, with Σ and Π the left and right adjoints to reindexing (Chapter 8). Kripke–Joyal truth is stage-relative and monotone under restriction along shifts of stage (Chapter 10). The universe hierarchy classifies fibrations (Chapter 12). Halimi's proposal, transplanted into the homotopic metatheory, is therefore not an add-on but a *recognition*: the semantics of natural language has the same shape as the semantics of dependent type theory, because both are the mathematics of indexed content. The type-theoretic contexts Γ of Part III and the pragmatic contexts X of this chapter differ in grain, not in kind; the Grothendieck construction converts either presentation into the other. And the Prospectus's remaining unformalized notion—the *prior language* upon which every apperception depends (Chapter 7)—is formalized here: the prior language is the context of reference X, and the novelty of a judgment is its Cartesian character, its universal property among all contents that factor through the shift.

Also formalized is the earlier claim (Chapter 8) that every "I" is indexed: Negarestani's $f : I \to I^*$ is a morphism of the base category of stances, and self-consciousness-for-another is reindexing of self-content along the shift to the other's context. The multi-agent circuit of Part II is a diagram in $\mathcal{C}$; its inferential network is the fibration above it.

Chapter 15

Descent: Amalgamation, Not Abstraction

15.1 Descent data

The second pillar concerns the construction of general meaning itself. The traditional picture: meaning is what is *abstracted* from the contextual contents of an expression—their common core, their invariant form. Halimi rejects the picture, and the mathematics of the rejection is Grothendieck's descent theory.

Definition 15.1. Given a context-shift $u : X' \to X$, the pullback $X' \times_X X'$ with its two projections $p_1, p_2 : X' \times_X X' \to X'$ encodes how X' overlaps itself when seen from above X. A **descent datum** above X' with respect to u is a pair $(f', \varphi_{f'})$: an object $f' \in \mathcal{S}_{X'}$ together with an isomorphism

$$\varphi_{f'} : p_1^*(f') \xrightarrow{\sim} p_2^*(f') \quad \text{in } \mathcal{S}_{X' \times_X X'},$$

satisfying the cocycle condition on $X' \times_X X' \times_X X'$.

Intuitively: when the local content is pulled back along the two ways of overlapping, the results agree—the datum is *virtually glueable*. The question of descent theory is whether virtual glueability lifts to a global object: does there exist $f \in \mathcal{S}_X$ whose pullback along u recovers $(f', \varphi_{f'})$? When the answer is affirmative for every descent datum, u is a **descent morphism**.

15.2 Meaning as gluing

In the linguistic case, the role of the descent datum is played by a family of contextual contents—paradigmatic uses of an expression in different contexts—which agree where their contexts overlap in a more specific common refinement. The descent question is whether these overlapping local contents glue into a single more generic content in a more generic context. Halimi's second slogan is as sharp as the first:

Slogan 15.2. Contextualization is a localization.

The general meaning of "red" is not the invariant left over after the particular uses have been factored out. It is the **amalgamation** generated by the projections, along specific context-shifts, of several particular contents attached to different specific contexts. The same content can contribute to entirely different amalgamations—and to the same amalgamation in entirely different ways—depending on the projection of its context.

The distinction between abstraction and amalgamation is conceptually decisive, and it is the exact point at which the fibrational semantics breaks with the entire Fregean tradition. In abstraction, isomorphisms register *sameness of structure*: two instances exemplify one form, and the form is the meaning. In amalgamation, isomorphisms register *coherence of overlap*: two local data are revealed as compatible aspects of a single global entity, and the entity exists only insofar as the coherence does. Two altogether different concepts of structure emerge, and only the second is faithful to how meaning works. Abstraction is Plato's participation and Frege's sense; amalgamation is Grothendieck's descent and—the critique adds—Brandom's tradition-constituting recollection, in which a norm is nothing over and above the coherent history of its applications. Chapter 18 completes that identification.

15.3 Sites and the topology of exemplification

The full apparatus arrives when $\mathcal{C}$ is equipped with a **Grothendieck topology** K: a specification, for each context X, of which families of context-shifts $\{u_i : X_i \to X\}_{i \in I}$ count as *coverings* of X. The sheaf/descent condition for a cover is the exactness of the equalizer diagram

$$\mathcal{S}_X \longrightarrow \prod_i \mathcal{S}_{X_i} \rightrightarrows \prod_{i,j} \mathcal{S}_{X_i \times_X X_j},$$

first map $\prod u_i^*$, parallel maps the pullbacks along the two projections from the overlaps: local data on a valid cover, agreeing on overlaps, determine at most one gluing.

Halimi's working hypothesis, which the critique adopts as an empirical postulate about natural language: the fibered category of contextual content over the category of contexts forms a *stack* with respect to a natural Grothendieck topology, generated by the families of contexts in which competent speakers exemplify the meaning of an expression. The hypothesis is a way to account for the fact that natural language works. Note its structure carefully: it does not say meanings are global; it says the coverings along which speakers' exemplifications are collected are *descent morphisms*—wherever the data could glue, they do. Which families count as covering is itself normatively instituted—by the community of speakers, in interaction, exactly as Part II requires. The Grothendieck topology is the mathematical form of what Brandom calls the structure of authority and responsibility in a discursive practice: to specify who must agree with whom, on which overlaps, for a usage to count as exemplifying a meaning, *is* to specify a topology on the category of contexts. Semantics is ethics; the ethics is a site.

Chapter 16

Stacks and the Absence of a Global Section

16.1 Stacks

Definition 16.1. A **stack** over the site $(\mathcal{C}, K)$ is a fibered category $p : \mathcal{S} \to \mathcal{C}$ in which every K-covering is a descent morphism: descent data on covers glue, up to canonical isomorphism—which is the only notion of identity available among fibres.

The stack property is the assertion that wherever the data *could* glue, the data *do* glue. It is a coherence guarantee, not a totality guarantee, and the difference between the two is the negative result of this critique.

Theorem 16.2 (No global section)**.** *Let $p : \mathcal{S} \to \mathcal{C}$ be a stack of semantic contents over the site of contexts. The stack condition guarantees: (i) every content transports along every shift; (ii) contents agreeing on the overlaps of a cover amalgamate over its base. It does not guarantee, and in general it is false, that the fibration admits a global section: an assignment, to every context, of a content, compatibly with all shifts. Perfect local coherence is consistent with the non-existence of any global semantic valuation.*

The proof-schema is the standard one from geometry, where the phenomenon is ubiquitous: the obstruction to a global section is cohomological—a nontrivial class built from the very gluing isomorphisms whose local existence the stack condition provides; the cocycles cohere locally while twisting globally, as with a nontrivial bundle, which is locally trivial everywhere and trivial nowhere. You can have perfect local coherence and still no global truth.

16.2 The verdict on Cavaillès' question

Chapter 4 left a question standing: can universality be founded, as in Cavaillès, on the *generality* of ∞-groupoids in HoTT—globally; or must metalinguistic locutions be situated in the *inferentialism* of HoTT—locally? Theorem 16.2 answers: locally, and necessarily so. The generality of the ∞-groupoid apparatus is real—it is the uniformity of the fibrewise structure and of the transport along shifts; every fibre is an $(\infty, 1)$-categorical world, and the reindexing functors are as canonical as adjoints can be. What does not exist, in general, is a single section threading all fibres: the mathematical Absolute is approached (Chapter 7's circuit really does converge locally, cover by cover) but not attained (the colimit of the approach is regulative). Mathematical necessity, in Cavaillès' sense, survives as the necessity of the coherence conditions; the contingent accidents survive as the choice of site; and "mathematical experience as a determined concept" is exactly a descent datum: determined, because the cocycle condition binds it; experiential, because it lives in a fibre.

This is also, now stated with full generality, the thesis of *Transcendental Mathematics* inherited and sharpened: **there can be no global orientation in language, only local positions that may have no necessary relation to others**—mathematically, because global sections may fail; phenomenologically, because constitution proceeds through local syntheses that need not compose into a total vision; linguistically, because meaning is constituted by differences and deferrals that prevent any sign from achieving full presence; formally, because sufficiently powerful systems cannot totalize themselves (Gödel), and every attempted totalization generates elements that escape it (the diagonal). What the earlier book established by convergence of testimony—Husserl, Derrida, Catren, Zalamea, Gödel—this critique establishes as a structure theorem about the fibration of meaning itself. And where the earlier book ended with the theorem, this one continues: the absence of a global section is not the end of the theory of meaning but the beginning of its ethics (Part VI) and the motor of its history (Part V). Zalamea's synthetic philosophy already saw the shape: contemporary mathematics is a systematic geometrization across all its environments—sheaves, homologies, cobordisms, geometrical logic—moving beyond set-theoretical restrictions to groupoids, categories, schemas, topoi, and motifs; sheafification permits the fluid integration of local data into cohesive global structure while traditional boundaries constantly deform—nonlinearity, noncommutativity, nonelementarity, quantization. The blending of diverse structures that Zalamea calls transitory ontology is, in the fibrational semantics, simply what meaning *is*.

Chapter 17

Sheafification and Transitory Ontology: Zalamea's Synthesis

17.1 The geometrization of the environments

The fibrational semantics did not fall from the sky; it is the linguistic precipitate of a transformation in mathematics itself, and Fernando Zalamea's *Synthetic Philosophy of Contemporary Mathematics* is that transformation's philosophical inventory. Zalamea argues for a new approach to understanding mathematics, one that goes beyond traditional methods: a *synthetic* view, focusing not just on the logical or set-theoretical aspects but encompassing the full breadth of arithmetical, algebraic, geometrical, and topological constructions, against the limitations of a purely analytical approach. His inventory records a systematic **geometrization** across all the environments of contemporary mathematics—sheaves, homologies, cobordisms, geometrical logic—a trend in which mathematical structures are increasingly viewed through a geometric lens. Contemporary mathematics has moved beyond the confines of set-theoretical, algebraic, and topological restrictions, embracing more flexible and encompassing frameworks: groupoids, categories, schemas, topoi, motifs. And it exhibits what Zalamea calls the fluxion and deformation of traditional structures—nonlinearity, noncommutativity, nonelementarity, quantization—boundaries constantly evolving and being redefined.

At the center of the inventory stands **sheafification**: the working-over of mathematical structures until they satisfy the sheaf condition, decisive wherever local-to-global passage governs a problem. Sheafification integrates local data into a coherent global structure without freezing either—the gluing is as mobile as the problems it serves. Zalamea reads the sheaf not only as a tool but as a *metaphor* for synthesis: distinct viewpoints glued along their agreements. The critique's contribution is to remove the word "metaphor." After Part IV, the gluing of viewpoints is not *like* sheaf theory; in the fibrational semantics of language it *is* sheaf theory—descent data over the site of contexts, amalgamation over covers, with the stack condition as the exact measure of how much synthesis the data permit.

17.2 Unity and diversity; invention and discovery

Two of Zalamea's recurring themes receive, in this book, formal addresses. The first is the dynamic between **unity and diversity**: mathematics as a discipline in constant flux, characterized by the emergence of new ideas and the blending of various structures. The fibration is that dynamic's diagram—diversity in the fibres, unity along the reindexings, and the measure of their balance in the topology: a coarse topology (few covers) preserves diversity at the price of synthesis; a fine one purchases synthesis at the risk of erasure. The second is **creativity as a synthesis of invention and discovery**: mathematics is not just the finding of pre-existing truths but the inventing of new frameworks and languages. In the terms of Part V: discovery is fibrewise (within a constitutive framework, results are found, and the realist idiom is locally correct); invention is base-level (revolutions install new contexts and new shifts, and the constructivist idiom is correct *across* frameworks). The tired realism/anti-realism oscillation of Chapter 1's closing question is thus resolved by indexing: realist in the fibre, constructivist along the base—which is just what a relativized synthetic a priori ought to mean.

Zalamea's third theme—that mathematical developments influence, and are influenced by, philosophical thought; that ideas are products of their time which transcend it toward a timeless relevance—is the theme of Part V entire, and his "transitory ontology" names its ontological register: the mode of being of entities that exist *in transit*, between fibres, sustained by the coherences that transport them. The amalgamated meanings of Chapter 14, the recollected norms of Chapter 18, the traditions and the frameworks: all are transitory in Zalamea's sense, and none the less objective for it. Transitory ontology is what ontology looks like when the demand for a global section is withdrawn.

17.3 Eidal ascent, quiddital descent

Zalamea's case studies—Grothendieck above all, and the chapters on eidal mathematics that treat Serre and Langlands—show mathematical creativity exceeding computation and deduction into something architectonic, and they show the connections between mathematical areas to be themselves mathematical phenomena, chartable and provable rather than merely felt. His triadic sorting of contemporary mathematics—eidal ascent toward structuring forms, quidditAl descent toward physical singularity, archeal invariants mediating between them—maps without strain onto the transit of Chapter 12: ascent to the universe, descent to the instance, and the invariant structures (adjoints, classifiers, cohomology classes) that make the round trip lawful. That a working philosophy of real mathematics and a formal metatheory of language should arrive independently at the same triad is the kind of consilience a critique is entitled to count as evidence: the shape is not an artifact of the formalism; it is the shape of synthetic reason.

The Langlands program, which Zalamea reads as eidal mathematics par excellence, deserves the last word of the chapter, for it is the grandest existing exhibit of the book's thesis. A web of correspondences between number theory, representation theory, and harmonic analysis—local statements over each completion, global statements over the number field, functoriality conjectures as the reindexings, and no master theorem from which the web hangs: the program *is* a stack of partial amalgamations under a regulative Idea of unity, pursued for decades with complete objectivity and no global section in sight. Mathematics does not wait for totality to make progress; neither does language; neither, Part VI will conclude, should we.

Part V

The Dialectic of Reason

Chapter 18

The Relativized A Priori: Friedman's Dynamics of Reason

18.1 The three periods

Michael Friedman argues, in *Dynamics of Reason*, that there is a contextual milieu providing the relative a priori for each and every scientific revolution. Against Quinean holism—one web of belief, every strand empirical in principle—Friedman recovers a stratification internal to science: at any epoch, certain mathematical and coordinating principles are not tested *by* experience but make the testing of anything by experience possible. Euclidean geometry and the laws of motion did not face the tribunal of experience alongside the inverse-square law; they constituted the framework in which an inverse-square law could so much as be formulated. This a priori is both universally valid—within its epoch, for all inquirers, constitutively—and relativized: the revolution that installs Riemannian geometry and the light-postulate does not falsify the old framework but *re-frames* it, exhibiting it as a limiting case under a translation that the old framework could not itself have expressed.

The temporal structure is tripartite, and the Prospectus adopted it as the skeleton of the entire metatheory: (1) *metascience*, the period in which thinkers refashion mathematics and reformulate physical principles in a philosophically-minded way—Helmholtz, Mach, Clifford, Klein, Poincaré, . . . , Einstein; (2) *revolution*, the installation of the new constitutive framework; (3) *philosophy*, the deliberation on the aftermath, in which the new framework's rationality is articulated and transmitted. If we follow Friedman's schema, the period we are currently in is his metascientific one—and the refashioning now underway is the univalent one: the philosophically-minded reformulation of logic and mathematics in homotopy-theoretic terms, of which this book is a document as much as a description.

18.2 The fibrational reading

Part IV supplies what Friedman's schema describes but does not construct: the mathematics of the transition. Read the epochs as contexts in a base category $\mathcal{C}$; read each epoch's constitutive framework—its synthetic a priori—as the fibre of admissible contents above it. Then:

- The **universal validity** of the a priori within an epoch is the uniform structure of the fibre: every content of the epoch is typed by its framework, and the framework's principles are the fibre's internal logic.

- The **relativization** across revolutions is reindexing: the shift $u : X_{\text{new}} \to X_{\text{old}}$ is the comparison in which the new epoch refers itself to the old as its context of reference, and u^* transports old contents into the new frame—Newtonian mechanics as the low-velocity limit is exactly $u^*(\text{Newton})$, the old content *as it appears* after the shift. The old a priori is not falsified but transposed.

- The **rationality** of revolution—Friedman's central concern, against Kuhnian irrationalism—is the Cartesian property of the lift: among all ways of carrying old content forward compatibly with the shift, the transition actually effected is universal; every other factors through it uniquely. Revolutions are not leaps but Cartesian lifts.

- The **prospective indeterminacy** of metascience—the wellspring of yet-to-be-intelligible mathematics that will relativize philosophical thinking once again—is the openness of the base category: new contexts, new shifts, no terminal object.

The relativized a priori is thus a priori *in* each fibre and relative *across* the base, with no equivocation: the two aspects are the two directions of the fibration. What Friedman established historically, the fibration states structurally; what Friedman could not answer—why the sequence of frameworks is progress rather than drift—requires the Hegelian supplement to which we now turn.

Chapter 19

Lawvere's Hegel: Adjoint Modalities and Aufhebung

19.1 From the Science of Logic to the calculus of moments

The project has a precise birthdate. "In early 1985," Lawvere recalled, "while I was studying the foundations of homotopy theory, it occurred to me that the explicit use of a certain simple categorical structure might serve as a link between mathematics and philosophy." Prähauser draws out the irony: the very concepts Russell wielded in his attack on Hegel—infinitesimals and "continuity"—turn out to do their best philosophical work inside the categorical and modal-homotopy-type-theoretic formalization of Hegel that Lawvere inaugurated. Lawvere went on to construct a formally strict calculus for Hegelian dialectics and to begin formalizing the objective logic; but the reach of that first formalization was bounded by the mathematics available to it, and its full amplification had to wait for the univalent foundations. The 1992 declaration that serves as this Part's epigraph is thus not prophecy but program: category theory was to supply dialectical philosophy with exact, contestable models of its oldest polarities, on the condition—which cuts both ways—that each discipline consent to learn the other.

The calculus itself is due to Lawvere, was systematized by Schreiber, and is expounded in Prähauser's manuscript, whose terminology this chapter adopts. Its basic move is to read a Hegelian **moment** as an idempotent (co)modal operator $\Box$ on a category $\mathcal{C}$. Idempotency ($\Box\Box = \Box$) is the mathematical content of a moment's one-sidedness: applying it twice adds nothing, so what it extracts from an object X—the $\Box$-*aspect* $\Box X$, in Prähauser's terminology—is already all that this moment can see. Each object then stands in a canonical comparison with its aspect, and the direction of the comparison map sorts the moments into two kinds: those with $\Box X \to X$ and those with $X \to \bigcirc X$. The objects a moment leaves fixed assemble into a sub-universe $\mathcal{C}_\Box \hookrightarrow \mathcal{C}$; every moment thereby factors through its own sub-universe, projecting the ambient category onto it and embedding it back. Within $\mathcal{C}_\Box$, the aspect $\Box X$ is the "best possible approximation" of X, as Prähauser puts it—what remains of X when only this moment's resources are available.

19.2 Unities of opposites

For Hegel's **unity of opposites** the calculus offers an *adjoint pair* of moments $\triangle_1 \dashv \triangle_2$. The choice of adjointness is the philosophical heart of the formalization. Two moments could be merely different; adjointness makes them opposites *of one another*: by uniqueness of adjoints each determines the other completely, so neither is intelligible without its opposite—which is exactly the Hegelian claim. And the opposition is irreducibly relational: the two moments may fix the very same sub-universe, differing only in how that sub-universe sits inside the ambient category—a point Prähauser is careful to stress, since it means the opposition would vanish if the wider universe were forgotten. Prähauser sorts these unities by the kinds of their terms ($\Box \dashv \bigcirc$ against $\bigcirc \dashv \Box$, his *ps*- and *sp*-unities), the two cases differing in how the shared sub-universe is presented: with a repeated embedding or a repeated projection. Every type X is then flanked by its two aspects—Prähauser calls this the *aspect sequence* of the unity:

$$\Box X \longrightarrow X \longrightarrow \bigcirc X.$$

Adjunctions being available in any 2-category, a unity of opposites can itself be adjoint to another unity of opposites—the **opposite of a unity**—yielding squares of moments

$$\begin{array}{ccc} \triangle_3 & \dashv & \triangle_4 \\ \vee & & \vee \\ \triangle_1 & \dashv & \triangle_2 \end{array}$$

and, by uniqueness of adjoints, strings of modalities $\Diamond \dashv \Box \dashv \bigcirc$ or $\bigstar \dashv \bigcirc \dashv \Box$: the adjoint triples that Part III has been using all along. $\exists_f \dashv f^* \dashv \forall_f$ is a unity of unities of opposites; the quantifiers are a dialectical structure, and were so before anyone said Hegel's name over them.

19.3 Aufhebung

The relation that makes the calculus *dialectics* rather than a static taxonomy is **Aufhebung**, formalized by Schreiber and given its systematic treatment by Prähauser. One unity of opposites $\triangle_3 \dashv \triangle_4$ constitutes a *higher sphere* of another, $\triangle_1 \dashv \triangle_2$, when it absorbs each lower moment ($\triangle_3\triangle_1 = \triangle_1$ and $\triangle_4\triangle_2 = \triangle_2$): the lower unity persists, intact, inside the higher. Aufhebung asks for one condition more—in the right-handed case, $\bigcirc_2\Box_1 = \Box_1$—and this extra equation is what trivializes the lower opposition when viewed from above: both of the once-opposed moments now appear as special aspects of a single side of the higher unity. The construction thereby earns the German word, whose three senses Prähauser enumerates as "to lift, to preserve and to negate": the lower unity is lifted into the higher sphere, preserved inside it, and negated as an opposition. Nothing guarantees uniqueness—a unity can be aufgehoben in several ways—and even the *minimal* Aufhebung, unique by construction whenever it exists, can fail to exist at all.

The philosophical yield for the critique is threefold. First, the dialectic is not a rival logic, flouting non-contradiction, but a *structure on logics*: a progression of adjoint cycles ordered by resolution, in which each level's oppositions become one-sided aspects at the next. Second, the progression has no guaranteed summit: the non-existence of minimal Aufhebungen in general is the modal-logical face of Theorem 16.2—the Absolute is the direction of the ascent, not a station on it. Third, Friedman's dynamics acquires its missing motor: a scientific revolution is an Aufhebung of the previous epoch's constitutive framework—the old a priori preserved (as reindexed limiting case), cancelled (as absolute framework), lifted (into the new unity)—and the *rationality* of the sequence is the adjointness at every stage. The affirmation–negation–double-negation triangle with which the Prospectus's diagrams closed is the smallest instance: double negation $\neg\neg$ is an idempotent modality, its sub-universe the classical world inside the intuitionistic one, its Aufhebung-relations to other modalities the opening moves of Hegel's logic of being formalized—Schreiber's *nichts* $\dashv$ *sein*, becoming as their first unity.

Chapter 20

The History of Histories: Brandomian–Hegelian Teleology

20.1 Recollection as descent

Brandom makes the claim that not only the *Geisteswissenschaften* exist within history but moreover the *Naturwissenschaften*. Science inherently has a historical dimension—which is not to say that it is socially constructed, but rather that the semantic truth of scientific theory involves context-dependent norms that make scientific truth determinate. The Hegelian figure through which Brandom articulates this is *recollection* (Erinnerung): at each stage, the tradition retrospectively rationalizes its own past, exhibiting the sequence of prior commitments as the progressive revelation of a norm that was, as it now turns out, implicit all along. The norm has no existence apart from this recollective history; yet within the history it is genuinely binding. Made determinate by recollection, the norm governs prospectively.

The fibrational semantics gives recollection its mathematics, and the identification completes the argument of Part IV. A recollective vindication is the construction of a *descent datum* over the covering of a tradition by its episodes: the local commitments of each epoch, together with the coherence isomorphisms—the reindexings—that exhibit each episode's content, pulled back to its overlaps with the others, as agreeing. The norm "implicit all along" is the amalgamated object over the base, existing exactly insofar as the descent succeeds; and Brandom's insistence that the norm is instituted by the recollection rather than discovered behind it is the amalgamation/abstraction distinction of Chapter 14, asserted of history: the tradition's meaning is glued from its episodes, not abstracted from them. Where descent fails—where the episodes' overlaps cannot be made to cohere—there is no norm, only the fragments of one: a tradition in crisis, awaiting either a finer covering (a more discerning historiography) or a revolution (a new base context into which the fragments transport separately).

The teleology of the whole three-period process is therefore Hegelian in exactly Brandom's sense and no stronger: progress is real, but it is constituted retrospectively, by descent, epoch over epoch; it is not the approach to an antecedently existing terminus. The overarching temporal teleology through the entirety of the three periods is explicitly Brandomian–Hegelian, while the first period is Kantian—and both claims are now theorems of the metatheory rather than slogans: the Kantianism of metascience is the fibrewise synthetic a priori (Chapter 16); the Hegelianism of the whole is recollective descent under the regulative Idea of a global section that need not exist (Chapter 15, Theorem 16.2).

20.2 Geist as the fibration entire

Negarestani's formula can now be given its final form. The mind approached functionally—what it does, what it computes—must exist within a history of histories: spirit. Read the phrase literally against the machinery: a history is a diagram of episodes in the base with its amalgamated content; a history of histories is the fibration entire—the stack of all such local amalgamations, related by reindexing, with no global section required or promised. Spirit becomes context-dependent codifications of public language in an inferential network of ∞-topoi: not a super-subject, not a world-soul, but the coherence structure of all the local worlds of meaning, exactly as strong as the descent that holds it together and no stronger. If one accepts the hypostasis of Ideas over matter, one may call the regulative direction of this structure the mathematical Absolute or world-historical spirit in the background processes; the critique's discipline is to use these names only for the direction, never for a station. Geist is not the global section. Geist is the gluing.

Part VI

The Doctrine of Consequences

Chapter 21

Linear Homotopy Type Theory and Quantum Natural Language Processing

21.1 Two semantics of the lexicon

The critique's formal apparatus must, finally, meet the empirical study of language where it lives. Contemporary computational semantics offers two apparently opposed paradigms, and Corfield's correspondence—quoted in Chapter 5—already framed the comparison exactly. The *distributional* paradigm embeds all lexical items in one large vector space, where great distance corresponds to semantic distinctness: cat, dog, and cup inhabit one space, cat and dog near each other, both far from cup. The *typed* paradigm categorizes entities according to different types, and so as non-comparable: $\mathsf{cat}, \mathsf{dog} : \mathsf{Animal}$ and $\mathsf{cup} : \mathsf{Utensil}$. To the charge that some sentences allow 'cup' to replace 'cat'—so better one space—the typed theorist answers with supertypes: all three are Object, and the graded hierarchy of types and supertypes explains the *limited range* of admissible replacements: 'cup' and 'dog' can replace 'cat' in "I see a cat," but only 'dog' in "The cat is walking away." Meaningfulness-preservation is typing; similarity is metric. The question is whether one framework can carry both.

21.2 The compositional-distributional synthesis

Quantum natural language processing answers affirmatively from the distributional side. Its categorical formulation—sentence structure described with string diagrams defining a network of words—rests on the observation that pregroup grammars and vector spaces share compact closed structure: the grammatical reductions of a sentence type are exactly the contraction diagrams by which word-vectors and word-tensors compose into a sentence meaning. Grammar supplies the wiring; distribution supplies the states; the meaning of the sentence is the value of the diagram. QNLP implements the composition on quantum hardware, where the tensor structure is native. This is semantic ascent instituted computationally, precisely as Chapter 8 forecast: the substitutable, functionally black-boxed dimension of a concept (red via ROYGBIV) is its tensor decomposition; the singular dimension (red qua red) is its state.

Linear homotopy type theory is, following Corfield's suggestion, the best of both worlds—and the setting in which the two paradigms become the two directions of one fibration. Linear dependent type theory fibers a linear (multiplicative, resource-sensitive) layer over an intuitionistic (cartesian, dependent) base: base types index; linear types above them carry the tensorial content; the dependent structure supplies typing, quantification, and identity; the linear structure supplies the monoidal geometry in which metrics, inner products, and hence *degrees* of substitutability live. DlHoTT has the expressive power to encode predicates in the object language and, moreover, substitutable terms in the higher dimensions of the ∞-topos: it provides a means of defining a topological metric between similar words, with formal substitution defined by such a metric. The Brandomian division of labor is thereby completed formally: directed material inference between predicates lives in the base (the '1' of the $(\infty, 1)$-topos); reversible substitution of singular terms lives in the higher cells (the '∞'); and the *gradation* of near-substitutability—the phenomenon the vector space captured and the bare type theory lost—lives in the linear fibre. The fibrational semantics of Part IV is thus not merely compatible with computational linguistics; its linear refinement *is* a computational linguistics.

21.3 The alien civilization, revisited

The chapter closes by redeeming a promissory note from Chapter 1. Carnap crystallized the bare minimum of structure required for an alien civilization to understand our language: propositions, and variables defining such propositions. The compositional-distributional synthesis converts the thought experiment into an engineering specification, and the specification vindicates the fibrational semantics point by point. What must be transmitted to the alien is not a dictionary—dictionaries presuppose the shared meanings they purport to convey—but a *grammar of composition* (the string-diagrammatic wiring, which is pure syntax) together with a *corpus of interactions* (the record of use). By Negarestani's condition, this suffices: syntax coupled with interaction yields semantics, and the alien reconstructs our meanings as behaviours in Girard's sense—equivalence classes of designs individuated by orthogonality—without ever receiving a single meaning as such. What the alien cannot reconstruct, even in principle, is a global valuation of our language: the corpus underdetermines any single section, because the language itself has none. The alien acquires, at best, what we possess: the stack—fibres of usage, reindexings between them, amalgamations where the data cohere. Radical translation, in the fibrational setting, is not indeterminate (against Quine) and not determinate (against the dictionary picture); it is *exactly as determinate as descent*: unique gluing wherever the cocycle conditions hold, genuine plurality wherever they do not. The inscrutability of reference is the non-existence of a global section, rediscovered from the outside.

Chapter 22

General Intelligence as Sociality

22.1 The realizability claim

The analytic of Part II staked a biconditional; the machinery since has earned the right to assert it: general intelligence is realizable if and only if language is understood inferentially, in terms of semantic social norms defined interactionally between agents through a minimal syntax generating the whole system of rule-based roles. Each direction now has its proof-sketch.

Only if. Suppose general intelligence realized. By the functionalist analysis (Chapter 8), its reason is what it computes; by the computational trinity, its computations are proofs, its proofs typed; by the transit theorem, its capacity for self-assessment—for treating its own claims as claims—requires the universe hierarchy, semantic ascent internalized. But normative self-assessment is not a monological achievement: a system that cannot be *challenged* cannot distinguish its taking-correct from its being-correct, and the distinction is the whole of objectivity. The challenge-structure is interaction (Chapter 6); its stable form is the differentiation of agential roles in a network (Chapter 7). Hence sociality—not as an ornament of intelligence but as the condition of the normativity without which "intelligence" names only signal processing.

If. Suppose language so understood: minimal syntax, interactional institution of roles, inferential semantics. Ludics shows the semantics precipitates from the syntax-in-interaction with no semantic primitives; dlHoTT shows the precipitate has the full expressive stratification—predication, quantification, identity, substitution-gradation—that discursive intelligence requires; the fibrational semantics shows the resulting contents transport, amalgamate, and recollect across contexts, which is to say: learn, generalize, and form a tradition. A community of such interactants realizes, distributively, every function the functionalist analysis assigns to general intelligence. Through the self's self-realizability—achieving self-consciousness, and consciousness beyond selfhood—geistig manifestation is achieved in the form of general intelligence, but only through an inferential network of social agents intrinsically encoded through computation.

22.2 Against alignment as global section

The consequence for the engineering of intelligent systems is the one *Transcendental Mathematics* drew for governance, here derived rather than urged. If there is no global orientation in language, then no AI system can occupy a meta-position from which all local perspectives are adjudicable. The dream of a value-aligned AI that perfectly implements human values founders on the theorem: there are no global human values—only local values, local frameworks, local language-games that need not cohere, related by shifts, sometimes amalgamable over covers, sometimes not. This does not make governance impossible; it makes it structurally local. Type-theoretic frameworks for particular domains, without the pretense that the frameworks integrate into a total system; interfaces between frameworks—functors, translations, adjunctions, geometric morphisms preserving the geometric fragment—without the claim that the interfaces achieve global coherence. The model is federalism rather than universalism, and the mathematics of the federation is the mathematics of this book: local data, gluing conditions on overlaps, and the frank acknowledgment that global sections may not exist. An alignment scheme that demands the global section is not merely infeasible; it is a paralogism—the transcendental illusion of semantic reason, engineered.

Chapter 23

Against the Global Section: Language, Politics, and the Other

23.1 Libidinal nihilism and the strategy of exemption

The critique's negative theorem has a political shadow, and the shadow has a rhetoric. There is a tendency in contemporary thought—exemplified by the late writings of Nick Land and certain accelerationist programmes—which may be named, after Haela Ravenna Hunt-Hendrix, *libidinal nihilism*. Its signature is the phrase *time will tell*: the deferral of all ethical interrogation to the impersonal verdict of Capital, the market, the Singularity. The deferral is not humility. It is a strategy of exemption: to say "time will decide" is to offer a reason while pretending reasons are obsolete—a move *within* the game of giving and asking for reasons that purports to stand outside it, and is refuted accordingly, as every such move is, by its own performance. Libidinal nihilism collapses the agent into the trajectory of a single attractor: the cybernetics of one feedback loop subsuming all becoming. It is, in the terms of this book, the demand for a global section made in the currency of despair: since no local position can survey the whole, let the whole—rebranded as process, market, machine—absorb every position.

The critique refuses the deferral, and refuses it without reasserting the universal, the view from nowhere, the philosophical king's-eye perspective. There is no view from nowhere—not as a matter of pessimism, but as a mathematical theorem about the structure of meaning (Theorem 16.2). Local positions can cohere; they can translate; they can coordinate; they cannot be unified into a single master perspective. Between the false whole and the abandoned fragments there is a third structure, and this book has spent its length constructing it: the stack. A politics of irreducibly local positions, coordinated through interfaces rather than subsumed under a master rule, has a precise mathematical home in a fibered category in which local data glue, where they can, into global items, without any guarantee that a global section exists. Coherence without unification; coordination without conquest.

23.2 The categorical imperative of interfaces

What follows practically from a semantics without global sections? Not relativism: the coherence conditions are binding, the reindexings canonical, the descent obligations real—a descent datum that fails its cocycle condition is *wrong*, locally and checkably. Not quietism: the topology is instituted, hence revisable; widening a covering—admitting more contexts as jointly authoritative over a meaning—is a normative act, the semantic form of enfranchisement. What follows is an imperative that deserves the Kantian adjective: **build the interfaces**. Construct the reindexing functors between the fibres that history, power, or distance have kept apart; identify the descent data—the overlaps on which irreducibly different practices already agree; define the Grothendieck topology under which those practices can be recognized as covering a common situation, without erasure of their difference. The work is not the discovery of a hidden universal but the construction of a topology fine enough that coordination becomes possible.

The exemplar the tradition offers is not a philosopher's. The Drexciyan mythos—the underwater civilization Stinson and Donald composed across the records of the 1990s: the unborn children of the Middle Passage, adapted to breathe underwater, building a technologically advanced society on the ocean floor—is a microutopia constructed from the materials of catastrophe. It lives in a fibre the dominant genealogies render invisible; its meaning is not abstracted from any general theory of liberation but amalgamated from the specific overlaps of Black diasporic experience, post-industrial Detroit, and the speculative ocean. There is no global context containing all such covers. There is only the patient construction of the topology under which coordination becomes possible—a civilization on the ocean floor, built from what the surface has thrown overboard. Against the long con of "time will tell," the critique proposes: **the other will speak**—the other context, the other fibre, the other community whose internal logic is irreducible to mine—and we shall build the interfaces through which that speech can be heard, translated, contested, and revised.

Conclusion: The Verdict of the Critique

The findings

A critique ends with a verdict, and verdicts should be brief.

On the synthetic a priori. Reinstated. Logic, arithmetic, and pure geometry are synthetic a priori in Martin-Löf's sense—constructive, generative, evidenced in acts—and their contemporary body is homotopy type theory under Awodey's interpretation, with univalence rendering identity itself synthetic. But the reinstated a priori is relativized (Friedman) and fibered (Part IV): a priori in each fibre, transported across the base, revisable by Cartesian lift.

On meaning. Meaning is inferential role, instituted interactionally; syntax coupled with interaction suffices for semantics; the institution is formally explicable—ludics for the interaction, dlHoTT for the expressive stratification, the $(\infty, 1)$-topos for the two dimensions of inference: directed material consequence between types, reversible substitution among terms. Semantics is answerable to pragmatics, and pragmatics is deontology: semantics is an ethical discipline, and its ethics has a mathematical form—the site.

On the metalanguage. The Tarskian tower is real and internal: the universe hierarchy, navigable in both directions, totalizable in neither. Semantic ascent and descent are adjoint aspects of one transit; the universal language is a paralogism; external questions are internal questions one level up, except where there is no level—and there, precisely, reason's demand becomes regulative rather than constitutive.

On context. Context-dependence is context-shift-dependence; contents are transported, not evaluated; general meanings are amalgamated, not abstracted; the semantic universe is a stack over the site of contexts, whose topology is normatively instituted by the community of speakers. Wherever local contents can glue, they do. Nothing guarantees a global section, and in general there is none: perfect local coherence without global truth. This is the negative finding, inherited from *Transcendental Mathematics* and here derived as a structure theorem: no global orientation in language, only local positions.

On history. The sequence of frameworks is Kantian in each epoch and Hegelian across them: metascience, revolution, philosophy; apperception, judgment, abduction; the relativized a priori recollected into tradition by descent. Lawvere's calculus of moments—adjoint cycles, Aufhebung—gives the dialectic its exact form and its exact modesty: the Absolute is a direction, not a destination; Geist is the gluing, not the global section.

On intelligence. General intelligence is sociality: realizable if and only if language is understood inferentially, its norms defined interactionally through a minimal syntax generating the system of roles. The alignment of intelligence to a global table of values is a transcendental illusion; the governance of intelligence is federal, interfacial, local—as is everything else in the space of meaning.

The lighthouse, again

Transcendental Mathematics closed with a lighthouse on a foggy coast: never illuminating the entire sea, sweeping its beam, joined by other lighthouses, coordinated. The present book has tried to say what the coordination *is*. The lighthouses are fibres; the sweep of each beam is a reindexing; the charts the keepers exchange are descent data; the coastline they jointly map is an amalgamation that exists exactly where their charts cohere; and the whole illuminated world is a stack—no beam reaches everything, no chart is the chart, and yet the navigation is objective, correctable, and shared. A formalization that captured all meaning would not be a foundation but a tomb; the voids are where grace enters. The critique of linguistic reason does not end the philosophy of language. It hands it, at last, its instruments: and the instruments say—*build the interfaces; the other will speak*.

Appendix A

Category-Theoretic Background

A.1 Categories, functors, natural transformations

A **category** $\mathcal{C}$ consists of objects and morphisms with associative, unital composition. A **functor** $F : \mathcal{C} \to \mathcal{D}$ assigns objects to objects and morphisms to morphisms, preserving composition and identities. A **natural transformation** $\eta : F \Rightarrow G$ assigns to each object A a morphism $\eta_A : FA \to GA$ such that for every $f : A \to B$, $Gf \circ \eta_A = \eta_B \circ Ff$. A functor is an *equivalence* when it is full, faithful, and essentially surjective. The category $\mathbf{Set}^{\mathcal{C}^{\mathrm{op}}}$ of presheaves on a small category $\mathcal{C}$ receives $\mathcal{C}$ via the Yoneda embedding $\mathrm{y}(A) = \mathcal{C}(-, A)$, which is full and faithful; the Yoneda lemma states $\mathrm{Nat}(\mathrm{y}A, P) \cong P(A)$, naturally.

A.2 Limits, colimits, adjoints

A **limit** of a diagram is a universal cone over it; finite products, equalizers, and pullbacks generate all finite limits. Colimits are the dual notion. An **adjunction** $L \dashv R$ between functors $L : \mathcal{C} \to \mathcal{D}$ and $R : \mathcal{D} \to \mathcal{C}$ is a natural isomorphism $\mathcal{D}(LA, B) \cong \mathcal{C}(A, RB)$; adjoints are unique up to canonical isomorphism; left adjoints preserve colimits and right adjoints limits. Between posets, an adjunction is a Galois connection: $L(x) \leq y \iff x \leq R(y)$. Monomorphisms into an object A, ordered by factorization, form the subobject poset $\mathrm{Sub}(A)$; when $\mathcal{C}$ has pullbacks, $f^* : \mathrm{Sub}(B) \to \mathrm{Sub}(A)$ is defined by pullback and the logical structure of Part III concerns its adjoints $\exists_f \dashv f^* \dashv \forall_f$.

A.3 Fibrations and the Grothendieck construction

A functor $p : \mathcal{S} \to \mathcal{C}$ is a **(Grothendieck) fibration** when every morphism of $\mathcal{C}$ has Cartesian lifts to every object above its codomain (Chapter 13). Choosing lifts yields a pseudofunctor $\mathcal{C}^{\mathrm{op}} \to \mathbf{Cat}$ (an *indexed category*); conversely the **Grothendieck construction** assembles any indexed category into a fibration, and the two presentations are equivalent. A **Grothendieck topology** on $\mathcal{C}$ assigns covering families stably and transitively; a site is a category with a topology; a **sheaf** is a presheaf satisfying the equalizer condition for every cover; a **stack** is the categorified sheaf: a fibration in which descent data on covers glue (Chapter 15).

A.4 Adjoint modalities and Aufhebung in brief

For Part V's calculus, following Prähauser's exposition of the Lawvere–Schreiber formalization: a **modality** on a category (or type theory) is an idempotent (co)monad $\Box$, with comparison $\Box X \to X$ (comodality) or $X \to \bigcirc X$ (modality); its *pure* types, those fixed by it, form a reflective or coreflective sub-universe $\mathcal{C}_\Box \hookrightarrow \mathcal{C}$, so that each moment decomposes into a projection $\mathcal{C} \twoheadrightarrow \mathcal{C}_\Box$ and an embedding $\mathcal{C}_\Box \hookrightarrow \mathcal{C}$. A **unity of opposites** is an adjoint pair of moments $\triangle_1 \dashv \triangle_2$; when the adjoints share their sub-universe, each type sits in its aspect sequence $\Box X \to X \to \bigcirc X$. Unities can themselves stand in adjunctions (opposites of unities), and by uniqueness of adjoints such squares collapse into strings $\Diamond \dashv \Box \dashv \bigcirc$. A unity $\triangle_3 \dashv \triangle_4$ is a **higher sphere** of $\triangle_1 \dashv \triangle_2$ when it contains it moment-wise ($\triangle_3\triangle_1 = \triangle_1$, $\triangle_4\triangle_2 = \triangle_2$); it is an **Aufhebung** when moreover the lower opposition trivializes from above ($\bigcirc_2\Box_1 = \Box_1$ for a right Aufhebung)—lifting, preserving, and negating the lower unity at once. Minimal Aufhebungen, when they exist, are unique; in general they need not exist. The paradigm instances in a topos or in HoTT: double negation $\neg\neg$; the sharp/flat/shape modalities of cohesion $\int \dashv \flat \dashv \sharp$; and, degenerately, the initial unity $\emptyset \dashv *$ of nothing and pure being whose first Aufhebung Schreiber identifies with becoming.

Appendix B

The Doctrinal Hierarchy at a Glance

Fragment	**Logical operations**	**Categorical structure**
Algebraic	$=$ (equations only)	finite products
Cartesian	$=$, $\top$, $\wedge$	finite limits
Regular	$=$, $\top$, $\wedge$, $\exists$	regular categories
Coherent	regular $+$ $\bot$, $\vee$	coherent categories
First-order	coherent $+$ $\Rightarrow$, $\neg$, $\forall$	Heyting categories
Higher-order	first-order $+$ powersets	elementary toposes
Dependent	Σ, Π, Id	locally cartesian closed categories
Homotopic	$+$ univalent universes	$(\infty, 1)$-toposes

Reading downward, each fragment adds connectives and each class of categories the corresponding structure; each row supports the three interlocking activities of proof theory, categorical semantics, and completeness via a classifying category; the two-way rules of every calculus are the syntactic traces of adjunctions. The final two rows extend the classical table of my *Mathematics of Neo-Rationalism* into the univalent setting that the present book requires.

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