Prolegomenon to a Treatise · Eric Schmid

ISBN: 978-3-946701-19-4

Reihe No. 3

First edition

Bauer Verlag, Frankfurt am Main 2022

Prolegomenon to a Treatise
on mathematical structuralism, constructive computationalism, de-ontologized metaphysics of hermeneutics, and the synthetic a priori

Eric Schmid

Forewords by
Rocco Gangle, Will Fraser, Michael Stumpf,
Connor Tomaka, Fernando Zalamea

Afterwords by
Alexander Boland, Laszlo Horvath,
Hunter Hunt-Hendrix, Inigo Wilkins, Tim Pierson

Postscript by Mattin

Co-edited by Ben Green

Bauer Verlag

Dedicated to my mother & Max Guy

Foreword one

Where are we? In at least two places at once: the conventional scene of a foreword to a theoretical text, and an unconventional space for reflections on what this text does and how it works. Eric Schmid gives us, his readers, a chance to think along fresh lines by activating—inside a fiendish theoretical armature: topos theory and homotopy type theory, Thom's catastrophes and Simondon's transductions, Laruelle's unilaterality, Zalamea's diagrams and sheaves, space-time Minkowski-style, Connes's tropical excursions, Deleuzian different/ciation, Negarestani's recursive deserts, Petitot's neurogeometries—new speeds, unprogrammed sparks, rigorous exuberances. There are two main moments in play: (1) the selection of the materials and the marshaling of their theoretical forces; (2) the deployment of this carefully arranged stock of concepts in the formulation of a prognosis. Schmid's abductive hypothesis is that of the theoretical synthesis of a revised form of structuralist realism foliated through various layers of physics and mathematics with a de-ontologized and radically pluralist metaphysics. This is not yet a fully-fledged program but a kind of promissory note for a philosophy-to-come. The text itself hearkens to novel commotions sounded from conceptually prospective avenues; it wires in to sublime channels and works to help make what's coming thinkable, that is, makeable. In my view, what is most important in Schmid's treatise is how it exercises a panoply of important sections from contemporary philosophy, science and mathematics in order to suggest how the epistemic relation between ‹abstract› and ‹concrete› that is manifest, in particular, in type theory (especially its timely homotopic incarnation) may be constructively employed and made effective without having to ground itself in correspondence to any correlative ontological relation between ‹form› and ‹matter›. The formal transcendence requisite for knowledge, any type of knowledge whatsoever, is no longer postulated—to condition such knowledge—as a necessary characteristic of the World or Real. Instead, it merely recurs and registers and freely regulates, like the formal-material mind imagined by Brouwerian mathematics, building knowledge constructively out of its indexical memorialization in the continuity of its own flux. This process of epistemic recursion does not need the ground or authoritative legitimacy of ontology to build up the knowledge of itself. There is no requirement for any primordial act of abstraction, only the local immersion each time in whatever abstract material. Jean-Luc Marion has offered us the saturated phenomenon of a God without Being, and Geoffrey Hellman has demonstrated the hardcore nominalist rigor of a mathematics without numbers; Schmid sketches the contemporary possibility of a science without ontology expressible via formalizations stripped of the sufficiency of form. This conception marks the idea of form itself without any duality external to form as such. At last, a real modernism within philosophy seems possible; at last, a final bow for the tradition and the end of the conservative need for bowing to it. As Schmid points out, even ontic structural realists are still Aristotelians at heart, and

the seemingly eternal gigantomachia between Platonists and Aristotelians, Scholastic realists and nominalists, formalists and materialists has always been philosophy's prepubescent funhouse mirror stage and a symmetrical show trial. What, then, remains unthought? Form as radical immanence—a new theoretical mode in which ‹mappings› from mathematics (for example) into other domains are rendered interestingly redundant, inert yet pluralized, like images of categorical objects under the Yoneda embedding. This is very close in spirit to what François Laruelle would intend by a ‹materiale› formalism (‹materiale›, not ‹material›—on analogy with the Heideggerian «existentiale»). We are confronted here with an image of old metaphysics as setup. It cons us but it also prepares us, like the preparation of a scientific apparatus for experimentation. What experiments, exactly, are we preparing for? Schmid doesn't entirely say (the text is a prolegomenon after all). But he shows how new possibilities for thought await us around the next corner, and how we might prepare ourselves at least for meeting them when they do make the turn. We should be grateful to the author of the following treatise for working as he does to open a space for a new kind of thinking and for providing the opportunity perhaps for new modes of writing: mathematical(ly), philosophical(ly), non-ontological(ly), epistemic(ally), constructive(ly)... Philosophy has always rightly positioned itself between the questions, ‹What are we doing?› and ‹What is to be done?›. A new space opens between these questions and their geophysical and neurogeometrical positioning. To paraphrase—or rather plug ourselves in for—Schmid citing Reza Negarestani: «We ask ‹where are we?› then we repeat the question over and over.» And now…

—Rocco Gangle

Foreword two

In such a hyperliberal, hyper-relativist world as negatively constrains us now, it seems to me that Eric Schmid's *Prolegomenon to a Treatise* offers an attempt to think a kind of sublation of complex relativism into a philosophy of the Absolute via a synthetic amalgam of mathematical and geometrical theories, ultimately advocating for what I see as a kind of mathematical *aesthesis*. His purported system would, as I see it, against ongoing theoretical production of *absolute relativization* dressed anew in the cultural fashion of the moment, try to thematize and conceive of *relative absolutization(s)* in the manner of Zalamean or Châteletian transits and akin to Fernando Zalamea's and Rodriguez Magda's notions of transmodernism. De-ontologized idealities sidestepping overt dogmatic idealism, and rationalist realism avoiding libidinal nihilism or what Alain Badiou terms democratic materialism, Schmid's method seeks to decant philosophy of its baggage on both sides in order to nimbly walk the crooked tightrope which contemporary truth so complexly renders.

I start on this broad, abstract note with its cultural and practical overtones because Schmid is first and foremost a cultural producer, an artist and musician steeped in the repertoire and unraveling of experimental and avant-garde art and music over the past century and of course more specifically and actively in the past decade. It's no small uncanny wonder that I too have drifted from my musical and artistic tendencies towards the magnetic draw of philosophical thinking, at an historical impasse which admits of no clear artistic way forward without thorough theoretical speculation, to account for the time's overwhelming complexity and underestimated libidinal entrainment. And, as is my experience as well, contemporary artistic culture is often and increasingly one of the most disappointing lenses through which to witness aforementioned (absolute) relativization and an embarrassingly democratic horizontalization of recycled (-because-only-horizontal) «sameness-with-a-twist.» But of course, where is the Real?

Anyone who has paid attention to Schmid's voluminous sound art over the years can attest that his works are highly conceptual, difficult, and oftentimes frustrating. It is my wager that they are so difficult and paradoxical precisely because the Real is so hard to access contemporarily, not only because of its being actively obscured by simulacra, homogenization, relativism, but also because it is often only accessible in the most complex and unexpected ways. Which, then again, if we are the good dialecticians we should be, would also be the most simple. Enter the *Prolegomenon*, where I think Schmid is trying to engage in the terrain of contemporary philosophical realisms in order to, with integrity, account for overwhelming complexity and to supplement an artistic practice with a robust theoretical toolkit. I commend this project enormously, even when I feel skeptical (or ill-informed) of certain of his theoretical maneuvers, because of its ambition to think otherwise—which is to think *through* the Real—philosophically and for artistico-practical ends. One can read the *Prolegomenon* as an initial outpost which

will continue to transform, while dialectically syncopating with Schmid's other unremitting artistic pole, in mutual and revisory conduction. So, there's almost a meta-transit emblematic in the work itself, of an autobiographical artistico-theoretical relay.

Utilizing such thinkers as Gilbert Simondon, Gilles Deleuze, René Thom, and Reza Negarestani, the *Prolegomenon* seeks to chart out the transit between the continuous and the discrete, whether conceptualized as ‹individuation› (Simondon), the two-pronged non-dialectical act of ‹different/ciation› (Deleuze), the discretization of the continuous via geometry (Thom), or ‹recursive localization› from local ↔ global (Negarestani), which latter concept I like to graft analogically onto, qua Deleuze, ‹repetitive different/ciation› from contingent ↔ necessary. A local ↔ global transitive method is what allows for creative traction on the Real: instead of reifying mutually enforcing global and local dogmas which sediment by both mutual exclusion and indistinction, one unbinds both given images from each other in order to skate the transits behind their mere shadows. So too, continuous ↔ discrete serves as a methodological analog or tautological complement to the relaying movement, whose poles must be stretched in order to invent the sutured topos of the Real. Methodological extrication affords epistemological traction and provisional ontological freedom.

Focus is placed however on the movement from the continuous to the discrete (continuous → discrete), as Schmid wants to avoid ontologizing a metaphysical continuity, an idealistic snare he thinks Deleuze and others fall back upon with notions like the ‹virtual›. Emphasis is placed instead on what discretization *affords*, his de-ontologized metaphysics being one which would depend on a methodologically synthetic mathematics to create its pure presuppositions, not unlike the ambitions of, but methodologically very different from, the contemporary work of Negarestani and Gabriel Catren, among others, not to mention neo-Sellarsians and their radicalized Kantian creed of the ‹Myth of the Given›. It's no small wonder that Schmid is an artist, an artificer, that what we are and know must be *made*, and here we all are, still, in the (post-)Kantian image, navigating the abyss of our making, continually in dire need for artificers and engineers alike to produce not only ourselves but our very conditions for knowledge otherwise. As Peter Wolfendale and Ray Brassier seek to unbind rationality from the human through the ends of the inhuman within the human, perhaps Schmid seeks to unbind universality from the local through the ends of the unlocalized within the local. Both of which, paradoxically, must be made.

We can think of the high structuralism of the group around the *Cahiers pour l'Analyse* and Jacques-Alain Miller's famous concept of ‹suture›, of the relation of 0 to 1, the conceptualization of articulating their simultaneous dependency and distinctness. And yet perhaps these mathematically loose articulations of paradox which have extended into contemporary Hegelian-Lacanianism, namely Žižekianism, have run their course, as they slightly update themselves à la mode of the theoretical and historical day. Schmid seems to ask: can't mathematics and geometry themselves

take us much more specifically further into the creative paradox, so we can almost (asymptotically) divine those positive transcendental invariants, of which we normally receive only the negative likeness—in fact, can we not theoretically prove that we can and do create these transcendental invariants ourselves?

How exactly this is to be done is necessarily unclear and germinal in the *Prolegomenon*, but Schmid's excitement around synthesizing multiplicitous mathematical and geometric theories certainly sparks a Romantic flare for ambition almost akin to the Jena Circle, most notably the transdisciplinary excesses of Novalis. Perhaps Schmid's *Prolegomenon* is to structural realism and synthetic mathematics as Novalis' *Fichte Studies* was to Fichteanism and German Romanticism: there's a sort of parallel gesture of multiscalar interrogation, revision and refinement. Indeed, I can't help thinking of the metaphor of the Nietzschean arrow and its being thrown across the centenaries of history, from the post-Kantian abyss, to the Nietzschean summit, to today's return to the hall of mirrors that is the Real with a resurgent generation of realist thinkers. The time for transits, and transdisciplinarity, is historically ripe again, and especially dire now as nihilist decadence threatens to fossilize into a kind of self-producing mechanical stagnation. It's almost as if Schmid's *Prolegomenon* is *itself* symptomatic of an historical conjuncture wherein art *must* (attempt to) think its concept in order to make a real impact. With the emergence of neo-rationalism in recent years, it seems an historical nexus has insinuated a need for attention to non-conceptual representings in more abstract sensibility with the aim to then bootstrap concepts for productive exit. Indeed, Schmid's jam-packed theoretical ecstasis conveys a real ambition for *figuring things out*.

The danger, however, is to get too lost in one's own conceptual maze of abstract concepts and methods, and one must carefully risk to walk the straight labyrinth between the Scylla of the rational and the Charybdis of the empirical: indeed, the very path that the Real itself treads. While the excursions into axiomatic mathematics, topos theory, computation, etc. are beyond my own ken at the moment, it's clear to me that it will remain for Schmid to work out the potential inconsistencies necessitated by glueing together such heterogeneous yet overlapping mathematical disciplines, namely in their functional, actual, or applied registers. At least someone is attempting to construct an epistemological, methodological toolkit, so sorely needed today with the conflation of epistemological with ontological problems and the collapse of methodological into metaphysical ones, the extrication of which, as Schmid seems to be trying to perform, yields new *formal* synthetic possibilities.

—Will Fraser

Foreword three (An exorcism of topological ghosts)

«Human beings are magical. Bios and Logos. Words made flesh, muscle and bone animated by hope and desire, belief materialized in deeds, deeds which crystallize our actualities.»
Sylvia Wynter, *The Pope must have been drunk, the King of Castile a Madman*

«Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics.»
Galileo Galilei, *The Assayer*

«Hence, if any physical fact happens to render a modification of our concepts necessary, the physicist will prefer to sacrifice the less perfect concepts of physics rather than to give up the simpler, more perfect, and more lasting concepts of geometry, which forms the solidest foundation of all his theories.»
Ernst Mach, *Knowledge and Error*

«...the ultimate shock to preconceived ideas lies ahead...»
John Archibald Wheeler, *Information, Physics, Quantum: The Search for Links*

Can we still imagine any-thing, individually? Collectively? This was the heart of the pressing questions of Fisherian ‹capitalist realism›. If you are a believer in imagination beyond what is, what does belief look like anymore? What would (or should) be the ethics of a genuinely imaginative mythopoeia? From the sulking husk of postmodernity, what pieces are to be cherished and uplifted to usher in the new and what is to be forgotten altogether? What can we work with as materials towards a new reality to be formed out of the puzzle pieces that surround us? Have we ever realized our true power(s) as a species?

A believer would doubt that we ever have, or at least believe that these power(s) had long since been forgotten since the onslaught of the epoch of Capital. So long forgotten that we romanticize such powers as utterly magical/mystical, beyond our reach, unthinkable. By what means could humanity unlearn or relearn its most profound and lost knowledge in the construction of a new world? Can we find ourselves ever again? These are the ethical questions at the core of any real anti-philosophical discourse today. The glass towers of academia have long since eroded their last semblance of connection to humanity, and the great journey of Reason has utterly failed, having all but damned the whole of mankind to an existential, postindustrial misery. Is there anything we can look to in this wreckage that is of any bona fide use in the absolute failure of philosophical and scientific inquiry to teach us anything necessary about ourselves and our situation, something that can redeem the efforts to know anything, leading to now? A believer would say there may well be… and what use is it not to believe after all in this n^{th} hour of Pascal's wager?

By what means is it possible to devise a de-mechanization of our worldview such that we recover from that which was over-mechanized while applying a more effective, fluid, dynamic, and robust logic, based in the very laws of Nature and not in the arbitrary demands of Capital (Pierre Klossowski's «trafficking in phantasms») and/or a scientistic quagmire (the trap of attempting to ever fully understand an objective reality)? In essence, how can we work with reality to Nature's will and logic? If reality is indeed enacted, and we know that an alienist/inhuman (and non-capitalist) logic to be a key to this enaction, then what does that logic look like, as inhuman? How can we think the inhuman?

To do so would be to think in terms of a geometric logic, Eric Schmid would contend. Put differently, following René Thom, «the inhuman logic of reality is geometric in nature.» Schmid presents a synthetic, non-philosophical science (art) that is beyond Nietzsche's «duping of linguistic habits,» by way of a renaissance of the synthetic a priori in mathematics, as well as of non-philosophy—paired together, we may have an epistemological escape route out of Nietzschean indeterminacy that is not in any way Kantian in pretension of the effability of ‹the Real›, only possible by the Laruellean decision of foreclosure of the Real to science. This is an important non-fatalistic turn in contemporary neorationalist discourse.

If our current technological matrix is quasi-magical in its power to control and exploit the subject, then perhaps we need a form of magic superior to it to free us from ourselves: an illusion stronger than technology's own illusory nature, an imagination unbounded by any illusion of freedom from the world, and thereby in cahoots and enacted with the logic of the world. As the world has its revenge on Babylon built-in (readymade as the technological), for eluding itself in science (the objective illusion of the world) from any respect for the indeterminacy and nothingness of the world, perhaps by way of over-consumption and the sheer ecstasy of value/communication, science must start anew with the quantum, freed from any delusions of objectivity towards the Real (of thinking any subject dissociated from the object) as well as freed from any scientific method towards Reason, and instead towards a de-programming harmoniousness of thought as object, back in line with the logics of Nature Herself and not of the mirror of the human mind. That logic, as science has told us, is geometric as well as poetic in nature. It is the logic of pure chance, of game or chaos, and of category theory, the only way to regain a responsible relationship with the nothingness of relativity (cause without cause), with a synthetic a priori. We believe that such a logic is in its becoming, gestating today, and may well serve us as an apotropaic magic against the ruse (curse?) of the technological pandora's box, which views itself as transcendent in rationale and computation, whether the ruse of cybernetics or artificial intelligence, a perfect illusion that must be disenchanted by a return to the world that attempts a re-unioning of forces, rather than their rote and doomed exploitation.

As quantum theory has shown us, what we construct as Reality (‹it›) is precisely that which we can construct from information about it—‹bits› (John Archibald Wheeler). In physics, with the most advanced string theory (Ed Witten's

M-theory) seemingly hitting its own event horizon of development (we will not be able to empirically confirm many of the higher dimensions it posits with our current, and likely nowhere-in-the-near-future, technology), scientists must begin to ask themselves new questions about what science itself means. Ludwig Wittgenstein taught us that meaning is use. This means that true science (art) must decipher what matters most at this critical juncture more than ever; for the rise of neorationalism in philosophy, we begin to see that information corollary to the Real is only as significant as the use value that it has to humanity in terms of furthering a most harmonious relationship with Nature. We must surrender to what we know to be the logic of Nature if we are to be granted further access to its inner workings, to learn a new way of being in the world. As Jean Baudrillard once wrote, «that might even be the definition of a radical thinking: a happy form and an intelligence without hope…what counts is the poetic singularity of the analysis.»[1] This too harkens to Baruch Spinoza: «there is no hope without fear, and no fear without hope.» In this regard, Schmid in his Treatise fearlessly attempts such a positively-hopeless intelligence and precisely such singularity in analysis.

To Schmid this also harkens to the thought of François Laruelle, who's ‹Future Christ› must think heretically to the utmost in order to spawn what he calls the «determinate Utopia of the Real.» As Laruelle posits «Heresies are the research and practice of a utopia, and the radical heresy of the Man-in-person is the discovery of the Real as *determinate* utopia,»

1 Jean Baudrillard, *The Perfect Crime* (London: Verso, 2008), 103.

and furthermore that, «Utopia is not always a delirium of the imagination, it can also be a radical poverty of representations, determining more profoundly an imagination and fiction that changes life, which *express* the Living-without-life such as it determines life.»[2] To this end, science must look at itself in the mirror of its history and become sentient of its own splendor, in a hope that future science can learn from its past follies as much as its past successes in a way that is beyond subjective, but that is committed to finding a stronger path of unity between postmodern humanity and the natural which we are dependent upon. This is an unthinkably enormous mission that begins today.

Where does Schmid's *Prolegomenon* fit into a broader world picture of a history of science? For me, Schmid provides a set of syntheses of disparate ideas from mathematics to (anti)philosophy, which could pose to us a cheat code to a new ecstasy of discovery: namely, that our logic should normatively be thought of as interchangeable with geometry. This flies in the face of much of our history of philosophical inquiry under Capital, which obsessed itself with mere psychological issues and synthetic phenomenology in pursuit of Reason. But those ideas can only take the observer-participant so far in the odyssey of the mind, by the very Nature of the Real as we have indirectly perceived it in science. As Paul Feyerabend suggested in his seminal text, I too concur that we must now say «Farewell to Reason» in its Enlightenment sense of its meaning as being an extension of our ability to exploit and control the world.

2 François Laruelle, *Future Christ—Lessons in Heresy* (London: Continuum, 2011), 69.

The *Prolegomenon* provides in part a loose foundation for what could serve as a catalyst for human thought in the future, in a way that at its best could spur fellow-minded ontologists or philosophers of mathematics into the epistemic-revolutionary power that could prove to be enough to begin to forge a new way of life and of living for humanity: one that achieves a deepened harmoniousness and understanding of how-to-live-with-and-within Nature in ways today largely unthinkable, yet no less critical as humanity faces its ultimate consequences for our ontological impoverishment and hierarchical capitalism which has had ruinous and potentially extinctive ramifications for our species. Similarly to the potent idealism of the Russian Cosmist movement for the ultimate redemption/salvific glory of humanity via science (a reawakening of all dead souls), this epistemic lodestone serves in the provision of a navigational toolset in a Baudrillardian desert of the Real (see chapter II on Reza Negarestani's «Where is the Concept?»). Schmid outlines the vector by which sapience should continue to understand itself in a ramifying fashion by way of the autonomy of Reason and in terms of the latter's questions/theories/concepts/orientation that will potentiate this bootstrapping, in the name of revolt and pure axiomatic heresy on all inquiry to date, a phase shift itself in epistemology, away from the binary relations of sapience controlling and exploiting Nature for its own ends, and on to an amphibology of interconnectivity of sapience to the object of its existence (God/Nature), perhaps in a Peircean sense of synechism:

As originally defined by Peirce, the pragmatic maxim states: «Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object.»[3] Peirce's unusual repetition of ‹conceive›, ‹conceivably›, and ‹conception› (as Zalamea notes) identifies a trifold schema in which some concept might unfold through its given actual (or local) space, its possible (or global) context, and finally the relational structure which contrasts the two. [...]

Peirce's thought is underlined by his hypothesis of the continuum, which speculates a generic gradation and correlation between the world and its appearance, uniting the sign and interpretants in a cascade which reveals the real operations of nature. Ultimately, it is a gambit on the asymptotic convergence of the ontological with the epistemic, revealed through a variety of multi-modal navigational strategies. Here we can recognize Zalamea's broader interest in forms of culture and the arts as contributive strategies towards investigating and mapping the real in its piece-wise approximation from the phenomena of the organism.[4]

Schmid accordingly beckons to a continuation and/or acceleration of an ongoing renaissance in contemporary philosophical rationalism (neorationalism and/or neopragmatism) by way of a turning-away from the scientism or decisionism of traditional epistemology and ontology. This could well be a key move, only possible by way of an incorporation of breakthroughs of de-ontologized reorientation to the object/the ineffability of the Real, allowing for a portal into pure mathematical abstraction (Schmid hints at Grothendieck/Galois/topos theory/future quantum mathematics), which in-the-last-instance makes possible a new modus operandi for sapience: a new form of mathematical-poetical-philosophical transcendence not previously attainable by traditional phenomenology in the vein of Husserl/Heidegger, whose thinking Schmid's *Prolegomenon* would deem predominantly conceptually nonsensical in their contents, being psychologically un-transcendent due to an unrealized mathematical rigor. Topos theory provides a radical way of seeing geometry as logic and logic as geometry: could this be the undergirding of a new pragmatic post-Marxism? (Science as art as insurrection against mechanization/dataism/computationalism/cyberneticized society by way of transcendental computationalism or neural materialism with semantic inferentialism?) Much like Fernando Zalamea, Schmid «seeks to free mathematics from its analytic prison, and mine its generative potential, uniting the domains of human creativity under the generous spirit of a new synthetic reason»[5]:

The apparently stable objects of elementary and advanced mathematics we may be familiar with, are merely the result of a variant ingenuity that binds these forms across the ontological and epistemic spectrum producing new invariances and techniques for further exploration and invention. It is then,

3 Charles Sanders Peirce, «Issues of Pragmaticism,» *The Monist* XV, no. 4 (1905), 481.

4 «Fernando Zalamea–Transmodernism,» Uberty, accessed July 28, 2021, https://uberty.org/items/recommended/fernando-zalamea-transmodernism/.

5 Ibid.

the constitution of these conceptual objects, beyond the concepts in themselves, which must be viewed as the work of real mathematics. In this sense, Zalamea provides an analysis of the creativity underlying mathematics, which should be understood as a unique mode of abstract navigation related to the discovery of purely formal invariances, but whose method of intuition is deeply intertwined with the human style of cognition. The ideality of mathematics and its applicability to more concrete problems provides a template for rigorously assessing the movement of creativity in general. […]

Zalamea describes Grothendieck's approach to problems as one of ‹immersion›, a technique which he likens to the softening of a nut, where after a suitable time in the proper medium, the «exterior softens and opens up ‹with a squeeze of the hands› […] like a ripe avocado.»[6] This strategy required the construction of the appropriate general categories, which allowed for abstract decomposition and localization of obstacles, before the solution emerged in the continual ascent and descent between the particular to the general. From this constant perspectival dislocation and attentiveness to context, Grothendieck was able to draw together the unity and multiplicity of these objects into a common geometrical structure, while pioneering paths for the production of further invariances and transformations of the resulting concepts. This Grothendieckian structure, which Zalamea identifies as the ‹sheaf›, is a central concept gluing the always partial and yet continuous fabric of Transmodern space, figuring the restless transits between domains of human activity. […]

Pragmatic synthesis is achieved through the recognition of the invariance and transformation of the sign as it accumulates

6 Fernando Zalamea, *Synthetic Philosophy of Contemporary Mathematics* (Falmouth/New York: Urbanomic/Sequence Press, 2012), 149.

interpretations. The connection which Zalamea recognizes with contemporary mathematics and, in particular, with Grothendieck's process of ‹sheafification›, should be somewhat evident at this point: for Grothendieck, the construction of the generic category reveals the web of relation between the particular and general.[7]

Along these lines, Schmid would favor a continued de-ontologized heretical practice of pure science (art!) perhaps only possible in academic fields such as pure mathematics or in more recent developments in neurophenomenology/neurogeometry (e.g., immanent geometry is the synthetic a priori, à la Jean Petitot by way of Immanuel Kant) in terms of making-intelligible biological insights into causality and the realm of chaos and/or category theory (phase shifts, Simondonian metastable systems of individuation, perhaps, or a geometrodynamics in further realizing the nature of space). To quote Baudrillard, «the real world […] is unthinkable […] We must break with it as critical thought once broke (in the name of the Real!) with religious superstition. Thinkers, one more effort!»[8] And furthermore, «reality asks nothing other than to submit itself to hypotheses. And it confirms them all. That, indeed, is its ruse and its vengeance.»[9] In the same way that Laruelle uses the ‹material› of philosophy and theology in order to develop axiomatic heresies in his *Future Christ*, Schmid too in his *Prolegomenon* is working

7 «Fernando Zalamea–Transmodernism,» Uberty, accessed July 28, 2021, https://uberty.org/items/recommended/fernando-zalamea-transmodernism/.

8 Jean Baudrillard, *The Perfect Crime* (London: Verso, 2008), 97.

9 Ibid., 99.

with the ‹material› of both post-Marxist philosophy as well as the philosophical insights of quantum physics and pure mathematics, to posit axiomatic mappings for the Laruelle-an ‹Future Christ›; in Negarestani's sense, this is the ongoing division of labor of the inhuman.

—Michael Stumpf

Foreword four

Pursuing traces at the edges of science and mathematics, Eric Schmid in his *Prolegomenon* locates a tension, an extreme informational obstacle with which our handling requires new procedures for adjustment and navigation. Through a radical transdisciplinary approach, Schmid grounds his analysis of the situation in topos and type theories, aimed at wedging access to new constructive possibilities through an intuitionist framework in homotopy type theory. Schmid's careful arrangement and deployment of this impressive array of interlocutors form a compelling preliminary topographical assessment of this informational landscape, and bring us to the precipice of how we might pursue remapping its contours. Schmid's *Prolegomenon* draws on the distinct history and philosophy of mathematics, and his efforts pay dividends towards revealing the philosophical underpinnings of its primary practitioners and the relevance of aesthetics and artists to future topological investigations. For my interests, of highest importance is Schmid's integration of Reza Negarestani's bidirectional navigation of the ‹local›/‹global› and the resulting blurring of boundaries between the two in the recursive desert which leads to the relevance of the specific work in both local, in Jean Petitot's neurogeometric schema, and global, in Guerino Mazzola's topos of music, horizons and their respective functions in Schmid's dawning procedure.

In outlining the way in which one could navigate the recursive desert we find ourselves in, as Negarestani notes, «is a desert for which no map and no compass yet exist. In order to navigate in this desert, first we have to inject a designated instability into it so as to disturb or qualitatively excite the epistemologically opaque homogeneity of this space.»[1] Negarestani's ontological frame, buttressed by Petitot's biomodalization, is used by Schmid in order to, in Schmid's paraphrasing of Negarestani, «unmoor the subject from fixed points and cancel any archetypal relation between subject and object.» The geometrization of logic used here, provided by Alexander Grothendieck and applied in Mazzola (see *The Topos of Music: Geometric Logic of Concepts, Theory, and Performance*), takes into account the fuzziness at the interactive boundaries of the local and global. Iannis Xenakis, composer and inventor of the granular synthesis technique, through his sound practice, and perhaps anachronistically, draws a line connecting Petitot and Mazzola's schemes. As Petitot describes a neurogeometry of vision, Xenakis and his granular synthesis and invention of the UPIC, maps a scheme based on graphical gesture, forwarding a form of intuitive composition taken up by contemporary composers such as Florian Hecker.

Schmid's own trajectory is in this vein, and it is here the tension present in his procedural formation is at its height. Of particular relevance in Schmid's sound practice to *Prolegomenon* are his albums *Pulsar Synthesis*, *Strange Attractor Synth*, and *Topos Theory*, where sound is put into communication with a broader mathematical intervention on the ‹in-

1 Reza Negarestani, «Where is the Concept?,» in *When Site Lost the Plot*, ed. Robin Mackay (Falmouth: Urbanomic, 2013), 231.

effable Real› and the undulating, screeching boundaries between a point and global manifold. This tension, at the crux of the Schmid's proposal for navigation, is best explored in *Pulsar Synthesis*, and in Schmid's own words, «This release questions the ‹subjective inflation› inherent to perception related to the question of whether the listener fills-in the holes or gaps of auditory experience by a rich phenomenology or does the subject inflate their cognitively poor sparse perception?»[2] *Pulsar Synthesis* functions two-fold in performing its resynthesis of the subject through the development of a sonic navigational procedure and, through its confrontation with the ineffable Real, produces a generative friction which has the capacity to manage invariance and stabilization in time.

—Connor Tomaka

2 Eric Schmid, *Pulsar Synthesis*, digital album, released by Mille Plateaux, February 2021, Bandcamp, accessed March 16, 2022, https://forceincmilleplateaux.bandcamp.com/album/pulsar-synthesis.

Foreword five (On Eric Schmid's *Prolegomenon*)

A *novel* approach (profiting from Eric Schmid's young age) to a *preliminary* problem (*Prolegomenon*) opens the richness of an *initial* enquiry, and it is to be saluted with part of the same enthusiasm offered in the reception of young Novalis's unsurpassable *Allgemeine Brouillon* (1798–1799). At the end, the future is of the youngsters, and Schmid's ambition to reflect on mathematics may well provide in a decade his flowering *Prolegomenon to a Treatise on mathematical structuralism, constructive computationalism, de-ontologized metaphysics of hermeneutics, and the synthetic a priori*. Of course, such quantity and depth of higher ideas in the title of the *Prolegomenon* point to universal goals that may not be truly realized, but remember that we are in the presence of an *initial* view of a universe (rather, a multiverse). As category theory informs us, an *initial* concept may *always* be projected into *any other* concept of the category, and Schmid's perception in his *Prolegomenon* may well be working in that direction: the richness of his structural-analysis-synthesis approach encompasses many different alternatives in mathematical thought, and *covers* (as does a Alexander Grothendieck topology), in an integral-unitary way, many differential-multivalent perspectives.

A *mixture* (recalling Albert Lautman's «mixtes») of (1) higher-order (that is, ∞-category-theory, non elementary-set-theory) structuralism, (2) many-layered back-and-forth processes between localization and globalization, (3) neurophysiological foundations of phenomenology,

and (4) complementarity of unitary/multivalent approaches to both knowledge and mathematics, offers Schmid a truly extended ground (a «shifting ground» in Maurice Merleau-Ponty's words) to *situate* some of the deepest problems in our present panorama of the ever-diverging philosophies of mathematics. The desire to *really contemplate diversity* is one of the most likeable characteristics of Schmid's *Prolegomenon*. When «diversity» stands at our order of the day, mostly through shallow intentions, Schmid's enterprise goes much farther and deeper, addressing directly the problems of stability and time, invariance and change, metaphysics and physics, universality and relativity, archetypical imagination and typed existence. Of course, a *sustained mathematical reflection* on these problems, as Schmid's *Prolegomenon* initiates, is a most welcome labor in our fleeting, chaotic and extremely superficial times.

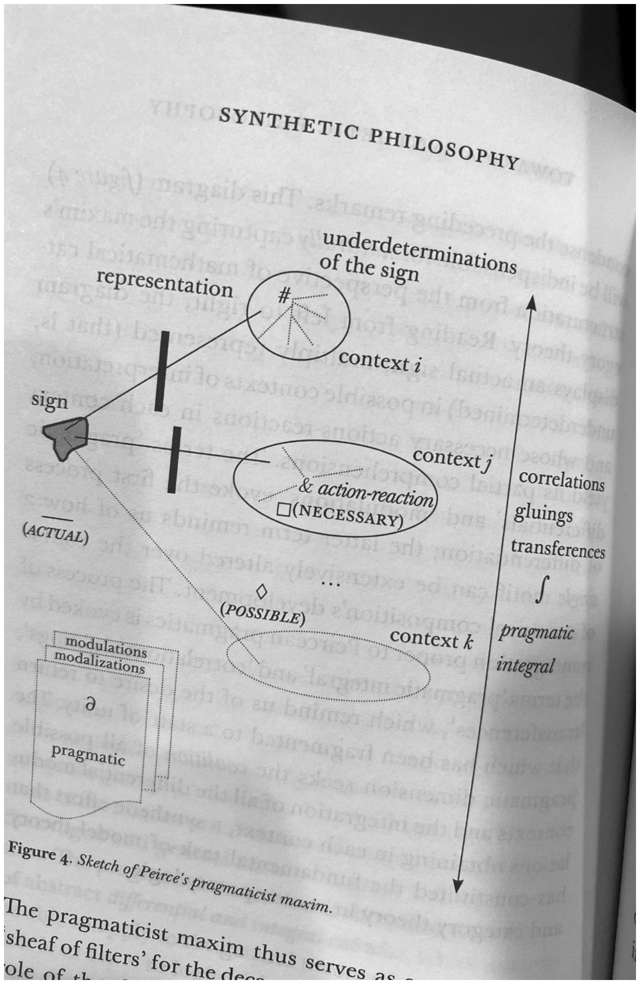
A characteristic of Schmid's effort is his *wide-ranging spectrum* of references, from the very Masters of mathematical thought (Poincaré, Hilbert, Mac Lane, Thom, Grothendieck, Connes, Voevodsky, etc.), to profound epistemologists (Piaget, Kripke, Simondon, Deleuze, Petitot, etc.), to some of the most exciting new thinkers of the 21st century (Negarestani, Laruelle). The many *mathematical* fields traversed by Schmid (mathematical logic, set theory, category theory, topology, number theory, algebraic geometry, differential geometry), show his desire to provide a working «foundational system which conceptually unifies ordinary mathematics,» at the same time *guided by geometrical considerations* (e.g., topos theory or homotopy type theory), *prior to logical im-*

peratives, and projected into the realm of our troubling philosophical times (e.g., locality, virtuality, de-ontologization). Schmid's *initial* perspective puts many tools at our disposal, but does not construct the purported system: it is an entirely *natural obstruction* in a *Prolegomenon*, which should be overcome in a future treatise. Nevertheless, for the moment, it represents a desired *new intention*—yet to be developed in an entire *new voice*—which stands as a nice counterpart to both «analytical philosophy» and the so-called «philosophy of mathematical practice,» which have been consistently separating themselves from mathematical creativity, oriented to either linguistic or sociological considerations, very far apart from the inventiveness of «real mathematics.»

—Fernando Zalamea

Keywords

Local-global navigation through Negarestani's ‹localization›, minimal metaphysics, Simondon's individuation of potentia within a metastable system, Einsteinian relativistic physics and its relationship to philosophical time, neurophysics, the Aristotelian ‹in the thing› realism of structure existing as concrete systems which *ontologically* instantiate the structure, the Axiom of Choice/Zorn's lemma/Banach-Tarski paradox, the universalist unity of mathematics from the Bourbaki approach, the complementarity of ‹axiom› and ‹theorem›, the multiplicity of ‹case(s)› through general ‹invariants› (topological, group-theoretic or categorical) which constrain the objects into varying algebraic structures, category theory as a philosophical structuralism, homotopy type theory as a foundation for mathematics, the Tarski axiom and Grothendieck's universes, arithmetic becomes geometric through topos theory and tropical geometry in the work of Connes and Consani, semantic realism as it is entangled with eliminative structuralism, eliminative structuralism of regimented propositions that have been de-ontologized, the ontology of the Tarski axiom, constructive computationalism, «arrows only» or the morphisms between objects solely.



N. Preface

Through my *Prolegomenon*, I am attempting to create a «transcendental method» which is arrived at from self-evident axiomatic truths (‹truth› with lowercase t) of mathematics and de-ontologized without a necessary background ontology (for examples of «background ontologies» see how the realm of the virtual supports the realm of actuality in Gilles Deleuze, or how the ‹ontic› is supported by the ‹ontological› in Martin Heidegger). Ultimately, I am interested in a ‹methodological dualism› between the ‹manifest image› of ‹being-in-the-world› (culture, the arts, politics, human affairs, soft sciences) versus the ‹scientific image› of the exact sciences. These two domains are incommensurable with each other. But more on that later. Cultural analysis is of the former realm and theoretical science is of the latter realm. Culture can never be as exact as science, but science is developed posterior to culture. But culture depends on science for «first principles» that undergird «reality.» For more on the distinction between the manifest image and the scientific image:

> «Philosophy and the Scientific Image of Man» (PSIM) describes what Sellars sees as the major problem confronting philosophy today. This is the «clash» between «the ‹manifest› image of man-in-the-world» and «the scientific image.» These two ‹images› are idealizations of distinct conceptual frameworks in terms of which humans conceive of the world and their place in it. Sellars characterizes the manifest image as «the framework in terms of which man came to be aware

of himself as man-in-the-world,»[0] but it is, more broadly, the framework in terms of which we ordinarily observe and explain our world. The fundamental objects of the manifest image are persons and things, with emphasis on *persons*, which puts normativity and reason at center stage. According to the manifest image, people *think* and they do things for reasons, and both of these «can occur only within a framework of conceptual thinking in terms of which [they] can be criticized, supported, refuted, in short, evaluated.»[1] In the manifest image persons are very different from mere things; things do not act rationally, in accordance with normative rules, but only in accord with laws or perhaps habits. How and why normative concepts and assessments apply to things is an important and contentious question within the framework.

The manifest image is not fixed or static; it can be refined both *empirically* and *categorically*. Empirical refinement by correlational induction results in ever better observation-level generalizations about the world. Categorial refinement consists in adding, subtracting, or reconceptualizing the basic objects recognized in the image, e.g., worrying about whether persons are best thought of in hylomorphic or dualistic categories or how things differ from persons. Thus, the manifest image is neither unscientific nor anti-scientific. It is, however, methodologically more promiscuous and often less rigorous than institutionalized science. Traditional philosophy, *philosophia perennis*, endorses the manifest image as real and attempts to understand its structure.

One kind of categorial change, however, is excluded from the manifest image by stipulation: the addition to the frame-

0 Wilfrid Sellars, «Philosophy and the Scientific Image of Man,» in *In the Space of Reasons*, eds. Robert B. Brandom and Kevin Scharp (Cambridge: Harvard University Press, 2007), 374.

1 Ibid.

work of new concepts of basic objects by means of theoretical postulation. This is the move Sellars stipulates to be definitive of the scientific image. Science, by postulating new kinds of basic entities (e.g., subatomic particles, fields, collapsing packets of probability waves), slowly constructs a new framework that claims to be a complete description and explanation of the world and its processes. The scientific image grows out of and is methodologically posterior to the manifest image, which provides the initial framework in which science is nurtured, but Sellars claims that «the scientific image presents itself as a *rival* image. From its point of view the manifest image on which it rests is an ‹inadequate› but pragmatically useful likeness of a reality which first finds its adequate (in principle) likeness in the scientific image.»[2] Is it possible to reconcile these two images? Could manifest objects reduce to systems of imperceptible scientific objects? Are manifest objects ultimately real, scientific objects merely abstract constructions valuable for the prediction and control of manifest objects? Or are manifest objects appearances to human minds of a reality constituted by systems of imperceptible particles or something even more basic, such as absolute processes (see «Foundations for a Metaphysics of Pure Process»)? Sellars opts for the third alternative. The manifest image is, in his view, a phenomenal realm à la Kant, but science, at its Peircean ideal conclusion, reveals things as they are in themselves. Despite what Sellars calls «the primacy of the scientific image,»[3] he ultimately argues for a «synoptic vision» in which the descriptive and explanatory resources of the scientific image are united with the «language of community and individual intentions,» which «provide[s] the ambience of principles and standards (above all, those

2 Ibid., 388.

3 Ibid., 400.

> which make meaningful discourse and rationality itself possible) within which we live our own individual lives.»[4, 5]

My hope with the *Prolegomenon* is to create an autonomous free-standing axiomatic system derived from synthetic judgements. But ultimately, we are faced with the problem of reconciling these two registers, cultural and scientific. My *Prolegomenon* articulates the a priori of the subject which is a universal condition of possibility to empirical experience. The phenomenological experience of the subject is bracketed into neuro-geometric or cognitive definitions of the subject. These soft sciences are of the realm of the manifest image. The scientific image is the realm of intensional mathematical systems, which is exact yet de-ontologized. It need not prove existence! Category theory is intensional. Homotopy type theory is intensional. There need not be a background ontology to support such intensional statements. The exact rigor of these systems is arrived at hypothetico-deductively. The intensional statements need not postulate the existence of its statements. For example, one need not posit the existence of the number 2, but one can intensionally define 2 + 2 = 4 and it is true by way of the axioms. For more on the distinction between intensional and extensional statements:

> For example, an intensional definition of the word ‹bachelor› is ‹unmarried man›. This definition is valid because being an unmarried man is both a necessary condition and a sufficient

4 Ibid., 408.

5 Willem deVries, «Wilfrid Sellars,» The Stanford Encyclopedia of Philosophy, accessed July 28, 2021, https://plato.stanford.edu/archives/fall2021/entries/sellars/.

> condition for being a bachelor: it is necessary because one cannot be a bachelor without being an unmarried man, and it is sufficient because any unmarried man is a bachelor.
>
> This is the opposite approach to the extensional definition, which defines by listing everything that falls under that definition—an extensional definition of ‹bachelor› would be a listing of all the unmarried men in the world.
>
> As becomes clear, intensional definitions are best used when something has a clearly defined set of properties, and they work well for terms that have too many referents to list in an extensional definition. It is impossible to give an extensional definition for a term with an infinite set of referents, but an intensional one can often be stated concisely—there are infinitely many even numbers, impossible to list, but the term ‹even numbers› can be defined easily by saying that even numbers are integer multiples of two.[6]

But as we'll see, homotopy type theory proves the existence of all its statements constructively (Curry-Howard correspondence). So, these are not fictitious objects! Constructive intuitionistic type theory allows us to demonstrate existence through the invocation of a particular ‹token›. Every constructive proof has a witness of the construction. We are building in real time in homotopy type theory.

Now, the manifest realm of ‹being-in-the-world› could be postulated elegantly by way of neuroscience/cognitive science/experimental psychology which ultimately depends on computational neuroscience, the latter of which is expressed in its purest form by a rigorous mathematical system

6 «Extensional and intensional definitions,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Extensional_and_intensional_definitions.

undergirding the extant logic of the applied (psychological) realm. Applied math vs. pure math… more or less! My *Prolegomenon* is attempting an exact science of the philosophy of being and structure through the questions of the foundation of mathematics and its ontology.

Now as an artist, my activity (of the cultural realm… or the manifest image) can interact with such an exact science (the scientific image) through what Wilfrid Sellars calls the «synoptic vision.» In the cultural sphere, I am aiming for a ‹total work of art› with my artistic activity, which incorporates emotional intimacy and radical transparency into the work itself through multiple modes of expression (the genres of painting, sculpture, poetry, sound, techno, computer music, art criticism, etc.) and interdisciplinary erudition (knowledge of Fluxus, sound poetry, noise music, contemporary art, philosophy, disco music, etc.). *Now here is the corollary* of what I have said: If one were to read my artistic activity *without* this scientific image, which is the exact science which undergirds all of my cultural activity, one would grossly misunderstand the full extent of my artwork.

The most important tool to the understanding of my *Prolegomenon* is Reza Negarestani's articulation of navigation between the ‹local› and the ‹global›, which is central to mathematics. Negarestani is summarizing the ideas, for example, of Jean Petitot, René Thom, Albert Lautman and Guerino Mazzola in this mathematical notion:

> What French mathematician and philosopher Jean Petitot calls ‹bimodalization› in mathematics is the unfolding of a

> generative synthetic environment between modal poles and for that purpose what is needed is a rigorous program to generalize concepts procedurally, or in other words, bimodalize them into their particular and general modes, local and global constructs.[7] There is a long trajectory of historical conceptual and philosophical problems behind this generalizing program of today's mathematics. It is not as if mathematics has some kind of obsession with generalization—no, these are responses to certain historical problems of philosophy and, in a more particular sense, mathematical problems that have been arrived at and which now mathematicians are working on. The generalization aspect of mathematics and the idea of bimodalization that facilitates the generativity of mathematics brings us back to the subject of this topic which is the formalization of the space of the universal and a robust identification of a local site as immersed in this space.
>
> I start the question of ‹approaching the space of the universal› by discussing the problem of localization. Reductively speaking, in order to approach a space whose full scope is not given, we need to start somewhere, which is to say, we need to find and identify a site through which we can approach this space without ever overstretching the resources of the site or being restricted to the immediate resources of its local horizon. This is the question of localization, which I believe is one of the most classical problems of philosophy. In fact, the question of localization undergirds the kind of ontological and epistemological justification you make about yourself, the subject and the world. Precisely because it is the localization that permits a robust approach and it is the robust approach—in the sense of navigation of different pathways—that forms knowl-

7 See Jean Petitot, «Local/Global,» in *Enciclopedia Einaudi* 4 (Lisbon: Impresa Nacional, 1986).

edge and makes sense of ontology. Localization is always an act of organization or configuration, it operates under the schema of ‹doing violence› to a landscape as Gilles Châtelet suggests.[8] A perturbation, the injection of disequilibrium, a designated alienation, a systematic distancing that brings about the possibility of qualitative organization of information from a homogenous information where nothing is given.[9]

8 See Gilles Châtelet, *Figuring Space: Philosophy, Mathematics, and Physics* (Dordrecht: Kluwer, 2000).

9 Reza Negarestani, «Where is the Concept?,» in *When Site Lost the Plot*, ed. Robin Mackay (Falmouth: Urbanomic, 2013), 230–231.

I. Introduction

«If I live on a surface, the world no longer faces me; and alterity is no longer lived as a confrontation.»
Gilles Châtelet as quoted by Vera Parlac

I think the classical post-Marxist analysis of the world is too parochial. It is predicated on the State and the unaccounted marginalized identity which has «no identity» within the State's imposed order of things/accounting apparatus. A good example is when a judge asked Louis Auguste Blanqui about his profession. Blanqui cheekily responded that he was a «proletarian,» only to be dryly reminded that «that is not a profession.» Defining your political practice in response to the hegemonic power structure is reactive, I believe. Similarly, the limits of modernism showed us that any formalizable logic which could be encoded with an arithmetic was incomplete. I remember once having an argument with Jonathan Gean where he said that if I was a formalist, I would necessarily be limited to the closed permutations of a chess board. Connor Camburn recently brought up Marcel Duchamp's figuration of the chess board. I remember from my Conceptual Art class the classic Duchamp work of three stoppages, which consists of three different means of demarcating a ruler or distance. If one were to get Heideggerian, one could think that any technology or framework inherently enframes (ineffable) Being into some methodological construction which determines our relationship to the world through a predetermined ready-to-hand and methodology.

Enframement is the means by which technology, which by way of its mere technical expression is merely material or matter, endows being with a technics. The incompleteness of any system points to the way in which any axiomatizable system has an aporia, which suggests the potential equiconsistence of infinitely many different frameworks for postulating some type of encoded language, which supplicates some accompanying metalanguage. But the problem with dissecting and deconstructing the power or privilege of the identities empowered by the State, is that the argument necessarily reacts to a political reality, which is contingent upon machination and history written by the victors. Even a historiography is limited by way of this dissection of a causal chain of events which merely occur within one register of time, the dominant historical mode which is inherently imperialist, patriarchal, etc.

An alternative means for considering formalism would be to continue the project first initiated by David Hilbert, by means of attempting to find a foundational system for geometry (*Grundlagen der Geometrie*) as opposed to the Fregean project of a foundational system for arithmetic (*Grundlagen der Arithmetik*). Only recently, with the work of topos theory from the 1960s–1970s and homotopy type theory after the work of Vladimir Voevodsky and Steve Awodey in 2000s–2010s, do we have an inherently geometric, or should I say topological, means of considering first principles. Rather than building a set-theoretical empire based on a history of antinomies, one is building a type-theoretic system predicated on weak equivalences of homotopic paths. So as opposed to the atomistic framework of propositions with the stricter sense of equality in set theory (i.e., double containment) one is building a constructive system which is based on an intuitionistic type theory which always necessitates a witness (as the proof) to the invocation of any construction (refer to the Curry-Howard correspondence). <u>A foundation of mathematics based upon topology is much more generative, as opposed to the modernist project of a chess board. The weak equivalence of any two paths in the homotopic sense provides an inherently spatial/intuitive sense for any axiomatic method and its resulting conclusions, forming a freestanding autonomous structure. G.H. Hardy said that pure math was aesthetics and such a self-evident hypothetico-deductive method for proving an extant logic upon the world we already intuitively understand is finally the reprise of Hilbert's formalist project of a system generated from the first principles of points, lines and spaces. Finally, the dream of Nicolas Bourbaki has been achieved by way of category theory. Thanks to Claude Chevalley, who insisted that Bourbaki consider category theory avant la lettre</u>: «Chevalley was a member of various avant-garde groups, both in politics and in the arts... Mathematics was the most important part of his life, but he did not draw any boundary between his mathematics and the rest of his life.»[10]

This *Prolegomenon* will examine the possibility for a foundational system which conceptually unifies ordinary mathematics (category theory, topos theory, or homotopy

10 Pierre Cartier, «Claude Chevalley,» *Notices of the American Mathematical Society* 31, no. 7 (November 1984): 775.

type theory are the options) or if we should pursue a constructive intelligence/computationalism instead. For that matter, this treatise will discuss structure in the sense of philosophical structuralism (not to be confused with French Structuralism) and whether category theory provides a structuralism as Awodey suggests. Recent research after the Univalence axiom has demonstrated a new contender: does homotopy type theory provide a foundation for mathematics or is it not important to be a foundation, as Awodey suggests, which would challenge us ontologically to pursue the constructivism of Martin-Löf type theory? We will borrow Negarestani's notion of ‹localization› and try to provide a more rigorous context for a ‹site› directly within topos theory, which has ramifications for any ontological framework. Moreover, we will discuss the necessary background ontology to any foundational ‹topos› which will depend on the use of the Tarski axiom.

We will address the issue of the ‹synthetic a priori› and whether we can, in the Kantian manner, treat geometry as the ‹a priori›, either at the macro-level, as Albert Einstein has shown, where the Minkowski spacetime develops a ‹spacetime› where the only invariant is the speed of light; or on the meso-level, «the relativity of simultaneity» where «*distant simultaneity*—whether two spatially separated events occur at the same time—is not absolute, but depends on the observer's reference frame»;[11] or on the micro-level (of neurons) where perception uses a cut loci which is a geometric

11 «Relativity of simultaneity,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Relativity_of_simultaneity.

form (from differential geometry), so that we can develop a «neurogeometry of vision,» as in the work of Petitot.

Furthermore, we will attempt to articulate some ‹self-evident structural Truth› on the ‹conceptual level› itself. In support of the *conceptual* level of any epistemology, I will be arguing for a de-ontologized metaphysics to accommodate the ineffability of the Real and the potentia of the conatus (as in Gottfried Wilhelm Leibniz or Baruch Spinoza), but rather than philosophize this ontology through some transcendent background (the realm of ‹virtuality› in Deleuze or ‹the ontological› for Heidegger), we shall wish to view logico-deductive abstraction itself (like in group theory) as a means to articulate intrinsic invariants of material which need not be embedded in a surrounding Euclidean space for its measurement a.k.a. thinking (see Gauss's Theorema Egregium). Through this immanent hypostatization of material which scientifically determines its own place/count reflexively (in the super-structuralist sense of Louis Althusser, Jacques Rancière or Alain Badiou) but is, simultaneously, completely hypothetical (since it is derived from axioms), one can consider an intensive definition that is completely relative (Alexander Grothendieck's perspective) which allows for ontological involvement, yet suspends its own background ontology through the complementarity of the duality (François Laruelle) between ‹theorem› and ‹axiom› (or for that matter, between ‹actual› and ‹virtual›). This Laruellean approach may be likened to eliminative structuralism, where «statements» get regimented/deflated into analytic propositions which are trivially/vacuously true for a system through assuming the hypothetical *hypostatization* of the theory. Because of the vacuity of any deflated statement, this approach flattens the underlying ontology of a philosophical theory through a quantifier parameter beforehand. I will discuss the presuppositions of Time that are based on a real-valued function which is parameterized and argue against ‹the time of philosophers›.

We will still attempt to demonstrate the underlying ontology of ‹technique› charting from the father of individuation Gilbert Simondon; to the Deleuzian system of becoming from the Idea («The Ideal Synthesis of Difference» in *Difference and Repetition*); to Laruelle's de-ontologized material experimentation uni-lateral to the Real; to Thom's discussion of the aporia of the technical opposition between continuity and discontinuity. The de-ontologized metaphysics of Laruelle, which is axiomatically derived (as opposed to a theological doctrine or philosophical transcendence) is explained here:

> Philosophers, Laruelle insists, do not know what they are doing. They are never doing what they say or saying what they are doing—even and especially when they purport to be able to legitimate their philosophical decisions in terms of some ethical, political or juridical end. The theoreticist idealism inherent in decision is never so subtle and pernicious as when it invokes the putative materiality of some extra-philosophical instance in order to demonstrate its ‹pragmatic worth›. To condemn Laruelle for excessive abstraction on the grounds that the worth of a philosophy can only be gauged in terms of its concrete, extra-philosophical (e.g., ethical, political or juridical) effects is

> to ignore the way in which extra-philosophical concretion invariably involves an idealized abstraction that has already been circumscribed by decision...
>
> In non-philosophy, radical axiomatic abstraction gives rise, not to a system or doctrine inviting assent or dissent, but to an immanent methodology whose function for philosophy no one is in a position to evaluate as yet. Ultimately, then, non-philosophy can only be gauged in terms of what it can do. And no one yet knows what non-philosophy can or cannot do.[12]

At the level of physics, an Aristotelian realism makes the most sense, which is known in the sciences as «ontic structural realism.» Why? Epistemology, at the level of structure, is either «in the thing» realism, «before the thing» Platonism or «after the thing» eliminative structuralism.[13] If one is of the realist camp, one would say that relativity exists in situ on the macro scale of large bodies, but that these structures don't appear in everyday life due to a scale problem. Similarly, catastrophic hysteresis means there is a lag between those infected with the SARS-CoV-2 virus between one week and the next, but only when a pandemic does in fact occur. But can we accept the entities described by physics as actually existing? Certainly it is true that special relativity is true as a framework. But as Negarestani has explained, Bas van Fraassen did not believe electrons really existed.[14]

12 Ray Brassier, «Axiomatic Heresy: The Non-Philosophy of François Laruelle,» *Radical Philosophy* 121 (September/October 2003): 34.

13 See Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford University Press, 1997).

14 Reza Negarestani (@NegarestaniReza), «No, atoms are not entities in the way we use that term in colloquial or traditional philosophical sense. I highly suggest the text that van Fraassen wrote on whether electrons are real or not,» Twitter, February 14, 2020, https://twitter.com/NegarestaniReza/status/1228459131419471882.

Negarestani has previously articulated an ontology defined by epistemology, which is not reducible to mere «ontological monism.» We will counter that categorical-structuralism provides the basis for a variant form of realism that is similar to «set-theoretic structuralism,» where «structures are isomorphism types (or representatives thereof) within the set-theoretic hierarchy.»[15] But the ontological commitments of such a system need not presuppose a «rich ontology of sets.»[16] The question still remains if category theory is isomorphism invariant. It is! But one could consider topos theory instead. But there is still the question of Grothendieck universes needed for topos theory. We will not be able to rigorously consider the ontology of topos theory unless we consider the Tarski axiom which presupposes an inaccessible cardinal (i.e., infinity). But, for example, Andrew Wiles in his proof of Fermat's Last Theorem did not need to even invoke these uncountable universes. The solution I believe to this foundational problem would be to pursue inquiry into a (more contemporary) field known as homotopy type

15 Geoffrey Hellman and Stewart Shapiro, *Mathematical Structuralism* (Cambridge University Press, 2018), 2.

16 See Erich Reck and Georg Schiemer, «Structuralism in the Philosophy of Mathematics,» Stanford Encyclopedia of Philosophy, accessed July 28, 2021, https://plato.stanford.edu/entries/structuralism-mathematics/, «However, there is also an important difference between standard if-then-ism and a category-theoretic approach in terms of the ontological commitments involved, as Awodey points out. According to standard if-then-ism, any mathematical statement can be translated into a universally quantified conditional statement, where the quantifiers are effectively meta-theoretic in nature, ranging over all set-theoretic systems of the right type. As such, the approach presupposes a rich ontology of sets in which such systems can be constructed. In contrast, along category-theoretic lines mathematical theorems do not involve such ontological commitments. There is no implicit generalization over the Bourbaki structures of a theory, e.g., over all groups, rings, or number systems. Rather, a mathematical theorem is ‹a schematic *statement* about a *structure* [...] which can have various *instances*›. These instances remain undetermined on purpose, unless a further specification of them is needed for the proof of the theorem in question.»

theory, which views things as instances, known as ‹tokens›.[17]

Does the recent work in category theory present an argument for structuralism, as a completely top-down (by its intensional nature) approach to math (as opposed to bottom-up constructions) or should we pursue the constructive computationalism of homotopy type theory? The «ontic structuralist realist» James Ladyman has suggested homotopy type theory is a foundation for mathematics.[18] Awodey has said it provides a philosophical structuralism.[19] Other examples of this notion of top-down classification in mathematics include Gaussian curvature (which is an invariant for different types of elliptic, hyperbolic, or Euclidean geometry), Klein's group-theoretic program of non-Euclidean geometry (which are group invariants, such as cross ratio, for different types of affine or projective geometry), and category theory. This top-down organization of mathematics presents a multiplicity of ‹case(s)› through general

17 See ibid., «A rejoinder should be added, however. McLarty's and Awodey's claim that all mathematical properties expressible in categorical set theory are isomorphism invariant has been contested, e.g., in Tsementzis (2017). In fact, Tsementzis argues that neither ZFC nor ETCS provide fully structuralist foundations for mathematics, since their respective languages do not, after all, exclusively allow for the formulation of invariant properties. Then again, both Makkai's FOLDS system (Makkai 1995, Other Internet Resources, 1998) and the Univalent Foundations program developed in homotopy type theory (Univalent Foundations Program 2013) seem to meet this condition.»

18 See James Ladyman and Stuart Presnell, «Does Homotopy Type Theory Provide a Foundation for Mathematics?,» *British Journal for the Philosophy of Science* 69, no. 2 (June 2018).

19 See James Ladyman and Stuart Presnell, «A Primer on Homotopy Type Theory Part 1: The Formal Type Theory« (Unpublished manuscript, University of Pittsburgh, 2014), 22, http://philsci-archive.pitt.edu/11157/1/HTT_Primer-PART-1.pdf, «Essentially, we are assuming that the mathematics founded in constructive logic that we study is no worse off as regards ontological questions than any other branch of mathematics on any other foundation. Further, we assume that whatever solutions can be found to philosophical problems regarding ontology of mathematics in other settings can equally well be applied here, in a way that is compatible with the foundational assumptions we've made. Homotopy type theory fits particularly well with structuralism in the philosophy of mathematics (see Steve Awodey ‹Structuralism, Invariance and Univalence›) but we do not pursue this in the present work.»

‹invariants› (topological, group-theoretic or categorical) which constrain the (group-theoretic or categorical) objects into varying geometries (Klein's method) or algebraic structures (category theory).

The «highest truth,» to build off Alexander Boland's point about «craftsmanship,» follows what he calls intuitive compositionality or Spinoza's intuition. In a categorical context, objects are literally only unique «up to isomorphism» and morphisms are the generalizing tool, which leads back to the question of «the conceptual level» mentioned before.[20] To qualify an idea by Boland where he states «I believe everything is a matter of aesthetics simply because all of epistemology is autopoietic,»[21] I would counter with «everything is a matter of aesthetics since all epistemology is conceptual. Conceptual mathematics and the axiomatic method are geometric. Therefore, everything, by way of epistemological delimiters, is a matter of aesthetics.» This is «conceptual poetry» (from the axioms) as opposed to «autopoiesis»; because instead of a homeostatic system (in the case of Niklas Luhmann and the imperfect model), there is a sense of «sheen» in the multivariate calculus of a manifold in which the «geometric intuition» is pure, as if pre-constituted. Pure mathematics is a matter of aesthetics according to the mathematician Hardy in *A Mathematician's Apology*. I am not arguing for Platonic Forms but rather for some form of Aristotelian realism, where logic and geometry converge,

20 Giuseppe Longo, «The Constructed Objectivity of Mathematics and the Cognitive Subject,» in *Quantum mechanics, mathematics, cognition and action. Proposals for a formalized epistemology*, eds. Mugur-Schachter Mioara and Alwyn Johannes van der Merwe (Cham: Springer Nature, 2003), 433–463.

21 Alexander Boland, personal website, accessed April 21, 2021, https://alexbo.land/.

and that structure takes place in exact terms within the Real itself but requires *hypothetical* actualization to confirm, such as the thought-experiment of Einstein's special relativity. There is no universal truth in the heavens. The Real is a necessary cause but the actuality need only be sufficient and not directly manifest (de-actualized worlds that have a unilateral source, the Laruellean Real). The conceptual invention of topological machinery, for example in a topos, allows for radical approaches between knowing subject and the ostensibly fixed object. ‹Base changes› allow for the freedom to move from one level of ‹open sets› to another in the navigation of a topos. The invention of this conceptual machinery which formalizes intuition at the highest level of abstraction is novel for «unify[ing] deep insights on arithmetic (number) and geometry (form).»[22] The conceptual invention of the epistemologically unmoored geometry, from the wellspring of the individuation/ontogenesis, is a becoming. These ‹germs› (in a ‹section› of a ‹stalk›) are «salient forms» which have been individuated from the «pregnance» emanating from «source-forms» (to use Thom's words from *Semio Physics*). This is morphogenesis, of the realm of aesthetic creation, individuated through germs of a ‹sheaf›. Dynamic perspectives. But through the axiomatic nature of such a geometry, we hope to defend a de-ontologized metaphysics that is not an «abusive hypostasis.» The axiomatic method allows for pure aesthetic fabrication (in the sense of Hardy), but one

22 Fernando Zalamea, «Multilayered Sites and Dynamic Logics for Transits between Art and Mathematics,» *Glass Bead, Site 0: Castalia, The Game Of Ends And Means* (2016): 13, https://www.glass-bead.org/wp-content/uploads/multilayered-sites-and-dynamic-logics-for-transits-between-art-and-mathematics_en.pdf.

that is freestanding and geometric, as opposed to Gottlob Frege's schematic assertion of God.

In imperfect cases, such as Luhmann's autopoietic systems (social or dynamical), these predictive determinisms have outliers which aren't predicted. The question ultimately remains whether or not all systems beyond thermodynamics (such as economics, for example) are ergodic? Boland has made the point in conversation that cybernetics is not ergodic. This is all to suggest that at a deeper level, there are cases when systems abide by larger «extant forms» and are not simply reducible to the recalibration of a system (predictions which only hold in 99.99% of cases, which is why there are outliers such as the Great Depression). That is, there are forms which are algebraically precise in a geometric/topological sense (for example, see topological invariants such as compactness, that every open cover has a finite subcover) and are therefore, more intuitive. We could take an economic space and ask if it is convergent. If it is metric, then the space will be compact when it is complete and totally bounded. So, if we can locate some upper bound for the supply and demand, and if all the limits are contained in the metric space, then it would be complete. So the value of a product (in economics) may follow some heuristic such as the «law of diminishing returns,» but this is only a principle and not naturalized since how can we define the topology of the space of utility (in the same sense, Moore's law is not exact in a mathematical sense either). Some truths exist in their algebraic exactness of terms, which can't be simplified to the heuristic of a system. One example is generalizing ba-

sic arithmetic to a topos wherein logic becomes inherently geometric. According to Alain Connes and Caterina Consani, one can think of the integers as geometric through the semiring of tropical integers as a sheaf of sets over a topos.[23] The case of the logic of catastrophe modeling or Monte Carlo simulation, which is the ‹autopoietic›, is prone to ‹error› and different from the more precise geometric formalism, that is still axiomatic.

At the formal level (in the manner of Hilbert's formal system for geometry), eliminative structuralism for categorical structuralism or what Elaine Landry calls ‹semantic realism›, which is ‹schematic› as opposed to ‹assertoric›, makes the most sense epistemologically, in the sense that objects can be viewed from the exterior rather than interior, through the categorical approach: all that matters is «arrows only» (morphisms between objects). The meaning of category theory can be understood intensionally. I will conclude the *Prolegomenon* with a postscript which discusses the social sciences and the mathematical presuppositions of cybernetics/economics which are problematic. For example, a Java program would not be able to compute certain functions due to uncountability. We therefore suggest to the reader interested in an alternative ontology (to this problematic one) to pursue the ontology advocated by Diedrich Diederichsen in his «Intimacy and Gesamtkunstwerk» in *Kai Kein Respekt* (predicated on «homosociality»).

23 See Alain Connes and Caterina Consani, «The Arithmetic Site,» *Comopte Rendus – Mathematique* 352, no. 12 (December 2014).

II. Topos through navigation between the local and global

> All this work culminated in another notion, thanks to Grothendieck and his school: that of a *topos*. Even though toposes appeared in the 1960s, in the context of algebraic geometry, again from the mind of Grothendieck, it was certainly Lawvere and Tierney's (1972) elementary axiomatization of a topos which gave impetus to its attaining foundational status. Very roughly, an elementary topos is a category possessing a logical structure sufficiently rich to develop most of «ordinary mathematics,» that is, most of what is taught to mathematics undergraduates. As such, an elementary topos can be thought of as a categorical theory of sets. But it is also a generalized topological space, thus providing a direct connection between logic and geometry.[24]

The foundational problem since Kurt Gödel's Incompleteness Theorem is whether any formal system is complete and/or inconsistent. A system is complete if «all truths expressible in Σ are theorems.»[25] A system is inconsistent wherein both P and Not-P are proven to be true. Moreover, the Peano Arithmetic (derived from the Peano axioms), if it is consistent, cannot prove its own consistency. This proof is shown by «translating the workings of Turing machines into arithmetic.»[26] Kripke-Joyal semantics has been shown to be complete. Kripke-Joyal semantics views propositions as true within possible worlds:

24 Jean-Pierre Marquis, «Category Theory,» Stanford Encyclopedia of Philosophy, accessed July 27, 2021, https://plato.stanford.edu/entries/category-theory/.

25 John Stillwell, *Concise History of Mathematics for Philosophers* (Cambridge University Press, 2019), 51.

26 Ibid., 62.

> The basic idea of the semantics is that a proposition is necessary if and only if it is true in all «possible worlds.» The idea is made precise as follows:
>
> A Kripke frame is a set W, the elements of which are called possible worlds, together with an accessibility relation R, that is, a binary relation between elements of W. A Kripke frame becomes a Kripke model when a valuation is given. A valuation *val* takes a world *w* and an atomic formula *P* and gives as value 0 or 1, to determine which atomic formulas are true at what particular worlds.[27]

Each reference frame provides a truth value of whether the atomic formula is true in such-and-such world.

What is most interesting about this «possible worlds» semantics is its use within topos theory.

> As part of the independent development of sheaf theory, it was realized around 1965 that Kripke semantics was intimately related to the treatment of existential quantification in topos theory. That is, the ‹local› aspect of existence for sections of a sheaf was a kind of logic of the ‹possible›. Though this development was the work of a number of people, the name Kripke–Joyal semantics is often used in this connection.[28]

Topos theory uses an intuitionistic Kripke logic and can be visualized according to Figure 2.

27 Sara Negri, «Kripke completeness revisited,» in *Acts of Knowledge: History, Philosophy and Logic*, ed. Giuseppe Primiero (Rickmansworth: College Publications, 2009), 237.

28 «Kripke-Joyal semantics,» Wikipedia, accessed July 18, 2021, https://en.wikipedia.org/wiki/Kripke_semantics#Kripke%E2%80%93Joyal_semantics.

$$F, G, H\colon C^{op} \longrightarrow Set$$
$$\sigma\colon F \rightarrowtail H,\ \tau\colon G \rightarrowtail H$$

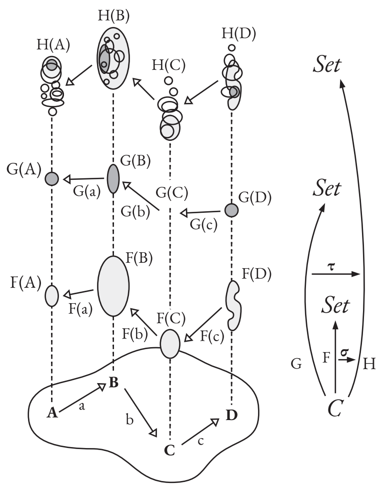


Fig. 2 *Topos of variable sets*

Each ‹stalk› (i.e., vertical line) over A, B, C, D, in *C* has a ‹germ› at various levels, i.e., the circles at the different levels; these are ‹sets› above the ‹site›. Here is the definition of a topos:

> [O]ne can also find three fundamental concepts in Grothendieck's toposes: at the *base*, the *Grothendieck topology J* in an adequate category *C* and its associated *site* (*C*,*J*); at the approximation level, a collection of *sheaves* with its gluing properties; at the *ideal* level (or point at infinity), the emerging *complete structure of the topos* with its limits, exponentials, and classifier object.[29]

We can begin to dissect what a topos is, it is a «category of sheaves of sets on a site *C*»; for a more precise exposition of a topos and the Grothendieck topology in more general cases, see footnote.[30]

Fernando Zalamea, philosopher of math, explains:

> In a similar vein, Grothendieck toposes (categories equivalent to categories of sheaves over abstract topologies, 1962) constitute *plastic sites*, specifically open to dynamic variations. Grothendieck toposes unify deep insights on arithmetic (number) and geometry (form). Beyond Cantorian, classical, static sets, the objects in a topos are to be understood as generalizations of *variable sets* [see Figure 2]. Instead of living over a rigid bottom, governed by classical logic, they live over a dynamic Kripke model, governed by *intuitionistic* logic. Beyond the classical example of the separated sheaf of holomorphic functions, a sheaf does *not* have to be separated in a general topos: *points*

29 Fernando Zalamea, «An Elementary Peircean and Category-Theoretic Reading of Being and Event, Logics of Worlds, and The Immanence of Truths,» *Filozofski Vestnik* 41, no. 2 (2020): 400.

30 See Luc Illusie, «What is a topos?,» *Notices of the American Mathematical Society* 51, no. 9 (October 2004): 1060, «To give a *topology* (sometimes called a Grothendieck topology) on *C* means to specify, for each object *U* of *C*, families of maps $(U_i \to U)_{i \in I}$, called *covering families*, enjoying properties analogous to those of open covers of an open subset of a topological space, such as stability under base change and composition. Once a topology has been chosen on *C*, *C* is called a *site*, and one can define a *sheaf of sets* on *C* in the same way as in the case in which *C* is the category of open subsets of a topological space: a sheaf of sets *E* on *C* is a contravariant functor $U \to E(U)$ on *C* (with values in the category of sets) having the property that for any covering family $(U_i \to U)_{i \in I}$, a section *s* of *E* on *U*, i. e., an element of $E(U)$, can be identified via the «restriction» maps with a family of sections s_i of *E* on the U_i's that coincide on the «intersections» $U_i \times_U U_j$. A *topos T* is a category equivalent to the category of sheaves of sets on a site *C* (which is then called a *defining site* for *T*).»

> *do not have to determine their associated objects*. We can even imagine objects without points, defined only through *flux processes*. A wonderful example is the topos of *actions* of monoids. Such a topos has an underlying classical logic (where the law of excluded middle holds and points are essential) if and only if the monoid is a group. Thus, when we deal with structures which are monoid non-groups, the logic of their action is just *intuitionistic*, *non-separated*, closer to topological fluxions, deformations, disruptions.[31]

One can think of sheaves as follows. Here is a «visualization of a sheaf in terms of stalks over each point of the space»[32]:

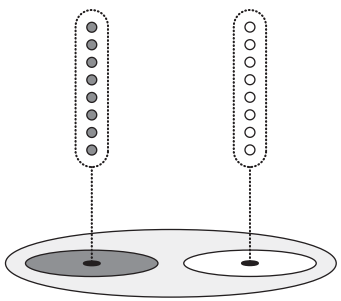
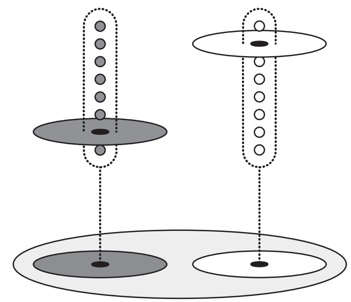


Fig. 3 to 5 *Sheaves with sections, germs and gluings*

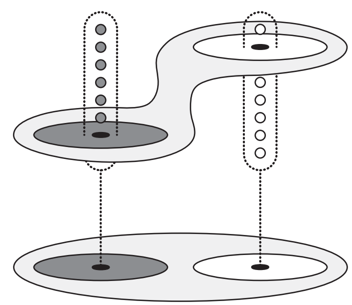
31 Fernando Zalamea, «Multilayered Sites and Dynamic Logics for Transits between Art and Mathematics,» *Glass Bead, Site 0: Castalia, The Game Of Ends And Means* (2016), https://www.glass-bead.org/wp-content/uploads/multilayered-sites-and-dynamic-logics-for-transits-between-art-and-mathematics_en.pdf.

32 «Talk:Sheaf (mathematics),» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki?title=Talk:Sheaf_(mathematics)#Some_visualization.

Next, one can take «particular sections (associated to a germ of the stalk) corresponding to each of the dark gray and white open sets of the space.»[33]



Then, one can «glue together two sections to create a section of the larger open set that they provide a cover of.»[34]



33 Ibid.

34 Ibid.

This is a radical breakthrough in the way one can view objects in an epistemological sense. The conceptual breakthroughs of topos theory are insinuated in Negarestani's essay «Where is the Concept?» on localization. Negarestani even uses the terminology of ‹site› and ‹topos› at times in the essay. One of the primary methods of epistemology in this *Prolegomenon* will be modeled on this approach of the localization of the concept, which Negarestani characterizes by its epistemological depth to «unmoor» the subject from fixed points and «cancel» any archetypal relation between subject and object, immersing the subject directly within the «navigational landscape»:

> Just like the desert that is one and the same and precisely because of its homogeneity we don't have access to its landscape, the monism of nature does not allow us to know nature without organizing an epistemic breakage. Ontologically, nature does not distinguish itself from itself. Monism is in this sense an ontological reality that demands a necessary epistemic strategy: Exactly because of this excess of informational homogeneity—a desert that is one and the same everywhere—we can't immediately approach nature or navigate it. The nature-culture division is an epistemic division, not an ontological one. From the possibility of epistemic traction, this division is necessary and far from rigid. It provokes approaches to nature hitherto unimagined. To claim that everything is nature is at best an indulgence in the vulgarity of the obvious and at worst, a complete blindness to the epistemic conditions through which we are able to progressively make sense of nature.
>
> The bimodalization of the universal to its global and local horizons is a navigational strategy which must be conceived

> through a local rupture, a regional discontinuity. To create or conceive this local rupture is the basic gesture behind the formation of the concept as a local site distinguished by its qualitatively differentiated information. It is the concept as a regional breakage or local disturbance in the qualitatively homogenous information that provokes approaches and pathways impossible in the absence of the epistemic rupture. Trying to understand nature without an epistemic division, solely through the ontological monism, is an appeal to mysticism. It results either in an ineffable conception of nature or an image of nature as a reservoir of meanings and stories about itself. Once we insist that the world is a repository of meanings, that it has stories to tell the subject without any demand for the subject to create a necessary epistemic condition, then we have already committed to conserve a stable relation between the knowing subject and the world. The world is always facing the subject as if it wants to tell a story, there is no need for the subject to destabilize its given status, to epistemically uproot itself so as to procedurally navigate the landscape. The subject of the world as a ready-made object of experience and a reservoir of meanings is quite stubbornly an anthropocentric and conservative form of subject even though it claims to be completely the opposite.
>
> Localization should be understood in terms of bringing about an epistemic condition that once rigorously pursued cancels any conserved relation between the knowing subject and the world, rather than anchoring the subject in any specific place, it unmoors the subject within a navigational landscape. This is the deracinating effect that registers itself as a condition of enablement insofar as it liberates epistemic possibilities which until now had remained captives of the tyranny of here and now—that is, the knowing subject tethered to a local do-

> main and a privileged frame of reference. Localization has obvious implications for thought not only because we ourselves are local instantiations within the terrestrial horizon, but also from an epistemological perspective: the concept as the space through which we gain traction on the world is a local horizon. As the most fundamental unit of knowledge, the concept is a local horizon, a locally organized space of information within a vast inferential economy and immersed within the general structure of knowledge. So the question of localization allows us a form of systematic study of the local context, and in particular a systematic analysis of conceptual behavior. In this sense, we can say that localization is the ultimate procedural framework of thought. It's a procedure—even a gradualist and stepwise procedure—because, as I shall argue, the local is not rooted. Its analysis is not a matter of zooming in and out on a specific point. Instead the examination of the local requires a procedure to follow it in a navigational context, in relation with other local horizons, via different directions and addresses. No axiomatic commitment at the level of the local makes sense unless through this procedure, which is to say, only when we localize parameters and orientations or generally speaking identify what makes a local domain local. The local is not a fixed point in space, it is a mobile framework immersed within a generic environment. Its internal analysis is always coupled with an external synthesis.[35]

To get a sense of what Negarestani exactly means by ‹localization›, one would need to familiarize oneself with the work of Thom. The assertion that «the bimodalization of

35 Reza Negarestani, «Where is the Concept?,» in *When Site Lost the Plot*, ed. Robin Mackay (Falmouth: Urbanomic, 2013), 231–233.

the universal to its global and local horizons is a navigational strategy which must be conceived through a local rupture» is a direct allusion to Thom. Thom was famous for his catastrophe theory where a local rupture creates an epistemological breakage in the fabric of homogeneous information. Catastrophe theory was thought to be a method for predicting catastrophes but the applications of the field proved to be not so fruitful. But the largest import the field had on mathematics was Thom's invention of the term «attractor,» which would be important in the field of chaos theory. One of the main reasons predictions and applications were not obtainable was that Thom's approach was completely topological and was an analogical hermeneutics of data and therefore a *qualitative* study of quantitative data. The motivation of this approach was to undertake a natural philosophy project which could unify the body and soul into a geometric object.[36] To get a better sense of the procedure of localization, we will examine one of the seven elementary catastrophes which Thom describes:

36 See Peter Tsatsanis, «On René Thom's Significance For Mathematics And Philosophy,» *Scripta Philosophiæ Naturalis* 2 (2012): 213–229.

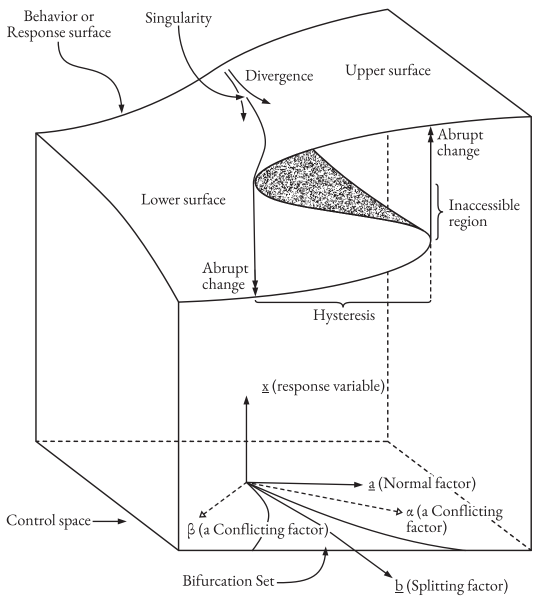


Fig. 6 *Cusp catastrophe*

This example is known as the ‹cusp catastrophe›. Here a local rupture creates a sudden jump between the upper stable region and the lower stable region. For Negarestani, this divergence creates the possibility, through epistemology, of grasping the problematic ontology of a monist space. In the picture, as the *x* and *y* coordinates change in the cusp region, the *z* coordinate is very unstable (an example of a catastrophe

would be a hurricane). Pictured is what is known as hysteresis or a lag between one state and the next state. A good example of hysteresis (a lag) in general would be the number of cases of SARS-CoV-2 virus between one week and the next week, where the number of cases detected grows as new testing detects newer cases. As Anthony Fauci states, «[w]hen you are dealing with a virus outbreak, you're always behind where you think you are.»[37] A true example of a catastrophe is «the change in shape of an arched bridge as the load on it is gradually increased. The bridge deforms in a relatively uniform manner until the load reaches a critical value, at which point the shape of the bridge changes suddenly—it collapses.»[38] The key to localization is the liberation of the subject from the tyranny of a fixed point or moment or perspective. As Negarestani says:

> [T]he deracinating effect that registers itself as a condition of enablement insofar as it liberates epistemic possibilities which until now had remained captives of the tyranny of here and now—that is, the knowing subject tethered to a local domain and a privileged frame of reference.[39]

The paradigm of a «conserved relation» between subject and object in the speculative sense is overthrown. The

37 Brian Resnick, «Scientists warn we may need to live with social distancing for a year or more,» *Vox*, March 17, 2020, https://www.vox.com/science-and-health/2020/3/17/21181694/coronavirus-covid-19-lockdowns-end-how-long-months-years.

38 «Catastrophe theory,» Encyclopedia Britannica, accessed March 9, 2022, https://www.britannica.com/science/catastrophe-theory-mathematics.

39 Reza Negarestani, «Where is the Concept?,» in *When Site Lost the Plot*, ed. Robin Mackay (Falmouth: Urbanomic, 2013), 233.

philosophy of armchair philosophers is over. Negarestani goes on to describe the recursive anatomy of the process of ‹localization›:

> The question ‹Where is the concept?› demands a methodology for approaching or seeing the concept from a perspective that is adjacent, rather than from a perspective that is fixed upon it. This is by way of a recursive procedure in which we say ‹Where is the concept?› and then we repeat this procedure. In other words, localization (*Where?*) is combined with recursion. Since localization is an analytico-synthetic procedure that unbinds new alternative addresses for a local site/concept, then repeating localization means that new paths branch from the existing paths. This diffusion of pathways or addresses for a local horizon is registered as a *ramified path structure* where new alternative addresses and opportunities for local-global synthesis are progressively unfolded. As I argued the concept as a local site is fringed with possibilities of alternative reorientations. The ramified path structure suggests a form of step-wise navigation via these possible reorientations through which the concept is simultaneously studied, traced, revised and constructed. [...]
>
> We ask ‹where is *x*?› then we repeat the question over and over. Every time we localize *x*, we see or approach it from a new address in the environment in which *x* is immersed. In our example of a point, the point can be seen not only according to new variable coordinates but also according to different layers of organization. It can be conceived arithmetically, geometrically, algebraically, topologically, and so forth. Accordingly, the operation ‹localization recursion› yields new ramifying paths and in doing so broadens the scope of navigation—that is to say, the constructive passage from the local to the global.

> An intuitive way to understand the procedure involved with recursive localization and its imports is as follows: Think of the planet Earth. When we are standing on the surface of the planet, we are occupying a location on its geodetic surface. From a local perspective, the geodetic surface appears to be flat and not curved. While occupying this point on the surface, if we ask ‹where is the earth?›, because of our immediate access to this local point where the global properties are perceived differently (i.e., locally), we would say the earth is a fixed sphere, we might even say it is just a flat surface. However, if we launch a perspective operator—a satellite—into the orbit and take pictures of the planet, upon compiling and integrating these pictures we will notice that the planet is fully mobile and it is spheroid. However, if we repeat this procedure from a broader neighborhood and take new orbital portraits or maps of the planet, then we will observe that not only is Earth spheroid, but also it is located within a celestial system held together by the gravitational force. This is a very intuitive and rather trivial understanding of how the product of localization and recursion works: By localizing the horizon and by way of reverse-engineering it from its orbit (i.e., possibilities of reorientation), rather than from its local fixed coordinates (i.e., information readily available by occupying a local section of it), recursive localization identifies the local horizon according to the site wherein it subsists. But from the perspective of recursive localization, the site is nothing but a cascade of ramifying paths and addresses. It is in the wake of these ramifying paths that the characterization of the local, its problems, imports and implications become a matter of navigation—that is to say, analysis and synthesis, remapping and reorientation, revision and construction.[40]

40 Ibid., 242–243.

«Recursive localization» becomes a process of «ramification» and «navigation.» Through identifying the horizon of my locale, one is able to «reverse-engineer» the pathways possible from my address of the setting (i.e., from the restricted and bounded information of a local site) and identify more and more global contexts and addresses to conceptualize my locale. A good example, as Negarestani explains in a video, is the Möbius Strip that locally appears one way but globally another (it has only one side).[41] Another example would be a torus, where locally, the surface appears to resemble the space of $\mathbb{R}^2$ the real number system, but if we zoom out, it actually appears to be a donut, which has certain homological properties different than a sphere, which is locally homeomorphic to $\mathbb{R}^2$ (i.e., it has two ‹holes› from the homology group $\mathbb{Z} \oplus \mathbb{Z}$).

> The Poincaré-Hopf Theorem relates local behavior of some differentiable object, namely C^∞ vector fields, to the global topological structure of the manifold. Numerous theorems of this general type, relating local invariants to global structure, have been discovered. These form a large but unified subject called *global differential geometry* or *differential topology*.[42]

It is important for epistemology to integrate all of these different scales into a synthetic navigation of the landscape. If one were to abide by classical notions of subjectivity, one would be still living in the paradigm of a «flat earth» because of the tyranny of a fixed point. The epistemology that is restricted to a local domain and abuses this framework is also the culprit behind why humans thought the universe was geocentric (as opposed to heliocentric), as there was not enough information from a more universal context.

41 «Reza Negarestani interview,» Vimeo, accessed April 2, 2020, https://vimeo.com/271301170.

42 Theodore W. Gamelin and Robert Everist Greene, *Introduction to Topology* (Mineola, New York: Dover Publications, Inc., 1999), 188.

III. Minimal metaphysics without ontology

> In genetic epistemology, as in developmental psychology, too, there is never an absolute beginning. We can never get back to the point where we can say, «Here is the very beginning of logical structures.» As soon as we start talking about the general coordination of actions, we are going to find ourselves, of course, going even further back into the area of biology. We immediately get into the realm of the coordinations within the nervous system and the neuron network, as discussed by McCulloch and Pitts. And then, if we look for the roots of the logic of the nervous system as discussed by these workers, we have to go back a step further. We find more basic organic coordinations. If we go further still into the realm of comparative biology, we find structures of inclusion ordering correspondence everywhere. I do not intend to go into biology; I just want to carry this regressive analysis back to its beginnings in psychology and to emphasise again that the formation of logical and mathematical structures in human thinking cannot be explained by language alone, but has its roots in the general coordination of actions.[43]

Jean Piaget was a structuralist psychologist who had even once spoken with Jean Dieudonné of the Bourbaki mathematicians. Piaget makes the argument that language cannot be the autonomous building block for logical and mathematical structures. Piaget noticed that children learn about basic notions of ‹commutativity› when counting pebbles. Children realize that when they count pebbles

43 Jean Piaget, *Genetic Epistemology*, trans. Eleanor Duckworth (New York: W. W. Norton & Company, Inc, 1971), 19.

one way or another way, they get the same result. This is a basic mathematical property known as commutativity where $a + b = b + a$. The equation is the same whether you add a first or if you add b first. Piaget says in his *Genetic Epistemology* that it is with children that we have the best chance of «studying the development» of logic:

> Since this field of biogenesis is not available to us, we shall do as biologists do and turn to ontogenesis. Nothing could be more accessible to study than the ontogenesis of these notions. There are children all around us. It is with children that we have the best chance of studying the development of logical knowledge, mathematical knowledge, physical knowledge, and so forth.[44]

But Piaget even suggests further, that as soon as we locate some beginning of logic, we have to go back even further to biology and the development of biological structure. Piaget espouses a philosophy similar to Simondon's idea of individuation (which we will discuss in the next chapter). As soon as we locate some individual element, we are already too late because the process of individuation/ontogenesis has already occurred. We can trace this reasoning back to the idea that the universe that we live in, consisting of discrete elements, goes back to a previous universe of continuous protoplasm. Genetic epistemology has no «absolute beginning.» Thom says that «the fundamental aporia of mathematics is this opposition of discrete-continuous»:

44 Ibid., 13–14.

[René Thom] For me, the fundamental aporia of mathematics is this opposition of discrete-continuous. At this same time, this aporia has a dominant role in all thinking.

[Emile Noël] You've said to me that, in the course of developing category theory, you were trying to relate apparent discontinuities to an underlying continuity. About a century ago there existed a controversy with regard to the central nervous system: Is it continuous or discontinuous? We all know that our perceptions are discontinuous. The question was answered by anatomy: Santiago Ramón y Cajal was right. Neurons are contiguous. That may have nothing to do with human psychology. On the other hand, what is your view on this matter?

[R.T.] To begin with, I don't agree at all with your statement that everyone knows that our perceptions are discontinuous. When I look at you, I'm seeing you in a continuous fashion!

[E.N.] I withdraw the term ‹perception›. I really mean ‹sensation›: The sensory receptors function in a discontinuous fashion.

[R.T.] Now you're talking like a neurophysiologist! However, I fall back on my primary intuition. What gives you the right to say that a researcher in neurophysiology has more insight than my own first impressions? I reject that argument.

[E.N.] Yet certain things must be accepted as givens. A few moments ago, you used the example of a film: Images pass in succession, and, thanks to a bit of ingenuity, an instrument, and to retinal after-imaging, one experiences continuity. But when I try to understand how the world works, I sense that there is a continuous basis upon which all things unfold in a

more or less discrete fashion. Where does one place discontinuity with regards to the relationship between the continuous and the discrete?

[R.T.] For myself, I'm happier with the notion that the discrete is manufactured from the continuous, rather than that continuity arises from the discrete. I realize of course that the standard model in contemporary mathematics is based on the definition of number given by Dedekind, by what are called Dedekind cuts. This makes it possible, in theory, to construct continuity directly out of arithmetic, that is to say, on the basis of the discontinuous. However, this process is in reality highly nonconstructive. It amounts to saying: The real numbers can be constructed by taking rational numbers and bringing them indefinitely close to one another. Then if one makes a cut, that is to say, a division of the rationals into two classes, in such a way that each rational in the first class is less than each rational in the second class, (with the understanding that the differences between them approaches zero), this can be taken as the definition of a real number.

It's the traditional method for making Gruyère cheese: You take some holes and start building the cheese around them. There isn't very much cheese but there are lots of holes! Finally one ends up with no cheese at all, only holes! How is it possible, with nothing but holes, to fabricate a continuous and homogeneous paste? I must admit that it goes beyond me...

The origins of all scientific thought can be situated in the paradoxes of Zeno of Elea: notably the story of Achilles and the tortoise. In it, one finds the fundamental opposition between the continuous and the discontinuous.[45]

45 René Thom, «René Thom: Interviews with Emile Noël,» ed. Peter Tsatsanis, trans. Roy Lisker (Unpublished translation, Ferment Magazine, 2010), 101–103, https://www.fermentmagazine.org/Stories/Thom/Predire.pdf.

In a later chapter, contrary to Santiago Ramón y Cajal, it will be shown that perception is continuous through the research of Petitot and that vision requires a geometric ideality which Petitot calls «neurogeometry» (or «neurophysics» if you like) and which can be observed experimentally at the level of neurons. I agree with Thom that the discrete is born out of the continuous. But as Thom describes, the typical construction of the real numbers, the Dedekind cut, is the other way around (the continuous out of the discrete). The Dedekind cut uses two sets in order to construct a real, possibly irrational number. The definition is as follows:

> Suppose that we already know about some real number x.
>
> When we could define a pair of sets (A_x, B_x), where
>
> A_x is the set of all rational numbers y such that $y < x$, and
>
> B_x is the set of all rational numbers y such that $y > x$.
>
> The way to think about this is that you are cutting the number line by an infinitely thin knife, at x, and A_x is all the numbers to the left of the knife and B_x is all the numbers to the right. Part of this definition is very good: The sets A_x and B_x are both subsets of rationals, so they do not directly refer to real numbers. However, the problem with this definition is that it depends on us already knowing about the number x. The idea behind Dedekind cuts is to just work with the pairs (A, B), without direct reference to any real number. Basically, we just look at all the properties that (A_x, B_x) has and then make these «axioms» for what we mean by a Dedekind cut.[46]

46 Rich Schwartz, «Dedekind Cuts» (Unpublished manuscript, Brown University, 2014), 3, https://www.math.brown.edu/~res/INF/handout3.pdf.

Deleuze makes use of the Dedekind cut to define his «ideality of difference» which is the «third synthesis of time.»[47] We will suggest an alternative to this construction, in the spirit of Thom, one that moves from continuous to discrete, to found geometry on the use of infinitesimals which was made rigorous in the work of Lawrence A. Lawver's «synthetic differential geometry,» a new development in 20th century mathematics.

In *Semio Physics*, Thom argues for, «the necessity of restoring by appropriate minimal metaphysics some kind of intelligibility to our world.»[48] If we are to found our metaphysics upon some topological truths, we could start for example with the extensive use of the «continuous» in much of Thom's topology. Whilst Thom was most famous for his construction of ‹cobordism›,[49] his greatest influence in the applied sciences may be his coinage of the term ‹attractor›. The ‹basin of attractor› is technically defined for continuous dynamics as open neighborhoods when t approaches infinity for the phase space.[50] A neighborhood is a basic definition within topology and states that, U being an open set containing x is equivalent to U being a neighborhood of x.[51]

47 Daniella Voss, «Deleuze's Third Synthesis of Time,» *Deleuze Studies* 7, no. 2 (May 2013): 194–216.

48 René Thom, *Semio Physics: A Sketch*, trans. Vendla Meyer (Boston: Addison-Wesley, 1990), xi.

49 See «Bordism,» Wolfram MathWorld, accessed March 20, 2022, https://mathworld.wolfram.com/Bordism.html, «A relation between compact boundaryless manifolds (also called closed manifolds). Two closed manifolds are bordant iff their disjoint union is the boundary of a compact $(n+1)$-manifold. Roughly, two manifolds are bordant if together they form the boundary of a manifold. The word bordism is now used in place of the original term cobordism»; For further discussion of the contemporaneity of ‹bordism› as a term, see Max Hopkins, «The Extraordinary Bordism Homology» (Unpublished manuscript, UC San Diego, 2016).

50 «Is a basin of attraction necessarily an open set?,» StackExchange, accessed March 9, 2022, https://math.stackexchange.com/questions/1755009/is-a-basin-of-attraction-necessarily-an-open-set.

51 James Munkres, *Topology* (Essex: Pearson Education, 2014), 94.

Open sets are the fundamental constituents of topology, which we can even generalize to the cases of sheaves and Grothendieck topologies (discussed in the previous chapter). If we are really to define some «minimal metaphysics» we would need to re-examine the notion of Time as continuously running in the manner of the real numbers. To make sense of the real numbers, and therefore time, we could arrive at some topological properties. The real numbers are a ‹connected› space. Moreover, the infinity utilized in the real numbers is an ‹uncountable› infinity. Time as the real number system is therefore time as a continuous backdrop. Should we accept this continuum of time which is used in any system (dynamical, or even not dynamical)?

Moreover, central to notions of topology are morphisms known as homeomorphisms, which are continuous bijections between two topological spaces. Discreteness can be thought of as direct sums or direct products of the groups found within the homology groups of topological manifolds. While smoothness on top of continuous morphisms can be thought, in the study of differential geometry, as a diffeomorphism between two manifolds. Thom argues topology, in catastrophe theory, can be a method of making qualitative analysis of data based on regional discontinuities, such as a jump (as was discussed in the previous chapter). These discontinuities, or singularities, can be studied through the method of analytic continuation (which is the standard means of expressing the Riemann hypothesis for example).

As Peter Tsatsanis writes, the genius of Thom's metaphysics was to theorize the «monism of body and soul» through a geometric object.[52] But the intelligible statements were completely qualitative, as opposed to quantitative, based on an *analogical* method of reasoning that Thomas Aquinas had once described. The process of morphogenesis, even in biology, is the actualization of potential states which can be articulated as the differential Process moving in the control space (other examples of mathematical processes include stochastic processes, such as Markov processes). For Thom, morphogenesis is ontologized biology; ontogenesis is ontologized structuralism. Thom goes on to say:

> Modern science has made the mistake of foregoing all ontology by reducing the criteria of truth to pragmatic success. True, pragmatic success is a source of pregnance and so of signification. But this is an immediate, purely local meaning. Pragmatism, in a way, is hardly more than the conceptualized form of a certain return to animal nature. Positivism battened on the fear of ontological involvement. But as soon as we recognize the existence of others and accept a dialogue with them, we are in fact ontologically involved. Why, then, should we not accept the entities suggested to us by language? Even though we would have to keep a check on abusive hypostasis, this seems the only way to bring a certain intelligibility to our environment. Only some realist metaphysics can give back meaning to this world of ours.[53]

52 See Peter Tsatsanis, «On René Thom's Significance for Mathematics and Philosophy,» *Scripta Philosophiæ Naturalis* 2 (2012).

53 René Thom, *Semio Physics: A Sketch*, trans. Vendla Meyer (Boston: Addison-Wesley, 1990), xi.

It seems that we should consider that entities imply «ontological involvement,» even linguistic entities, in order to ascribe intelligibility to the world. And Thom is saying that meaning exists through a «realist metaphysics.» In my Twitter interaction with Negarestani, he has commented upon this Thom quote, saying: «Thom was a great mathematician but he himself mistook metaphysical theses with epistemical ones. This is why his name now in math and biology societies equals the patient zero of charlatanism.»[54] In order to be sure we are not hypostatizing an ontology which is supported by theology or some abusive axiom, I would suggest that our minimal metaphysics be ‹de-ontologized› in the manner of Laruelle's realism, which I will discuss in chapter VII on eliminativism. But the question then remains whether we are of the realism camp or eliminative camp. I would argue that we accept «ontic structural realism» while simultaneously deriving a «topological hermeneutics,» that is analogical in order to ascribe intelligibility to the world, but the necessity of such a metaphysics need be suspended. So, the real first cause would not be an Aristotelian realism of structure, even though physics is real and as a structure is *ontic* (i.e., there is an a priori on the level of relativity: light is absolute), but rather the real cause of ontology is the ineffability of the Real which can not be philosophized and can only be hypothesized in the manner of intuitionistic logics topoi and Grothendieck universes which require the Tarski axiom. But even with this ontology, Negarestani has said

54 Reza Negarestani (@NegarestaniReza), Twitter, February 25, 2020, https://twitter.com/NegarestaniReza/status/1232401640130564096.

we need to be careful: «Zalamea is a gentleman. But is this enough? Is this what we actually want: a sheaf of romantic vagaries where all math fields become the expressions of the sublime? Sheaves everywhere=tender-heart ontological hypothesization?»[55] Therefore, I would counter that the inaccessible cardinal assumed by topos theory is actually (according to my investigations) not invoked in the contemporary mathematics of algebraic geometry and number theory. Thom is arguing for some minimal metaphysics. But we can, in a eliminativist way, desublimate/de-ontologize this metaphysics for the purposes of a hermeneutics (of geometric logic, of physics, etc.) and treat any «ontological involvement» as merely an axiom which we do not need to invoke (for example, Wiles's non-usage of Grothendieck universes)! Through the axiomatic approach of topos theory, one is able to conceive of arithmetic or number as inherently geometric or form. We will discuss this breakthrough in the work of Connes in chapter XI, whose work in non-commutative geometry (mathematical physics) has been shown as means of expressing quantum field theory (theoretical physics). Is mathematics the hermeneutics of physics? The more challenging question remains if we should accept the ontology of physics; that is, do we accept that physical entities really do exist?

55 Reza Negarestani (@NegarestaniReza), Twitter, April 9, 2020, https://twitter.com/NegarestaniReza/status/1248366257541451777.

IV. Individuation

If we are to follow the thinking of Piaget, who said there is no «absolute beginning,» then we must consider the father of this thinking of individuation: Gilbert Simondon. Simondon's thinking had a great influence on the likes of Deleuze, Thom, Châtelet, Laruelle and Stiegler.[56] Below Simondon explains the metaphysical logic of individuation in his seminal essay «The Genesis of the Individual»:

> [H]ylomorphic theory decrees that the individuated being is not already given when one comes to analyze the matter and form that will become the *sunolos* (the whole): we are not present at the moment of ontogenesis because we have always placed ourselves at a time before this process of ontogenetic formation actually takes place. The principle of individuation, then, is not grasped at the point where individuation itself occurs as a process, but in that which the operation requires before it can exist, that is, a matter and a form. Here the principle is thought to be contained either in the matter or the form, because the actual process of individuation is not thought to be capable of *furnishing* the principle itself, but simply of *putting it into effect*. Thus, the search for the principle of individuation is undertaken either before or after individuation has taken place, according to whether the model of the individual being used is a physical one (as in substantialist atomism) or a technological and vital one (as in hylomorphic theory). In both of these cases, though, there remains a *region of uncertainty* when it comes to dealing with the process of individuation, for this process is seen as something that needs to be explained, rather than as something in which the explanation is to be found: whence the notion of a principle of individuation. Now, if this process is considered as something to be explained, this is because the received way of thinking is always oriented toward the successfully individuated being, which it then seeks to account for, bypassing the stage where individuation takes place, in order to reach the individual that is the result of this process. In consequence, an assumption is made that events follow a certain chronology: first, the principle of individuation; then, this principle at work in a process that results in individuation; and finally, the emergence of the constituted individual. On the other hand, though, were we able to see that in the process of individuation other things were produced besides the individual, there would be no such attempt to hurry past the stage where individuation takes place in order to arrive at the ultimate reality that is the individual. Instead, we would try to grasp the entire unfolding of ontogenesis in all its variety, and *to understand the individual from the perspective of the process of individuation rather than the process of individuation by means of the individual.*[57]

Deleuze was particularly influenced by Simondon's «pre-individual fields,» utilizing this idea in his articulation of «becoming-actual» out of the virtual. Simondon argues that an atomist metaphysics is wrong for attempting

56 See «Gilbert Simondon,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Gilbert_Simondon, «Gilbert Simondon: une pensée de l'individuation et de la technique (1994), the proceedings of the first conference devoted to Simondon's work, further charts his influence on such thinkers as François Laruelle, Gilles Châtelet, Anne Fagot-Largeau, Yves Deforge, René Thom, and Bernard Stiegler (the latter having placed Simondon's theory of individuation at the very heart of his ongoing and multi-volume philosophical project).»

57 Gilbert Simondon, «The Genesis of the Individual,» in *Incorporations*, eds. Jonathan Crary and Sanford Kwinter (New York: Zone Books, 1992), 299–300.

to locate the origin of structure in the individual atom. For the atomistic individual takes place after the process of individuation has already occurred. Instead, there is a pre-individual field from which the atom was individuated. We can turn to Michael Atiyah's explanation of this problem, whereby our world of the discrete (atoms) was individuated from protoplasm.[58] Similarly, the Aristotelian hylomorphic theory is wrong for attempting to locate the origin before individuation in the metaphysical notions of ‹matter› and ‹form› which individuation puts into place. Simondon makes the argument that we should make the principle of individuation itself the explanation for individual elements. Individuation is «a never-ending ontological process.»[59] Deleuze's own philosophy of actualization is based on Simondon's principle of individuation. Refer to Deleuze's fifth chapter of *Difference and Repetition*: «The Asymmetrical Synthesis of the Sensible.» For Simondon, one can think of individuation as the process by which a metastable system, with a multiplicity of potential states, stabilizes.

58 Michael Atiyah, «Michael Atiyah–Did we invent number theory? (89/93),» Youtube, accessed July 28, 2021, https://www.youtube.com/watch?v=Xu-SjsOAJbg.

59 «Individuation,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Individuation.

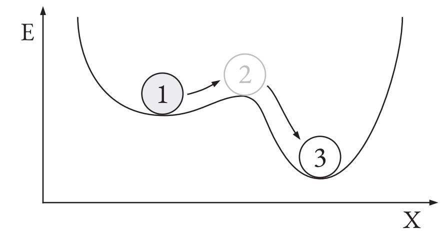


Fig. 7 *Metastable state of weaker bond (1), transitional ‹saddle› configuration (2), stable state of stronger bond (3)*

This potentia is the wellspring from which the conatus actualizes. Simondon makes the argument that there is an ontology to this process of individuation. This question concerned Leibniz in his theory of bodies in motion, as Gilles Châtelet explains (for Leibniz was a deist).[60] How could stationary bodies get out of rest? This relation between states is completely non-thetic. The system is an immanent system which resolves (unlike the Hegelian dialectic) by way of its own metastability. Central to Simondon (and Deleuze) is the notion of the ‹singularity› of the thing, which can be thought of as the specificity of the thing. Simondon writes:

> To a certain extent, the idea of a *principle of individuation* has been derived from a genesis that works backward, an ontogenesis «in reverse,» because in order to account for the genesis of

60 See Gilles Châtelet, *Figuring Space: Philosophy, Mathematics and Physics* (Dordrecht: Kluwer Academic Publishers, 1993).

> the individual and its defining characteristics one must assume the existence of a first term, a principle, which would provide a sufficient explanation of how the individual had come to be individual and account for its singularity (haecceity)—but this does not prove that the essential precondition of ontogenesis need be anything resembling a first term. Yet a term is itself already an individual, or at least something capable of being individualized, something that can be the cause of an absolutely specific existence (haecceity), something that can lead to a proliferation of many new haecceities. Anything that contributes to establishing relations already belongs to the same mode of existence as the individual, whether it be an atom, which is an indivisible and eternal particle, or prime matter, or a form. The atom interacts with other atoms through the *clinamen*, and in this way it can constitute an individual (though not always a viable one) across the entire expanse of the void and the whole of endless becoming. Matter can be impressed with a form, and the source of ontogenesis can be derived from this matter-form relation. Indeed, if haecceities were not somehow inherent within the atom, or matter, or indeed form, it would be impossible to find a principle of individuation in any of the above-mentioned realities. *To seek the principle individuation in something that preexists this same individuation is tantamount to reducing individuation to nothing more than ontogenesis. The principle of individuation here is the source of haecceity.*[61]

Haecceity for Simondon is to be found in the principle of individuation itself and ontogenesis cannot be merely reduced to a metaphysical atomism. It is through the individuation of the cell that the cell acquires its unique singularity. It is through individuation that the work of art acquires its aura. It is through individuation, through ‹becoming› (in the Deleuzian sense), that the actual becomes manifest from the wellspring of the virtual. For Deleuze, this actualization (‹differenCiation›) is based upon the realm of virtuality (‹differenTiation›), pre-individual singularities of the Idea. The latter is defined in the fourth chapter of *Difference and Repetition* and the former in the fifth chapter. The Idea is defined as the «relations of the universal»—difference-in-itself. The ‹a priori› for Deleuze is the «pure past,» that pre-exists the empirical present and is transcendental and metaphysical (in the pre-individual Idea), which the third synthesis then «makes extensive and expresses» (i.e., differenCiation) through its «correspondence» (unification) between present, «larval subject» («empirical ground») and «undifferenciated Idea of ‹pure past›»[62]:

> However, actualization happens as individuating difference is the determination of differentiation in the Idea, and not specification in the concept. So determination must be distinguished from specification and Idea from conceptuality. How so? Determination is to be understood as the distinction in the Idea as the pure virtual realm, whereas specification in concepts already refers to given differentiated concepts. The Idea as pure virtual is non-conceptual difference *in-itself*, insofar as it is merely latent in the whole, not already specified by

61 Gilbert Simondon, «The Genesis of the Individual,» in *Incorporations*, eds. Jonathan Crary and Sanford Kwinter (New York: Zone Books, 1992), 299–300.

62 Daniel Sacilotto, «Heidegger and Deleuze on Death (Nihil Unbound—Chapter IV),» Being's Poem, blog entry, March 8, 2010, http://bebereignis.blogspot.com/2010/03/heidegger-and-deleuze-on-death-nihil.html.

conceptual identity. Thus actualization achieves an *individuated difference* by determining an *immanent difference in the Idea*. Determination in actualization determines a) extrinsic difference of instances contracted in the present and b) the intrinsic difference between degrees of contraction in memory; the difference between present and past is determined as the difference between *extensive repetition* related to successive instants (living present of habit, expressing thought or larval subject) and *intensive repetition* of co-existing levels in the past (past as whole, contraction of time (pure past) by space/dilution of space by time (pure past)/expressed Idea of the virtual undifferenciated being).[63] It is a third synthesis of these two moments, as expressing and expressed which guarantees their correspondence, and their unitary being. Thought determines in the passivity of the larval subject (present as contracted past, empirical ground), the Idea is undifferenciated being (pure past as transcendental/metaphysical ground); between them is *the pure empty form of time* which transcendentally guarantees that the indeterminate (Idea) can become determinate.[64] This is a purely logical, non-chronological time void of empirical content (living present of habit) or metaphysical substance (contractions/dilations of ontological memory) and guaranteeing the correspondence between thinking and being, expressing and expressed. Brassier writes:

«Accordingly, it establishes the correlation between the determination of thought as individuating difference borne by the intensive thinker, and the determinability of being as differentiated but undifferenciated pre-individual realm. Thus it is the third synthesis of time which accounts for the genesis

63 Gilles Deleuze, *Difference and Repetition*, trans. Paul Patton (New York: Columbia University Press, 1994), 169.

64 Ibid.

> of ontological sense as that which is expressed in thought, and which relates univocal being directly to its individuating difference as the expressed to its expression. In this regard, it is indissociable from the transcendent exercise of the faculties through which the Idea is generated.»[65, 66]

The future, Deleuze's third synthesis of time, is defined as the «fractured I» through «Kant's definition of time as pure and empty form, Friedrich Hölderlin's notion of ‹caesura› and the mathematical definition of Dedekind cut.»[67] Daniela Voss argues that «[o]f central importance is the notion of the cut, which is constitutive of the third synthesis of time defined as an a priori ordered temporal series separated unequally into a before and an after.»[68] The future (or third synthesis) is the means of differenCiating the pre-Individual Idea, the latter being ideal differenTiation (the second synthesis) of the realm of problematic multiplicity/mathematical differentials/pure difference. DifferenCiation is a psychic Simondonian individuation (based on biology), linking «the expressed to its expression»; «thinking and being, expressing and expressed»:

> Thus Deleuze's account of spatio-temporal synthesis begins by ascribing a privileged role to organic contraction in the first synthesis of the present, proceeds to transcendentalize memory

65 Ray Brassier, *Nihil Unbound* (London: Palgrave Macmillan, 2007), 179.

66 Daniel Sacilotto, «Heidegger and Deleuze on Death (Nihil Unbound—Chapter IV),» Being's Poem, blog entry, March 8, 2010, http://bebereignis.blogspot.com/2010/03/heidegger-and-deleuze-on-death-nihil.html.

67 Daniella Voss, «Deleuze's Third Synthesis of Time,» *Deleuze Studies* 7, no. 2 (May 2013): 194–216.

68 Ibid.

as cosmic unconscious in the second synthesis of the past, and ends by turning a form of psychic individuation which is as yet the exclusive prerogative of homo sapiens into the fundamental generator of ontological novelty in the third synthesis of the future… Transcendental access to the sense of being is internalized within experience through the transcendent exercise of the faculties that generates Ideas as the intensional correlates of larval thought (albeit a ‹sense› which is indissociable from non-sense).[69]

The Dedekind cut is defined as a virtual point (real number or cut) that exists between two sets of rational numbers (see previous chapter). The Dedekind cut is discussed in chapter four of *Difference and Repetition* on differenTiation:

> Three principles which together form a sufficient reason correspond to these three aspects: a principle of determinability corresponds to the undetermined as such (dx, dy); a principle of reciprocal determination corresponds to the really determinable ($^{dy}/_{dx}$); a principle of complete determination corresponds to the effectively determined (values of $^{dy}/_{dx}$). In short, dx is the Idea—the Platonic, Leibnizian or Kantian Idea, the ‹problem› and its being.
>
> The Idea of fire subsumes fire in the form of a single continuous mass capable of increase. The Idea of silver subsumes its object in the form of a liquid continuity of fine metal. However, while it is true that continuousness must be related to Ideas and to their problematic use, this is on condition that it be no longer defined by characteristics borrowed from sensible or even geometric intuition, as it still is when one speaks of the interpolation of intermediaries, of infinite intercalary series or parts which are never the smallest possible. Continuousness truly belongs to the realm of Ideas only to the extent that an ideal cause of continuity is determined. Taken together with its cause, continuity forms the pure element of quantitability, which must be distinguished both from the fixed quantities of intuition [*quantum*] and from variable quantities in the form of concepts of the understanding [*quantitas*]. The symbol which expresses it is therefore completely undetermined: dx is strictly nothing in relation to x, as dy is in relation to y. The whole problem, however, lies in the signification of these zeros. Quanta as objects of intuition always have particular values; and even when they are united in a fractional relation, each maintains a value independently of the relation. As a concept of the understanding, *quantitas* has a general value; generality here referring to an infinity of possible particular values: as many as the variable can assume. However, there must always be a particular value charged with representing the others, and with standing for them: this is the case with the algebraic equation for the circle, $x^2 + y^2 - \mathbb{R}^2 = 0$. The same does not hold for $ydy + xdx = 0$, which signifies ‹the universal of the circumference or of the corresponding function›. The zeros involved in dx and dy express the annihilation of the quantum and the quantitas, of the general as well as the particular, in favour of ‹the universal and its appearance›. The force of the interpretation given by Bordas-Demoulin is as follows: it is not the differential quantities which are cancelled in dy/dx or 0/0 but rather the individual and the individual relations within the function (by ‹individual›, Bordas means both the particular and the general). We have passed from one genus to another, as if to the other side of the mirror: having lost its mutable part or the property of variation, the function represents only the

69 Ray Brassier, *Nihil Unbound* (London: Palgrave Macmillan, 2007), 201.

immutable along with the operation which uncovered it. That which is cancelled changes in it, and in being cancelled allows a glimpse beyond of that which does not change. In short, the limit must be conceived not as the limit of a function but as a genuine cut [*coupure*], a border between the changeable and the unchangeable within the function itself. Newton's mistake, therefore, is that of making the differentials equal to zero, while Leibniz's mistake is to identify them with the individual or with variability. In this respect, Bordas is already close to the modern interpretation of calculus: the limit no longer presupposes the ideas of a continuous variable and infinite approximation. On the contrary, the notion of limit grounds a new, static and purely ideal definition of continuity, while its own definition implies no more than number, or rather, the universal in number. Modern mathematics then specifies the nature of this universal of number as consisting in the ‹cut› (in the sense of Dedekind): in this sense, it is the cut which constitutes the next genus of number, the ideal cause of continuity or the pure element of quantitability.

In relation to *x*, *dx* is completely undetermined, as *dy* is to *y*, but they are perfectly determinable in relation to one another. For this reason, a principle of determinability corresponds to the undetermined as such. The universal is not a nothing since there are, in Bordas's expression, ‹relations of the universal›. *dx* and *dy* are completely undifferenciated [*indifferencies*], in the particular and in the general, but completely differentiated [*differenties*] in and by the universal [emphasis added].[70]

70 Gilles Deleuze, *Difference and Repetition*, trans. Paul Patton (New York: Columbia University Press, 1994),171–172.

Deleuze demonstrates a tendency to idealism in defining both the differential unconscious of the pure past and the «fractured I» of the future, upon the ideal «static» continuous number which is the «universal in number»—not to be thought of as individual elements but rather the «relations of the universal»: pure differentials. Continuity belongs to «the realm of Ideas,» insofar as it is not dependent upon the senses or geometric intuitions, i.e., the ideal cause of continuity is undifferenciated and undetermined with respect to (individual) *x* or *y* (*dx* to *x*, or *dy* to *y*). As an «ideal cause of continuity,» it is completely determined and «differentiated in and by the universal» (*dx* to *dy*). This may beg the question of Badiou's critique of Deleuze where he makes him «a philosopher of the One.»

But we are not critiquing the idealism of Deleuze in this *Prolegomenon* (we will address Laruelle's de-ontologization of idealism in chapter VII). We are merely pointing out that any kind of logic of the individual should not be based on atomistic propositions, for these terms have already been individuated. Deleuze similarly denies the primacy of the value or the variable, but falls back on a Platonism. Consider the statement: «The tree has a trunk, branches and leaves.» Yes, this statement is true, but are these the fundamental building blocks? The tree was born out of an acorn and its interactions with the earth! And what of the soil? Piaget believed cognitive functions are «epigenetic.»[71] But also, there is no

71 See Jean Piaget, «The Epigenetic System and the Development of Cognitive Functions,» in *Brain Development and Cognition: A Reader*, 2nd ed., eds. Mark H. Johnson, Yuko Munakata and Rick O. Gilmore (Hoboken, Wiley-Blackwell, 2008), 29–35.

«absolute beginning» and basic logic which can be traced to the child's development, its coordination of actions, can be traced even further back to biology. We are hoping to found logic upon geometry itself, as Thom had proposed.

V. Aristotelian realism/Einsteinian relativity

> The *in re* structuralism («in the thing»), or modal structuralism (particularly associated with Geoffrey Hellman), is the equivalent of Aristotelian realism (realism in truth value, but anti-realism about abstract objects in ontology). Structures are held to exist inasmuch as some concrete system exemplifies them. This incurs the usual issues that some perfectly legitimate structures might accidentally happen not to exist, and that a finite physical world might not be «big» enough to accommodate some otherwise legitimate structures.[72]

> «*Il n'y a donc pas un temps des philosophes.*» Einstein's reply—stating that the time of the philosophers did not exist—was incendiary.
>
> What Einstein said next that evening was even more controversial: «There remains only a psychological time that differs from the physicist's.»[73]

In the early 20th century, there was a debate between Albert Einstein and Henri Bergson. Einstein argued that the time of philosophers did not exist and said there were only two modes of time, psychological, which erred, and the time of physicists. According to Einstein's relativity, time is dependent upon one's inertial frame of reference. And the only thing that is absolutely invariant is the speed of light *c*.

72 «Structuralism (philosophy of mathematics),» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Structuralism_(philosophy_of_mathematics).

73 Jimena Canales, «This Philosopher Helped Ensure There Was No Nobel for Relativity,» *Nautilus*, April 18, 2016, https://nautil.us/this-philosopher-helped-ensure-there-was-no-nobel-for-relativity-4554/

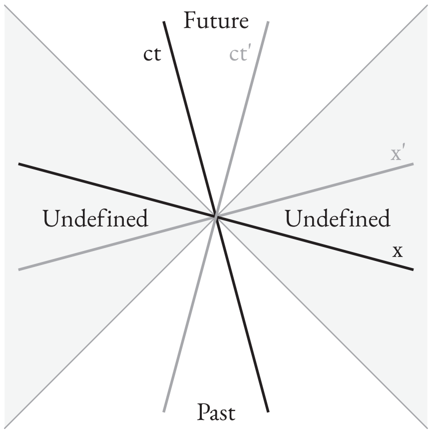


Fig. 8 *Past and future relative to the origin. For the grey areas a corresponding temporal classification is not possible*

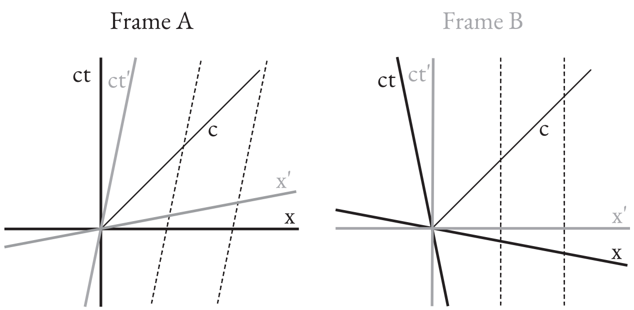


Fig. 9 *Minkowski diagrams for two coordinate systems, Frame A and Frame B*

For each inertial frame of reference, here Frame A and Frame B, the only thing invariant is that one's frame is 90° ($\pi/2$ radians) and the light cone appears at 45°. As a Stanford course explains: «In the following figure we display several different inertial reference frames as observed in Alice's frame. Notice that the light cone will always appear at 45° in each of these frames, that is in each frame light is observed to propagate at *c*.»[74]

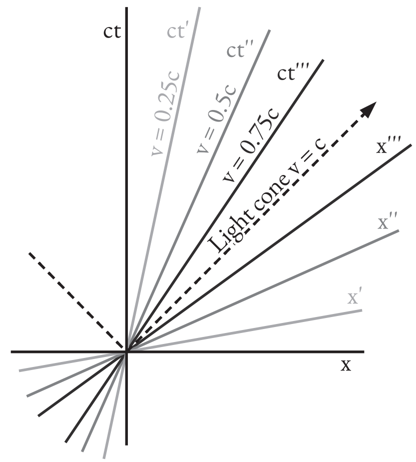


Fig. 10 *Minkowski diagram for three coordinate systems relative to Alice's frame. Each vector is the velocity at which one is traveling relative to the speed of light. For the speeds relative to the system in black v=0.25c, v=0.5c, v=0.75c*

74 Gary Oas, «SPCS Physics: Summer Institute—References, Problems, and Solutions—Special and General Relativity,» *Lecture 3: Spacetime Diagrams, Spacetime, Geometry* (Unpublished handout, Stanford University, 2015), 5.

So space and time, which are an interwoven four-dimensional Minkowski spacetime (which is a four-dimensional hyperbolic geometry), are relative to light![75] And the speed of light c is the only absolute!

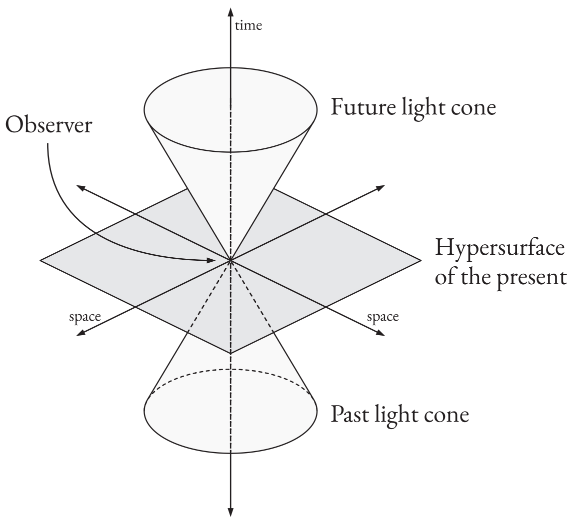


Fig. 11 *Diagram of a light cone with future, past and present*

75 See «Minkowski space,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Minkowski_space, «Where v is velocity, and x, y, and z are Cartesian coordinates in 3-dimensional space, and c is the constant representing the universal speed limit, and t is time, the four-dimensional vector $v = (ct, x, y, z) = (ct, \mathbf{r})$ is classified according to the sign of $c^2t^2 - r^2$. A vector is <u>timelike</u> if $c^2t^2 > r^2$, <u>spacelike</u> if $c^2t^2 < r^2$, and null or <u>lightlike</u> if $c^2t^2 = r^2$.»

Each vector is the velocity at which one is traveling relative to the speed of light. «Notice that for reference frames which move very fast the ct' and x' axes approach each and would actually meet for frames moving at c.»[76] So something moving at the speed of light would appear «lightlike» (as opposed to «timelike» or «spacelike»). Future and past are delimited by the light cone, which is the speed of light c = 299792458 m/s.

Einstein's argument was that the time of philosophers did not exist. Atemporal time as «backdrop» (Eternity) does not exist. Einstein disagreed with Bergson's idea of duration. We would also argue that Heidegger's time is too idealist and his metaphysics of time is non-existent according to physical reality. The problem of his ontology is to found a temporality upon purely psychological notions. Jean Wahl had criticized Heidegger for trying to create a *nunc stans* through a unification of his three ekstaces of time (fallenness, thrownness and projection).[77] Space and time can be only experienced through the embodied vectors of ct and x. The ramifications this has for epistemology are tremendous. Our finite world is too small to exemplify these macro structures, but they really do exist once we are at that scale and are exemplified in a completely in situ manner. Therefore, the only means of comparing two spaceships traveling at different speeds would be to compare their respective vec-

76 Gary Oas, «SPCS Physics: Summer Institute—References, Problems, and Solutions—Special and General Relativity,» *Lecture 3: Spacetime Diagrams, Spacetime, Geometry* (Unpublished handout, Stanford University, 2015), 5.

77 Jean Wahl, *Human Existence and Transcendence*, trans. William C. Hackett (University of Notre Dame Press, 2016).

tors to the speed of light. Light is the only absolute. In this sense, there is an a priori faktum, namely the absolute which is the speed of light. The hyperbolic geometry, whether the Klein model or Poincaré half plane, usually has an absolute which is the ‹point at infinity›. For the Minkowski model, the speed of light is taken to be this absolute. But on a micro- or meso-level, can we conceive of a parameterized time, as we have in the previous chapters where we discussed the ideas of Thom? Aristotle viewed time as a series of ‹now-points›. Is not the axis of time equivalent to the real number line? Kevin Davey has argued that we do not need real-value decimal expansions in physics, that physics need not be mathematically rigorous. The experience of time is relative to one's velocity. But the dimension of time (the four-dimensional vector $v = (ct, x, y, z) = (ct, r)$) is still *continuous*. In this sense we can still correctly assume the time of Simondon's individuation where a cell individuates and becomes formed in real time (some continuously running variable such as t), but then we run into problems of different scales, micro, meso, macro.

Another question is the issue of particle physics. Zeno's paradox presents the problem of infinite subdivision. If we follow the physicists, is the universe constituted of subatomic particles? Negarestani has been careful to make the point that atoms in the colloquial sense are not the same as atoms in the scientific sense. He has said, van Fraassen asks whether or not electrons really do exist. From my own conversations with a mathematician, I have gleaned that two electrons are identical, but no two protons are identical. Can we make

claims about the identity of two things, if we are not even allowed this relation of identity? I will address the problems of object-oriented programming and cybernetics in chapter XII of this *Prolegomenon*, whereby virtual objects are constituted and compared to one another and discuss the problems of computing when the computer needs to compute a function which is uncountable.

Can we accept the structure (expressed by mathematical physics) that has been deployed in physics as really existing? For example, string theory makes use of the notion of compactification into a string, thanks to the work of the mathematician Theodor Kaluza. Another mathematical theory employed in theoretical physics is group theory, which is drawn upon in particle physics, specifically, spin of subatomic particles through the three-dimensional rotation group $SO(3)$. As William Chin of DePaul University notes, «group theory appears in physics in many other places as do other algebraic structures such as Lie algebras and C*-algebras.»[78] We leave these questions of the existence of physical entities to philosophers of science and we will choose to focus instead on the questions of structure (its epistemology and ontology) in the philosophy of mathematics. We will argue that ‹neurophysics› does in fact exist, though.

78 William Chin, e-mail to author, March 13, 2022.

VI. Neurophysics

> In other words, it is only by restricting phenomenal reality to its most elementary form (essentially, the trajectories of material bodies, fluids, particles, and fields) that we have been able to carry through the program of reconstruction and computational synthesis. For the other classes of phenomena, this project has long come up against unsurmountable epistemological obstacles.
>
> At this point, it was taken as self-evident that there was an unavoidable scission between phenomenology (being as it appears to us in the perceived world and the cognitive faculties that process it) and physics (the objective being of the material world). However, we may say that it is not so much self-evident as a straightforward prejudice. In any case, this disjunction transformed the perceived world into a world of subjective-relative appearances—mental projections—with no objective content and belonging to psychology. Beyond psychology, the most that could be attributed to these appearances in the way of objectivity was a logical form of objectivity to be found in the theories of meaning and mental contents, from Bolzano and Frege, Husserl and Russell, to contemporary analytical philosophy.
>
> We may say that the current work aims to go beyond this scission by developing *a mathematical neurophysics of the phenomenology of the perceived world and common sense.*[79]

Petitot has argued for a «mathematical neurophysics» in order to unify the divided fields of phenomenology and physics. The fields studying perception were taken to be too subjective and were deemed «psychology» and could only

79 Jean Petitot, *Elements of Neurogeometry* (Cham: Springer Nature, 2017), 38.

achieve some objectivity within the fields of logic and analytic philosophy. Similarly, the field of physics could not explain the perceived world as it immediately appears to us and could only explain situations on either a macro-scale or at the most fundamental level. Petitot argues that this division between phenomenology and physics is a prejudice. Petitot constructs a theoretical computational machine which uses λ-calculus for the operation of «neural hardware» and differential geometry for the operations of «geometric software.» Petitot shows that *V1* neurons have been experimentally demonstrated to construct a cut locus (from differential geometry) through brain imaging:

> Our last example concerns the cut locus of a figure, also called the generalized symmetry axis or ‹skeleton›. Following the psychologist Blum, Thom always stressed its fundamental role in perception (see Figure [12]).
>
> Once again, imaging can show us the neural reality of the construction of this inner skeleton, for which there is no trace whatever in the sensory input, the latter consisting merely of an outer contour. Figures [13] and [14], produced by David Mumford's disciple Tai Sing Lee, illustrate the response of a population of simple *V1* neurons, whose preferred orientation is vertical, to textures with edges specified by opposing orientations. Up to around 80–100 msec, the early response involves only the local orientation of the stimulus. Between 100 and 300 msec, the response concerns the overall perceptual structure and the cut locus appears. These experiments are rather delicate to carry out, and they are much debated, but the detection of cut loci seems to be well demonstrated experimentally.

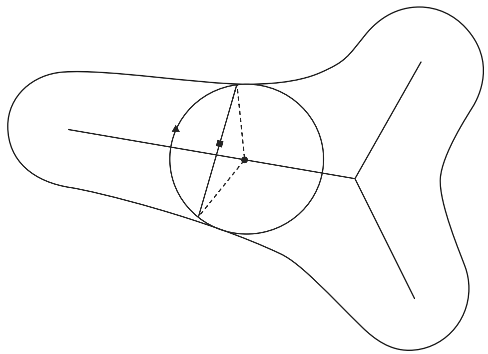


Fig. 12 *Example of a cut locus*

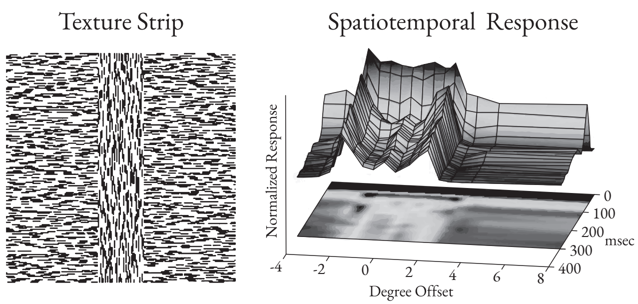


Fig. 13 *Response to a stimulus whose form is specified by opposing textural orientations*

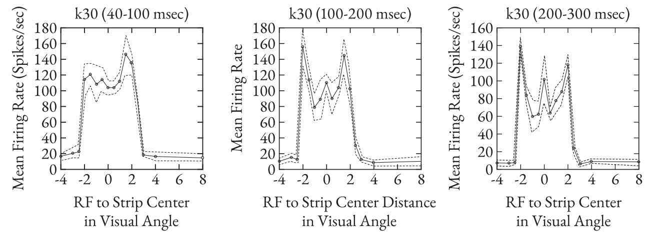


Fig. 14 *Recording of the construction of the cut locus*

All these examples share the fact that the geometry of the percept is *constructed*—Husserl would say ‹constituted›—from sensory *data which do not contain it*, whence it must originate somewhere else. Put another way, they all involve subjective Gestalts. This is indeed why we chose them, because, as claimed by Jancke et al., these subjective global structures ‹reveal fundamental principles of cortical processing›, the kind of principles that interest us here.

The origins of visual perceptual geometry can be found in the functional architecture which implements an immanent geometry, and it is the latter that provides the focus of neurogeometry.[80]

The geometry of perception is constructed from sensory data that does not directly exemplify this geometry (the sensory input only, i.e., texture strip, contains a rough contour of it).[81] Therefore the cut locus is rather created from mental

80 Ibid., 41–42.

81 Ibid., 41.

Gestalts. This is an immanently experienced neurogeometry which does not appear out in the world, but is, rather, subjective. Petitot argues that this immanent geometry is the «synthetic a priori.» Kant had said that geometry is «a priori» and Petitot's conclusions seem to verify this tenet experimentally. One need only examine the texture strip and the corresponding spatiotemporal response pictured in Figure 14 to see how this geometry is completely immanent to the subject.

VII. Semantic realism as it is entangled with eliminative structuralism

Elaine Landry employs category theory to find a compromise position between *ante rem* and eliminative realism.[82] Rather than aiming to establish the existence of mathematical objects or structures metaphysically, through an *ante rem* approach, or an *in re* approach in which they arise from a process of abstraction, Landry attempts to provide a linguistic basis for our talk about these structures and objects.

«[T]he only reality that categories need to be taken as part of is ‹linguistic reality›, that is, the reality that concerns us with *what we say*. Interpreted along semantic realist lines, categories are not claimed to exist independently of their linguistic use and even when they are held as «objects,» they are only taken to exist in the sense of being required to *talk about* the way things are in a given structure.»[83]

This position she dubs *semantic realism*. Objects and structures are established as existing in as far as is necessary to provide our linguistic utterings with content. She aims to secure this measure of realism by analysing structures as they behave in category theory. She investigates the manner in which we refer to a singular object in a particular structure, expressed in the language of category theory, and the way in which this changes if we in turn approach the structure from a general perspective. From all this, she finds that the former analysis

82 See Elaine M. Landry, «Category Theory as a Framework for Mathematical Structuralism,» *Proceedings of the Canadian Society for the History and Philosophy of Mathematics* (1998). [Joost Vecht:] «Landry herself refers to the latter as *in re* structuralism, following Shapiro's older method of classifying all non-*ante rem* variants of structuralism as ‹in re›. I will continue using the term ‹in re› to refer to the position arguing that mathematical structure are to be found in the world specifically.»

83 Ibid., 138.

> leads us to an *ante rem* interpretation of structures, whereas the latter is best captured on an eliminative interpretation.[84]

> Another option is to deny that structures exist at all. Talk of a given structure is just convenient shorthand for talk of all systems that are isomorphic to each other. [...] Views like this are sometimes called *eliminative* structuralism, since they eschew the existence of structures altogether.[85]

Following the semantics of Charles Sanders Peirce who argues for a metaphysics, I suggest we philosophically pursue a synthesis between ‹in the thing› realism and ‹after the thing› realism, which is called ‹semantic realism›. We shall discuss eliminative structuralism as it relates to Laruelle now. Reiterating the eliminativist philosophy of mathematics, Negarestani talks about the Yoneda Lemma used in category theory:

> The approaching of the concept or the local site from its adjacent environment and alternative perspectives is the gesture of the Yoneda Lemma in category theory. Yoneda Lemma is a phenomenologically trivial tool, but it nevertheless possesses a formidable power to reverse-engineer local concepts by way of their neighborhood, by way of their outside. A point is nothing but the pointer that points to it. The actual mark is a pointer endowed with a limit, just like the mark that the tip of a pencil leaves on a piece of paper. Once the point is understood as a pointer, the concept of point can be made via an infinite recursive descent: A point is a point is a point is a point ... *ad infinitum*. Each pointer can be decomposed to a concatenation of different sets of pointers or addresses. The concept of the point is nothing but an alternating collection of gestural/perspectival pointers (arrows or morphisms). There is indeed a functionalist underside to this definition of the concept qua a local site: If what makes a thing a thing is not what a thing is but what a thing does, then we can decompose this activity or behavior (the behavior of the concept) into operative perspectives or possible activities that make the behavior of the concept in an inferential network. The study of the concept and its construction overlap, as they become part of a controlled exploratory approach.[86]

While Negarestani seems to indicate that structure is merely shorthand for the isomorphic systems, he seems to actually be more of a functionalist than an eliminativist, as his recent work *Intelligence & Spirit* confirms. One can think of the quintessential functionalist Hilary Putnam, who claimed that mind was a Turing machine. The key import of the Yoneda limit for Negarestani is the functional decomposition of the morphisms into «the behavior of the concept» within an «inferential network,» i.e., not what the concept is, but «what a thing does.» In order to differentiate these two schools from one another (eliminative structuralism and functionalism) we will examine a more precise definition of the former:

84 Joost Vecht, «Categorical Structuralism and the Foundations of Mathematics» (Unpublished manuscript, University of Amsterdam, 2015), 41, https://www.andrew.cmu.edu/user/awodey/preprints/awodeyVhellman.pdf.

85 Geoffrey Hellman and Stewart Shapiro, *Mathematical Structuralism* (Cambridge University Press, 2018), 2.

86 Reza Negarestani, «Where is the Concept?,» in *When Site Lost the Plot*, ed. Robin Mackay (Falmouth: Urbanomic, 2013), 226.

> The eliminativist holds that mathematical statements *just are* (or are best interpreted as) generalizations like these and she accuses the [Sui Generis] structuralist of making too much of their surface grammar, trying to draw deep metaphysical conclusions from that. [...]
>
> In general, any sentence Φ in the language of the arithmetic gets regimented as something like the following:
>
> In any natural number system S, $\Phi[S]$, (Φ')
>
> where $\Phi[S]$ is obtained from Φ by restricting the quantifiers to the objects in S, and interpreting the non-logical terminology in terms of the relations of S.
>
> In a similar manner, the eliminative structuralist [...] regiments—and deflates—what seem to substantial metaphysical statements, [...] [f]or example, «the number 2 exists» becomes «in every natural number system S, there is an object in the 2-place of S.»[87]

The closest continental thinker to this school is then Laruelle. For Laruelle, his non-philosophy is a means of suspending the transcendence (the «metaphysical conclusions» mentioned above) of any philosophical system and arriving at a transcendental which consists of the duality between ‹axiom› and ‹theorem›. For example, if Laruelle were to deconstruct Deleuze's philosophy, this duality would be the pair of ‹actual› and ‹virtual› taken together. This in effect, de-ontologizes the view from the «plane of immanence» which supposedly lives in the realm of virtuality

87 Geoffrey Hellman and Stewart Shapiro, *Mathematical Structuralism* (Cambridge University Press, 2018), 3–4.

(as we discussed before, this is an idealist transcendence since Deleuze's second synthesis is based on an ideality—the relations of differential calculus). Laruelle's radical immanence is a means for performative philosophy which is in favor of material experimentation (the «derived a priori» of the «unilateral duality» which takes the non-philosophizable Real as its condition) as opposed to Deleuze's plane of immanence (which has a formal a priori, namely the process of differenTiation described in his fourth chapter of *Difference and Repetition*).

But there still remains a problem with eliminative structuralism. In order to avoid statements which are entirely vacuous such as «1 + 3 = 7,» Geoffrey Hellman suggests a «structuralism without structures» which goes further to posit the necessity of the existence of a mode for each statement:

> [A] sentence such as «2 + 3 = 5» is analyzed as follows:
>
> Necessarily, for all relational systems M, if M is a model of the Dedekind-Peano axioms, then $2_M + 3_M = 5_M$.
>
> To avoid the non-vacuity problem, he adds the following assumption:
>
> Possibly, there exists an M such that M is a model of the Dedekind-Peano axioms.[88]

88 «Structuralism in the Philosophy of Mathematics,» Stanford Encyclopedia of Philosophy, accessed January 26, 2022, https://plato.stanford.edu/entries/structuralism-mathematics/#Cons1980ResnShapHellEtc.

Superficially, Hellman's «structuralism without structure» may sound like Laruelle's «given-without-givenness.» But as opposed to Hellman, Laruelle's non-philosophy is not «philosophically necessary,»[89] where for Hellman, the model is possibly necessary! John Ó Maoilearca writes:

> Non-philosophy is an unknown quantity with ‹an immanent methodology whose function for philosophy no one is in a position to evaluate as yet›. Laruelle himself expresses an allied point: there is no basis upon which non-philosophy can be commended that is itself *philosophically necessary*:
>
> «There is then no imperative fixing a transcendent, onto-theo-logical necessity to ‹do non-philosophy›: this is a ‹posture› or a ‹force-(of)-thought› which has only the criterion of immanence as its real cause—which takes itself performatively as force-(of)-thought—and the occasion of its *data*; which contents itself to posit axioms or hypotheses in the transcendental mode and to deduce or induce starting from them.»[90, 91]

Laruelle then sounds actually closer to the axiomatic method that was first championed by Hilbert. In the famous Hilbert-Frege debate, Hilbert thought axioms were ‹schematic›, while Frege thought axioms were ‹assertoric›. Frege famously uses the axiomatic method to assert the existence of God (in a letter to Hilbert):

89 John Ó Maoilearca, «Laruelle's ‹Criminally Performative› Thought: On Doing And Saying In Non-philosophy,» *Performance Philosophy* 1 (October 2015): 164.

90 François Laruelle, *Principles of Non-Philosophy*, trans. Nicola Rubczak, Anthony Paul Smith (London: Bloomsbury 2013), 198–199.

91 John Ó Maoilearca, «Laruelle's ‹Criminally Performative› Thought: On Doing And Saying In Non-philosophy,» *Performance Philosophy* 1 (October 2015): 164–165.

> We imagine objects we call Gods.
>
> Axiom 1. All Gods are omnipotent.
>
> Axiom 2. All Gods are omnipresent.
>
> Axiom 3. There is at least one God.[92]

But to be fair to Hilbert, his system of geometry was freestanding and theology is not freestanding, let alone formalizable; geometry is formalizable.[93] One should consider the axiomatic method in the schematic sense (rather than assertoric), where the existence of such entities need not be necessary. Category theory provides a structuralism somewhere between ‹in the thing› realism and ‹after the thing› eliminativism. If we accept the ‹schematic› nature of category theory, we begin to approach a constructive intelligence through postulating existence. This notion introduces the later work in homotopy type theory, which after the Univalence axiom, postulates equivalence as an axiom. «In other words, identity is equivalent to equivalence. In particular, one may say that ‹equivalent types are identical›.»[94] For an entirely formalist account in the tradition of classical structuralism par excellence, see the following chapter on Bourbaki. Colin McLarty refers to this as «Bourbaki ideology.» André Weil, the de facto leader, for example, worked directly with Claude

92 Gottlob Frege, *Philosophical and Mathematical Correspondence* (Oxford: Blackwell, 1980), quoted in Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford University Press, 1997), 163.

93 Ibid.

94 The Univalent Foundations Program, *Homotopy Type Theory: Univalent Foundations of Mathematics* (Princeton: Institute for Advanced Study, 2013), 4.

Lévi-Strauss on anthropology for a study (helping him with the group theory). We suggest a synthesis between ‹ontic structural realism› with respect to category theory and ‹eliminative realism› for now. But ultimately the high structuralism of axiomatic formalism seems most interesting. The acerbic wit of Weil and his coterie is just one autobiographical example of how the ‹mother structures› (though informal) unified all of mathematics despite themselves. But new research on the Univalence axiom suggest the possibility of an equivalence between Martin-Löf type theory and homotopy theory. This system is predicated on an intuitionism since all constructions are local. We leave it up to the reader to decide on whether category theory, topos theory or homotopy type theory provides a philosophical structuralism as we narrate this story in the coming chapter. Or is the axiomatic method itself (though schematic) enough to provide a pure structuralism as Jean-Pierre Marquis has written.

VIII. The axiomatic method of Bourbaki

As the legendary Nicolas Bourbaki, a pseudonym for a collective of (mostly French) mathematicians who were (independently) extremely influential to mathematics,[95] write in their unofficial manifesto written by Dieudonné:

> It must therefore be out of the question to give to the uninitiated an exact picture of that which the mathematicians themselves can not conceive in its totality. Nevertheless it is legitimate to ask whether this exuberant proliferation makes for the development of a strongly constructed organism, acquiring ever greater cohesion and unity with its new growths, or whether it is the external manifestation of a tendency towards a progressive splintering, inherent in the very nature of mathematics, whether the domain of mathematics is not becoming a tower of Babel, in which autonomous disciplines are being more and more widely separated from one another, not only in their aims, but also in their methods and even in their language. In other words, do we have today a mathematic or do we have several mathematics?[96]

The division of mathematics into many different fields which were not united was a big ideological problem for Bourbaki. Bourbaki, with their treatise *Éléments de mathématique*,

95 See «Nicolas Bourbaki,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Nicolas_Bourbaki, «Over time the founding members gradually left the group, slowly being replaced with younger newcomers including Jean-Pierre Serre and Alexander Grothendieck. Serre, Grothendieck and Laurent Schwartz were awarded the Fields Medal during the postwar period, in 1954, 1966 and 1950 respectively. Later members Alain Connes and Jean-Christophe Yoccoz also received the Fields Medal, in 1982 and 1994 respectively.»

96 Nicholas Bourbaki, «The Architecture of Mathematics,» *The American Mathematical Monthly* 57, no. 4 (April 1950): 221.

believed in a single unified mathematics that could be axiomatically constructed from sets. Bourbaki continues on to answer their own question about the unity of mathematics:

> After the more or less evident bankruptcy of the different systems, to which we have referred above, it looked, at the beginning of the present century as if the attempt had just about been abandoned to conceive of mathematics as a science characterized by a definitely specified purpose and method; instead there was a tendency to look upon mathematics as «a collection of disciplines based on particular, exactly specified concepts,» interrelated by «a thousand roads of communication,» allowing the methods of any one of these disciplines to fertilize one or more of the others. Today, we believe however that the internal evolution of mathematical science has, in spite of appearance, brought about a closer unity among its different parts, so as to create something like a central nucleus that is more coherent than it has ever been. The essential aspect of this evolution has been the systematic study of the relations existing between different mathematical theories, and which has led to what is generally known as the «axiomatic method.»[97, 98]

The axiomatic method of Bourbaki was in the tradition of Hilbert's axiomatic program. They wished to found a geometric system (starting from sets). But in Bourbaki's treatise, there is no mention of Gödel's Incompleteness Theorem.[99] Also, the usual critique levied against the Bourbaki school of

97 Léon Brunschvicg, *Les étapes de la philosophie mathématique* (Paris: Alcan, 1912), 447.

98 Nicholas Bourbaki, «The Architecture of Mathematics,» *The American Mathematical Monthly* 57, no. 4 (April 1950): 222.

99 See A. R. D. Mathias, «The Ignorance of Bourbaki,» *The Mathematical Intelligencer* 14, no. 3 (June 1992): 4–13.

mathematics is that they did not even include category theory in their axiomatic system.[100] Contrary to many claims that Bourbaki's axiomatic system is dated, a paper by Maribel Anacona, Luis Carlos Arboleda and F. Javier Pérez-Fernández states:

> [W]e study the axiomatic system proposed by Bourbaki for the Theory of Sets in the *Éléments de Mathématique*. We begin by examining the role played by the sign τ in the framework of its formal logical theory and then we show that the system of axioms for set theory is equivalent to Zermelo–Fraenkel system with the axiom of choice [ZFC] but without the axiom of foundation. Moreover, we study Grothendieck's proposal of adding to Bourbaki's system the axiom of universes for the purpose of considering the theory of categories. In this regard, we make some historical and epistemological remarks that could explain the conservative attitude of the Group.[101]

According to these authors, Bourbaki's system is equivalent to ZFC set theory (for the most part). Also Grothendieck had proposed his universes (which are used in category theory/topos theory) in a Bourbaki paper![102] As an alternative to set theory, we will discuss category theory as a system for philosophical structuralism.

100 Ibid.

101 Maribel Anacona, Luis Carlos Arboleda and F. Javier Pérez-Fernández, «On Bourbaki's axiomatic system for set theory,» *Synthese* 191, no. 17 (November 2014): 4069.

102 Michael Artin, Alexander Grothendieck and Jean-Louis Verdier, *Séminaire de Géométrie Algébrique du Bois Marie* «Theorie des Topos et Cohomologie Etale des Schemas» (Bures-sur-Yvette: IHÉS, 1963), 185, http://library.msri.org/books/sga/sga/4-1/4-1t_185.html.

IX. Non-Euclidean geometry; the multiplicity of case(s) through general invariants (topological, group-theoretic or categorical) which constrain the objects into varying algebraic structures; category theory as a philosophical structuralism

The fifth axiom of Euclid is equivalent to the statement that parallel lines do not intersect.[103] The first mathematicians to provide a fully mathematical system of non-Euclidean geometry, which suspends the parallel postulate, were the mathematicians János Bolyai and Nicolai Ivanovich Lobachevskii. Non-Euclidean geometry is completely equiconsistent to Euclidean geometry.[104] After these advances, the philosophy of geometry was later taken up by the mathematician Poincaré:

> [Poincaré's] concern in his *On the foundations of geometry* was with epistemology.
>
> Poincaré argued that the mind quickly realises that it can compensate for certain kinds of motions that it sees. If a glass comes towards you, you can walk backwards in such a way that the glass seems unaltered. You can do the same if it tilts or rotates. The mind comes to contain a store of these compensating motions, and it realises that it can follow one with another and the result will be a third compensating motion.

103 See «Parallel postulate,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Parallel_postulate, «If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.»

104 John Head, «Syllabus: Math-UA.0270 / V63.0270: Transformations and Geometry,» New York University, accessed 28 July 2021, https://math.nyu.edu/courses/fall11/V63_0270_syllabus.html.

> These mental acts form a mathematical object called a group. However, the mind cannot generate compensating motions for other motions it sees, such as the motion of the wine in the glass as it swirls around. In this way the mind comes to form the concept of a rigid body motion, that being precisely the motion for which the mind can form a compensating motion.
>
> Poincaré then considered what group the group of compensating motions could be, and found that, as Helmholtz had suggested and Lie had then proved, there was a strictly limited collection of such groups. Chief among them were the groups that come from Euclidean and non-Euclidean geometry, and as abstract groups they are different. But which one was correct?
>
> Poincaré's controversial view was that one could never know. Human beings, through evolution and through our experience as infants, pick the Euclidean group and so say that space is Euclidean. But another species, drawing on different experiences, could pick the non-Euclidean group and so say that space was non-Euclidean. If we met such a species, there would be no experiment that would decide the issue... This twist on the Kantian doctrine of the unknowability of the *Ding an sich* (the thing in itself) and our confinement to the world of appearances, was congenial to Poincaré as a working physicist, but there is an important distinction to make. The viewpoint just explained is Poincaré's philosophy of geometrical conventionalism. He advocated conventionalism in other areas of science, arguing that what we call the laws of nature (Newton's laws, the conservation of energy, and so forth) were neither empirical matters open to revision nor absolute truths but were well established results that had been elevated to the role of axioms in present scientific theories.

> They could be challenged, but only if a whole scientific theory was being challenged, not idly when some awkward observations were made. Faced with a satellite that did not seem to be obeying Newton's laws one should, said Poincaré, consider some as-yet unnoticed force at work and not seek to rewrite Newton. But a new theory can be proposed, based on different assumptions that rewrite a law of nature, because these laws are not eternal truths—we could never know such things. And if a new theory were to be proposed, one can only choose between the new and the old on grounds of convenience.[105]

Poincaré was chiefly concerned with epistemology. In his philosophy of geometry, there is no way to know if Euclidean or non-Euclidean geometry is the fundamental explanation of space. He believed that we were conditioned to see the world in the Euclidean way. This equiconsistency at, not only at the mathematical level, but also the epistemological level, should lead us to consider the fundamental import of Felix Klein's Erlangen program. Klein's program for geometry was to first unify all geometries through projective geometry. But secondly and more importantly,

> Klein proposed that group theory, a branch of mathematics that uses algebraic methods to abstract the idea of symmetry, was the most useful way of organizing geometrical knowledge; at the time it had already been introduced into the theory of equations in the form of Galois theory.[106]

105 Jeremy Gray and José Ferreirós, «Epistemology of Geometry,» Stanford Encyclopedia of Philosophy, accessed July 28, 2021, https://plato.stanford.edu/entries/epistemology-geometry/#HelPoi.

106 «Erlangen program,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Erlangen_program.

The only formula that proves to be invariant in each respective space (through Möbius transformations) within the respective geometries of hyperbolic, Euclidean or spherical is that of ‹cross ratio›. Foote has shown a formula that is common to each geometry.[107] The cross ratio is invariant under Möbius transformations. Klein's program can be seen as a method of instantiating different geometries through the use of different groups. Conceptually, this can be thought of as the multiplicity of ‹case(s)› whereby general ‹invariants› (group-theoretic) constrain the objects into varying geometries. The Gaussian curvature is another example by which an invariant determines and attenuates a surface into varying, constrained geometries:

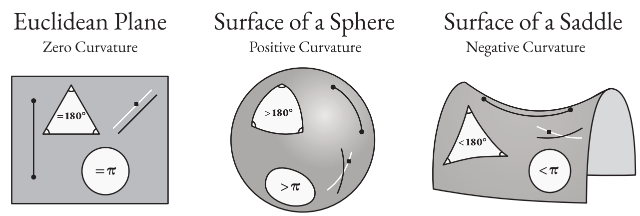


Fig. 15 *Three different geometries: euclidean, elliptic and hyperbolic. Note the curvature of each respective surface with an embedded geometry*

107 Robert L. Foote and Xidian Sun, «An Intrinsic Formula for the Cross Ratio in Spherical and Hyperbolic Geometries,» *The College Mathematics Journal* 46, no. 3 (May 2015): 182-188.

Even the question of the shape of the universe can be considered according to various curvatures. The formalism of algebra (group theory) in geometry/topology has been tremendously fruitful in the fields of algebraic topology and algebraic geometry. Following Gauss's Theorema Egregium, this method provides a means by which the intrinsic invariants of material are central (de-ontologized metaphysics) as opposed to the extrinsic objectivation of the thing in an embedded space (pre-ontology). As Petitot has asked: does embedded space exist without neural networks? Moreover, according to Poincaré's epistemology, we can think of the axiomatic method as a conventionalist approach which need not state anything ontological. But there is something to be said of the schematic (as opposed to assertoric) approach to axioms, which Hilbert espoused, for autonomous formal systems.

Saunders Mac Lane and Samuel Eilenberg, the inventors of category theory, wrote when they introduced the notion of the ‹category› in their seminal paper: «This may be regarded as a continuation of Klein's Erlangen program, in the sense that a geometrical space with its group of transformations is generalized to a category with its algebra of mappings.»[108] Category theory was thus thought of as in the same tradition as Klein's program! Because of its abstraction of specific algebraic structures to categories of objects which relate to one another through morphisms, category theory can be thought of as a philosophical structuralism:

108 Samuel Eilenberg and Saunders MacLane, «General Theory of Natural Equivalences,» *Transactions of the American Mathematical Society* 58, no. 2 (September 1945): 237.

> Two facets of the nature of mathematical objects within a categorical framework have to be emphasized. First, objects are always given in a category. An object exists in and depends upon an ambient category. Furthermore, an object is characterized by the morphisms going in it and/or the morphisms coming out of it. Second, objects are always characterized up to isomorphism (in the best cases, up to a unique isomorphism). There is no such thing, for instance, as *the* Natural numbers. However, it can be argued that there is such a thing as *the concept* of Natural numbers. Indeed, the concept of Natural numbers can be given unambiguously, via the Dedekind-Peano-Lawvere axioms, but what this concept refers to in specific cases depends on the context in which it is interpreted, e.g., the category of sets or a topos of sheaves over a topological space. Thus, it seems that sense does not determine reference in a categorical context. It is hard to resist the temptation to think that category theory embodies a form of structuralism, that it describes mathematical objects as structures since the latter, presumably, are always characterized up to isomorphism. Thus, the key here has to do with the kind of criterion of identity at work within a categorical framework and how it resembles any criterion given for objects which are thought of as forms in general. One of the standard objections presented against this view is that if objects are thought of as structures and only as *abstract* structures, meaning here that they are separated from any specific or concrete representation, then it is impossible to locate them within the mathematical universe.[109]

Category theory can be thought of as a means to instantiate varying algebraic structures through either the category of sets, the category of groups, the category of rings, the cate-

109 Jean-Pierre Marquis, «Category Theory,» Stanford Encyclopedia of Philosophy, accessed July 28, 2021, https://plato.stanford.edu/entries/category-theory/.

gory of topological spaces, etc. The only thing that is «positive» is the morphisms which relate objects within these categories, which can be thought of as the structural relations between objects. As Chin notes: «There are ‹functor categories› where the objects are functors and the functors are natural transformations between functors, to put the shoe on the other foot.»[110] The interiors of the objects are not necessary!

> What is remarkable about this apparatus is that the vast bulk of structures arising in mathematics can be captured in the spare language of «arrows only,» without «looking inside» the objects (which, recall, are typically whole structures or spaces e.g., algebraic, metric, topological, etc.).[111]

Typical set membership relationships need not be exemplified. Everything can be viewed in terms of «arrows only» (morphisms) as is typically shown in commutative diagrams. But the problem with such generality is that «it is impossible to locate [objects as specific or concrete structures] within the mathematical universe.» The solution to this dilemma is *type theory*:

> A slightly different way to make sense of the situation is to think of mathematical objects as *types* for which there are tokens given in different contexts. This is strikingly different from the situation one finds in set theory, in which mathematical objects are defined uniquely and their reference is given directly. Although one can make room for types within set theory via equivalence classes or isomorphism types in general, the *basic* criterion of identity within that framework is given by the axiom of extensionality and thus, ultimately, reference is made to specific sets. Furthermore, it can be argued that the relation between a type and its token is *not* represented adequately by the membership relation. A token does not belong to a type, it is not an element of a type, but rather it is an instance of it. In a categorical framework, one always refers to a *token* of a type, and what the theory characterizes directly is the type, not the tokens. In this framework, one does not have to locate a type, but tokens of it are, at least in mathematics, epistemologically required. This is simply the reflection of the interaction between the abstract and the concrete in the epistemological sense (and not the ontological sense of these latter expressions.) [112]

110 William Chin, e-mail to author, March 13, 2022.

111 Geoffrey Hellman and Stewart Shapiro, *Mathematical Structuralism* (Cambridge University Press, 2018), 43.

This top-down approach of category theory is explained by Awodey in his response to Hellman. We can think of it as the impartiality of abstraction through invoking invariants (similar to Klein's program) from a bird's eye view. Awodey defends mathematical structuralism. We will not yet comment upon the ontology of such a philosophical structuralism (we will address that very complicated issue in the next chapter), but we note that structuralism, by way of its *schematic* nature, does not require the existence of some universe of, for example, all possible rings. It requires, say, only one ring. In this sense, it has no background ontology. Let's take Awodey's example:

112 Jean-Pierre Marquis, «Category Theory,» Stanford Encyclopedia of Philosophy, accessed July 28, 2021, https://plato.stanford.edu/entries/category-theory/.

> (i) in any ring, if $x^2 = -1$ then $x^5 = x$, and
>
> (ii) the complex numbers are by definition a ring with an element i such that $i^2 = -1$, and having a couple of other distinctive properties.
>
> The foundationalist may now object that [they], too, can show by the same simple proof that:
>
> (i′) any ring in his universe with an element such that $x^2 = -1$ also has $x^5 = x$, and
>
> (ii′) [their] complex numbers are a ring in which i is a root of -1[113]

Awodey (responding to Hellman) argues that category theory can provide a «philosophical structuralism.» The question everyone should be asking is whether «category theory is a framework for philosophical structuralism?» A «mathematical structuralism» proper is obvious. The point made by Awodey is that category theory favors a top-down approach over the bottom-up foundationalist approach. A set theorist needs to define all of the necessary structures and axioms in order to do the usual mathematics, while the category theorist need not make a «universal quantification» over the universe of all possible structures. The category theorist can define the objects through morphisms, in relation to which objects are rather instances or tokens. For example, in order to define $i^2 = -1$, the category theorist can say «given any ring» as opposed to defining the universe of all possible ring structures

113 Steve Awodey, «An answer to Hellman's question: ‹Does category theory provide a framework for mathematical structuralism?›» (Unpublished manuscript, Carnegy Mellow University, 2003), 4, https://www.andrew.cmu.edu/user/awodey/preprints/awodeyVhellman.pdf.

and fixing some given structure, such as the complex numbers. Awodey makes the argument that the category theorist is making a «schematic statement about structure.» This can be generalized to structures that are «formally similar» but not identical (the key to category theory's premise of «up to isomorphism») such that you have some polynomial in indeterminate X. This is even generalizable to any semiring, since it need not require additive inverses.

> Now one and the same simple equation proof shows that $i^5 = i$ in the complex numbers and that any element satisfying (1) in a semi-ring with unit also satisfies $x^5 = x$. Moreover, it also shows that e.g., any *set* X such that $X^2 + X + 1 \cong X$ also has a canonical isomorphism $X^5 \cong X$, obtained by essentially the same calculation. And the same holds for any object X in any category with finite products and coproducts that are distributive, say, the category $\mathrm{Sets}^{\mathbb{C}}$ of all set-valued functors on a small category $\mathbb{C}$, as well as for countless other situations that are «formally similar.»[114]

But schematic statements are not about «universal quantification» over some universe of, say, all possible semirings, nor even about some «universal quantification» (which usually requires the quantification to be restricted over a «fixed domain»). He uses the example of a fixed quantification of a value, such as for *all real* numbers $x^2 + 1$ is a real number *versus* the indeterminate case, such as for a polynomial ring $\mathbb{R}[x]$ with real coefficients, where it can be proved that there does not exist a polynomial $p(x)$ such that $p(x)^2 = x^2 + 1$. Awodey argues:

114 Ibid., 6.

> In the case of the quantified statement, we consider what is true for an arbitrary real number, while in the indeterminate case we, in effect, consider what holds for an arbitrary number x, in an arbitrary ring over $\mathbb{R}$. One could say, rather speculatively, that the difference in both cases seems to be related to that between what is true for all of a fixed range of values, as opposed to what can be proved for an indeterminate value.[115]

When a theorem invokes «a finitely generated abelian group» it does not specify the elements of the group, so the statement is schematic in the sense of what the structures «consist of» are «undetermined.» Similarly, Awodey says the fundamental group of any topological space acts in a certain way on a universal covering space. Building off Awodey's suggestion, if one makes a statement relating a topological space such as S^1 to its covering space $\mathbb{R}$, and in computing the fundamental group[116] through the fiber of S^1, one is not *necessarily* defining what points are in the topological space. One is relating a group $\mathbb{Z}$ to the cover $\mathbb{R}$ based on an isomorphism between the index of the fiber and the integers (there are countably many elements stacked on top of each other).[117] Awodey isn't argu-

115 Ibid., 8.

116 See «Fundamental Group,» Wolfram MathWorld, accessed March 20, 2022, https://mathworld.wolfram.com/Bordism.html, «The fundamental group of an arcwise-connected set is the group formed by the sets of equivalence classes of the set of all loops.»

117 See Anne Thomas «Covering Spaces» (Unpublished manuscript, University of Chicago, 2004), 1, http://math.uchicago.edu/~womp/2004/athomas04.pdf, «Definition: *A covering space or cover of a space X is a space $\tilde{X}$ together with a map $p : \tilde{X} \to X$ satisfying the following condition: every point $x \in X$ has an open neighborhood $U_x \subseteq X$ such that $p^{-1}(U_x)$ is a disjoint union of open sets, each of which is mapped by p homeomorphically onto U_x.* You can visualize the pre-image of the neighborhood U_x as a «stack of pancakes,» each pancake being homeomorphic to U_x. Some more terminology: sometimes the space X is called the *base space*, the map p is called the *covering map* or *projection*, and the pre-image $p^{-1}(x)$ of some point x in the base space is called the *fiber over x*. Examples: [...] The map $p : \mathbb{R} \to S1$ given by $p(t) = e^{it}$ is a covering map, wrapping the real line round and round the circle. The pre-image of a little open arc in the circle is a collection of open intervals in the real line, offset by multiples of 2π.»

ing that category theory can provide an autonomous alternative to set theory for the foundations of mathematics. He doesn't think there will be some «true topos,» in which «all of mathematics» can be done. Rather, Awodey thinks category theory is the best modern approach of mathematics after Évariste Galois et al.: «In category theory, such notions as relation, connection, property, and operation are all subsumed under the primitive notion of a morphism.»[118]

Awodey rather thinks there needs to be a different interpretation of categorical mathematics, which we observe, after homotopy type theory, is the top-down approach. He says the top-down approach need not have to define where higher and higher levels come from: for example if one has some group G and then a category $\mathbb{C}$ and then one has Group ($\mathbb{C}$) which is the category of all groups in that category. Everything is an ‹instance› or ‹type› and need not be presupposed to exist; what matters is the relations between objects (as also stated by Jean-Pierre Serre[119]):

> Thus rather than saying, for example, «now suppose this particular solar system is an atom in some huge piece of matter in an enormous solar system,» one is instead saying «now suppose this particular configuration of bodies occurs, not as a solar system, but as an atom in some piece of matter in a solar system.»

118 Steve Awodey, «An answer to Hellman's question: ‹Does category theory provide a framework for mathematical structuralism?›» (Unpublished manuscript, Carnegy Mellow University, 2003), 10, https://www.andrew.cmu.edu/user/awodey/preprints/awodeyVhellman.pdf.

119 Martin Brandenburg, «What is the source of this famous Grothendieck quote?,» Math Overflow, accessed March 28, 2022, https://mathoverflow.net/questions/133020/what-is-the-source-of-this-famous-grothendieck-quote, «Serre writes ‹...comme Grothendieck nous l'a appris, les objets d'une catégorie ne jouent pas un grand rôle, ce sont les morphismes qui sont essentiels.› See Jean-Pierre Serre, ‹Motifs,› *Asterisque* 198–200 (1991), pp. 335. Although this quote is from 1991, of course it refers to the 1950s and 1960s.»

The former assumption indeed requires additional (outrageous) existence assumptions, while the latter requires none. A configuration is only assumed as a structure from the start, and so it can be specialized by assuming it to occur in more special situations. The schematic character of statements about structures is clearly essential to this approach; whatever we were saying about $\mathbb{C}$ originally (e.g., that there is a group in it), can still be said about $\mathbb{C}$ after we have put it into some ambient category $\mathcal{E}$, because we weren't assuming anything particular about it in the first place.[120]

120 Steve Awodey, «An answer to Hellman's question: ‹Does category theory provide a framework for mathematical structuralism?›» (Unpublished manuscript, Carnegy Mellow University, 2003), 11, https://www.andrew.cmu.edu/user/awodey/preprints/awodeyVhellman.pdf.

X. The ontology of topos theory and the ontology of homotopy type theory

> Lawvere from early on promoted the idea that a category of categories could be used as a foundational framework. This proposal now rests in part on the development of higher-dimensional categories, also called weak *n*-categories. The advent of topos theory in the seventies brought new possibilities. Mac Lane has suggested that certain toposes be considered as a genuine foundation for mathematics. Lambek proposed the so-called free topos as the best possible framework, in the sense that mathematicians with different philosophical outlooks might nonetheless agree to adopt it. He has also argued that there is no topos that can thoroughly satisfy a classical mathematician. [...]
>
> This matter is further complicated by the fact that the foundations of category theory itself have yet to be clarified. For there may be many different ways to think of a universe of higher-dimensional categories as a foundations for mathematics. It is safe to say that we now have a good understanding of what are called $(\infty,1)$-categories and important mathematical results have been obtained in that framework. An adequate language for the universe of arbitrary higher-dimensional categories still has to be presented together with definite axioms for mathematics. [121]

There have been many different theories (which are all related) which have been proposed as a foundation for mathematics outside of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC): the category of categories, topos theory,

121 Jean-Pierre Marquis, «Category Theory,» Stanford Encyclopedia of Philosophy, accessed July 28, 2021, https://plato.stanford.edu/entries/category-theory/.

homotopy type theory. We will in this chapter discuss the ontological commitments of topos theory. We note that homotopy type theory, as Ladyman has said provides a foundation for mathematics.[122] As Awodey, a leading structuralist, writes:

> The recent discovery of an interpretation of constructive type theory into abstract homotopy theory suggests a new approach to the foundations of mathematics with intrinsic geometric content and a computational implementation. Voevodsky has proposed such a program, including a new axiom with both geometric and logical significance: the Univalence axiom. It captures the familiar aspect of informal mathematical practice according to which one can identify isomorphic objects.[123]

Homotopy type theory is completely intensional in the manner of functional programming languages (such as Haskell) which make use of lambdas. Everything is built constructively through ‹tokens› of a ‹type›, so there need not be ridiculous ontological commitments (say, the existence of huge universes). What are the ontological commitments of topos theory? Moreover, if ZFC set theory can not be proved to be consistent within the system of ZFC itself, what are the alternatives for a formal system?

> The consistency of ZFC does follow from the existence of a weakly inaccessible cardinal, which is unprovable in ZFC if ZFC is consistent. [...]

122 See James Ladyman and Stuart Presnell, «Does Homotopy Type Theory Provide a Foundation for Mathematics?,» *British Journal for the Philosophy of Science* 69, no. 2 (June 2018).

123 Steve Awodey, «Structuralism, Invariance, and Univalence,» *Philosophia Mathematica* 22, no. 1 (February 2014): 1.

> If consistent, ZFC cannot prove the existence of the inaccessible cardinals that category theory requires. Huge sets of this nature are possible if ZF is augmented with Tarski's axiom.[124]

> Tarski–Grothendieck set theory is an axiomatic set theory. It is a non-conservative extension of ZFC and is distinguished from other axiomatic set theories by the inclusion of Tarski's axiom, which states that for each set there is a Grothendieck universe it belongs to. Tarski's axiom implies the existence of inaccessible cardinals, providing a richer ontology than that of conventional set theories such as ZFC. For example, adding this axiom supports category theory.[125]

By way of the Tarski axiom, one is able to prove the consistency of ZFC. The Tarski axiom requires the existence of a Grothendieck universe for each set. But the Tarski axiom requires an inaccessible cardinal (i.e., size of infinity; Georg Cantor was the first to formalize cardinality and countability) which is not provable within ZFC, if ZFC is consistent. This axiom by way of the existence of such an uncountable cardinality, therefore makes an ontological commitment. Wiles, who proved Fermat's Last Theorem, makes use of much of Grothendieck's mathematics in his proof of Fermat's Last Theorem. I am not at a technical level to study his proof, but the question remains, does Wiles's work, which makes use of étale cohomology, require these uncountable universes? To answer this question, I defer to an expert:

124 «Fraenkel set theory,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel_set_theory.

125 «Grothendieck set theory,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Tarski%E2%80%93Grothendieck_set_theory.

The basic contention here is that Wiles's work uses cohomology of sheaves on certain Grothendieck topologies, the general theory of which was first developed in Grothendieck's *Séminaire de géométrie algébrique du Bois Marie* and which requires the existence of an uncountable Grothendieck universe. It has since been clarified that the existence of such a thing is equivalent to the existence of an inaccessible cardinal, and that this existence—or even the consistency of the existence of an inaccessible cardinal!—cannot be proved from ZFC alone, assuming that ZFC is consistent. Many working arithmetic and algebraic geometers however take it as an article of faith that in any use of Grothendieck cohomology theories to solve a «reasonable problem,» the appeal to the universe axiom can be bypassed. Doubtless this faith is ~~predicated on~~ abetted by the fact that most arithmetic and algebraic geometers (present company included) are not really conversant or willing to wade into the intricacies of set theory.

And read this:

As a result, in order to figure out if Wiles's proof uses universes, or whether it's relatively easy to avoid them, you'd need to either read the proofs yourself or find someone who was both deeply familiar with the details of the proof, and someone who cares a lot about details. One person who comes quickly to mind is BCnrd [Brian Conrad]. BCnrd was one of the mathematicians who proved the Modularity Theorem, which showed that all elliptic curves over $\mathbb{Q}$ are modular. This is a strengthening of Taylor and Wiles's result, which applied only to semi-stable elliptic curves, and the proof involved understanding and building on Taylor and Wiles's work. BCnrd is also famous for his attention to detail and for consulting

underlying foundational sources; he is the author of a book dedicated to simplifying and correcting the presentation of Grothendieck duality in Hartshorne's book *Residues and Duality*. [...]

BCnrd says there's really no issue at all in Wiles's proof. All of the specific things that Wiles uses stay far away from any of the difficult issues where you might be worried about needing to invoke universes.[126]

Brian Conrad, a leading algebraic geometer, says that Grothendieck universes can be and were avoided in Wiles's proof of Fermat's Last Theorem. Therefore, the ontological commitments of Tarski's axiom can be bypassed. Should we accept such an ontology for topos theory (as was practiced by Grothendieck)? These questions are ultimately existence questions. But even category theory itself does not have the power to even defend the existence of its own categories!

The second argument against the autonomy of categorical foundations has been called the «mismatch objection.» It concerns the general status of category theory or topos theory; and it is based on the distinction between two ways of understanding mathematical axioms, namely as ‹structural›, ‹algebraic›, ‹schematic› or ‹Hilbertian›, on the one hand, and as ‹assertoric› or ‹Fregean›, on the other hand. As Hellman argues, foundational systems such as classical set theory need to be assertoric in character, in the sense that their axioms describe a comprehensive universe of objects used for the codification of other mathematical structures. Zermelo-Fraenkel set theory is

126 «Inaccessible cardinals and Andrew Wiles's proof,» Math Overflow, accessed July 28, 2021, https://mathoverflow.net/questions/35746/inaccessible-cardinals-and-andrew-wiless-proof/35772.

> an assertoric, ‹contentual› theory in this sense. Its axioms (e.g., the power set axiom or the axiom of choice) make general existence claims regarding the objects in the universe of sets.
>
> In contrast, category theory represents a branch of abstract algebra, as its origin reveals. Thus it is, by its very nature, non-assertoric in character; it lacks existence axioms conceived as truths about an intended universe. For example, the Eilenberg-Mac Lane axioms of category theory are not ‹basic truths simpliciter›, but ‹schematic› or ‹structural› in character. They function as implicit definitions of algebraic structures, similar to the way in which the axioms of group theory or ring theory are «defining conditions on types of structures.» This point is related to another argument against the autonomy of category theory that Hellman calls the «problem of the ‹home address›: where do categories come from and where do they live?»[127] Given the «algebraic-structuralist perspective» underlying category theory and general topos theory, its axioms make no assertions that particular categories or topoi actually exist. Classical set theories, such as ZFC with its strong existence axioms, have to step in again in order to secure the existence of such objects.[128]

We can define axiomatically what a group or ring is by means of abstract algebra, but can we prove that these structures exist? Category theory as a foundational system requires both axioms that are «algebraic axioms» (such as abstract algebra) but also ones that «consist of positive assertions concerning the existence and inter-relations of categories

127 Geoffrey Hellman, «Does Category Theory Provide a Framework for Mathematical Structuralism?,» *Philosophia Mathematica* 11, no. 2 (June 2003): 136.

128 Erich Reck and Georg Schiemer, «Structuralism in the Philosophy of Mathematics,» Stanford Encyclopedia of Philosophy, accessed July 27, 2021, https://plato.stanford.edu/entries/structuralism-mathematics/.

and topoi.»[129] Awodey describes examples of the former axioms but says nothing about the latter.[130] Is homotopy type theory the solution? We leave the true answer to that question to the philosophers of mathematics. What is at stake is between the diametric opposition between formalism and intuitionism, as well as idealism and materialism, Platonism and realism and eliminativism, as well as rationalism and empiricism. Is there a third way based on abduction? Peirce's abduction is the notion to achieve the best explanation based on the hypothesis available. Do we necessitate a Kantian synthesis, i.e., the transcendental method? A Sellarsian «synoptic vision?»

Do we follow intuitionistic logic or formalist mathematics? Kripke-Joyal is complete but it necessitates fallibility scenarios. Do we accept the axiomatic method? Do we accept physical entities as really existing or do we follow van Fraassen and say they do not? Shall we pursue computationalist intelligence, Sellars's polemic on the ‹Myth of the Given› and Paul Churchland's thesis that all psychological processes are reducible to neuroscience or instead pursue «libidinal nihilism?» Do we follow Negarestani's idealist turn or Nick Land's myopia of materialism? Or is Zalamea the third way. I believe so.

129 Geoffrey Hellman and Stewart Shapiro, *Mathematical Structuralism* (Cambridge University Press, 2018), 44.

130 Ibid.

XI. Arithmetic becomes geometric

Connes has said that even a child could treat the Natural numbers as a topos, where one replaces the addition of two numbers with the infimum and the product of two numbers with their sum.[131] This is known as a tropical semiring (from the field of tropical geometry). Connes says the tropical semiring could be viewed as a topos:

> We show that the non-commutative geometric approach to the Riemann zeta function has an algebraic geometric incarnation: the «Arithmetic Site.» This site involves the tropical semiring $\bar{\mathbb{N}}$ viewed as a sheaf on the topos $\widehat{\mathbb{N}^{\times}}$ dual to the multiplicative semigroup of positive integers.[132]

Zalamea has explained the significance of the work of Grothendieck toposes, «Grothendieck toposes unify deep insights on arithmetic (number) and geometry (form).»[133] Topos is a completely geometric object, where, typically in contrast, arithmetic is logical (such as for example, Peano Arithmetic, which is first- or second-order, depending on the axioms). Topos provides a radical way of seeing geometry as logic and logic as geometry:

131 Alain Connes, «Alain Connes – The power of the simplest arithmetic examples of Grothendieck toposes (part 1),» Youtube, accessed July 28, 2021, https://www.youtube.com/watch?v=dDm0wxnawJE&t=140s.

132 Alain Connes and Caterina Consani, «The Arithmetic Site,» *Comopte Rendus – Mathematique* 352, no. 12 (December 2014): 971.

133 Fernando Zalamea, «Multilayered Sites and Dynamic Logics for Transits between Art and Mathematics,» *Glass Bead, Site 0: Castalia, The Game Of Ends And Means* (2016): 13, https://www.glass-bead.org/wp-content/uploads/multilayered-sites-and-dynamic-logics-for-transits-between-art-and-mathematics_en.pdf.

> The history of category theory offers a rich source of information to explore and take into account for an historically sensitive epistemology of mathematics. It is hard to imagine, for instance, how algebraic geometry and algebraic topology could have become what they are now without categorical tools.
>
> Category theory has led to reconceptualizations of various areas of mathematics based on purely abstract foundations. Moreover, when developed in a categorical framework, traditional boundaries between disciplines are shattered and reconfigured; to mention but one important example, topos theory provides a direct bridge between algebraic geometry and logic, to the point where certain results in algebraic geometry are directly translated into logic and vice versa. Certain concepts that were geometrical in origin are more clearly seen as logical (for example, the notion of coherent topos). Algebraic topology also lurks in the background. On a different but important front, it can be argued that the distinction between mathematics and metamathematics cannot be articulated in the way it has been. All these issues have to be reconsidered and reevaluated.[134]

For an exposition of the relationship between logic and geometry and how algebraic geometry creates this bridge, we defer to the experts who wrote the book *Sheaves in Geometry and Logic: A First Introduction to Topos Theory*.[135] As the mathematician and philosopher René Thom writes in his book *Semio Physics*:

134 Jean-Pierre Marquis, «Category Theory,» Stanford Encyclopedia of Philosophy, accessed July 27, 2021, https://plato.stanford.edu/entries/category-theory/.

135 See Saunders MacLane and Ieke Moerdijk, *Sheaves in Geometry and Logic: A First Introduction to Topos Theory* (Cham: Springer Nature, 1994).

> We will not be looking for the foundation of geometry in logic. On the contrary, we shall be seeing logic as a derived activity… a rhetoric. We shall be trying, not so much to convince, as to suggest representations and to extend the intelligibility of our world. Instead of building geometry in a logical manner, we will seek to base what is logical on geometry.[136]

The aim of this treatise was to follow Thom's philosophical program of geometry as the foundation (instead of logic). Recent work in homotopy type theory, which is based upon topological notions, would most likely confirm this philosophy. We conclude with a postscript critiquing the logic of object-oriented programming and economics.

136 René Thom, *Semio Physics: A Sketch*, trans. Vendla Meyer (Boston: Addison-Wesley, 1990), 2.

XII. Postscript: the Axiom of Choice and the Banach-Tarski paradox

The full strength of the Axiom of Choice does not seem to be needed for *applied* mathematics. Some weaker principle such as Countable Choice or Dependent Choice generally would suffice. To see this, consider that any application is based on measurements, but humans can only make *finitely many* measurements. We can extrapolate and take limits, but usually those limits are sequential, so even in theory we cannot make use of more than countably many measurements. The resulting spaces are separable. Even if we use a nonseparable space such as L^∞, this may be merely to simplify our notation; the relevant action may all be happening in some separable subspace, which we could identify with just a bit more effort. (Thus, in some sense, nonseparable spaces exist *only* in the imagination of mathematicians.) If we restrict our attention to separable spaces, then much of conventional analysis still works with Axiom of Choice replaced by Countable Choice or Dependent Choice. However, the resulting exposition is then more complicated, and so this route is only followed by a few mathematicians who have strong philosophical leanings against Axiom of Choice.

A few pure mathematicians and many applied mathematicians (including, e.g., some mathematical physicists) are uncomfortable with the Axiom of Choice. Although Axiom of Choice simplifies some parts of mathematics, it also yields some results that are unrelated to, or perhaps even contrary to, everyday «ordinary» experience; it implies the existence of some rather bizarre, counterintuitive objects. Perhaps the most bizarre is the Banach-Tarski paradox: It is possible to take the 3-dimensional closed unit ball,

$$B = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$$

and partition it into finitely many pieces, and move those pieces in rigid motions (i.e., rotations and translations, with pieces permitted to move through one another) and reassemble them to form two copies of *B*.

At first glance, the Banach-Tarski result seems to contradict some of our intuition about physics—e.g., the law of conservation of mass, from classical Newtonian physics. If we assume that the ball has a uniform density, then the Banach-Tarski paradox seems to say that we can disassemble a one-kilogram ball into pieces and rearrange them to get two one-kilogram balls. But actually, the contradiction can be explained away: Only a set with a defined ‹volume› can have a defined mass. A volume can be defined for many subsets of $\mathbb{R}^3$—spheres, cubes, cones, icosahedrons, etc.—and in fact a volume can be defined for nearly any subset of $\mathbb{R}^3$ that we can think of. This leads beginners to expect that the notion of ‹volume› is applicable to *every* subset of $\mathbb{R}^3$. But it's not. In particular, the pieces in the Banach-Tarski decomposition are sets whose volumes cannot be defined.[137]

137 See Eric Schechter, «Axiom Of Choice,» Vanderbilt University, accessed July 28, 2021, https://math.vanderbilt.edu/schectex/ccc/choice.html, «More precisely, *Lebesgue measure* is defined on some subsets of $\mathbb{R}^3$, but it cannot be extended to all subsets of $\mathbb{R}^3$ in a fashion that preserves two of its most important properties: the measure of the union of two disjoint sets is the sum of their measures, and measure is unchanged under translation and rotation. The pieces in the Banach-Tarski decomposition are not Lebesgue measurable. Thus, the Banach-Tarski paradox gives as a corollary the fact that there exist sets that are not Lebesgue measurable. That corollary also has a much shorter proof (not involving the Banach-Tarski paradox) which can be found in every introductory textbook on measure theory, but it too uses the Axiom of Choice. Here is a brief sketch of that shorter proof: Work in «the real numbers modulo 1»—that is, the number system that you get if you cut the interval [0,1) out of the real line and loop it around into a circle, so that 0 and 1 are the same number. (Like the way that 0 and 12 are the same on a circular clock.) In that number system, multiplication and division don't really work very well any more, but addition and subtraction still work fine, and so does Lebesgue measure. Let's call that number system *T*; its entire measure is 1. Now, the Axiom of Choice is used to «construct» a rather peculiar subset of *T*—let us call it *C*—with the property that the sets $C+r = \{x+r : x \text{ in } C\}$ are all disjoint from each other, for different values of the rational number *r*. The union of these sets is all of *T*. Now, if *C* were measurable, then so would each *C+r*, and they would all have the same measure, and their measures would add up to the measure of *T*—that is, they would add up to 1. But how many of these *C+r* are there? There are a countable infinity of them. If the measure of *C* were zero, their sum would be zero. If the measure of *C* were positive, their sum would be infinite. You can't get 1, either way.»

The Axiom of Choice is controversial to some mathematicians. It states that it is possible to have a «choice function,» which chooses an element from an infinite amount of collections, infinitely many times. The Axiom of Choice leads to the Banach-Tarski paradox, which roughly says one ball can be decomposed and reassembled into two balls, which seems to contradict the law of conservation of mass. But what is important to note is, these sets do not have volume, so it does not lead to any contradiction in physics since these sets are not Lebesgue measurable. The Axiom of Choice can be replaced by less controversial cases such as countable choice or dependent choice. Most physicists deal with cases that require only finitely many measurements. Stephen Hawking at least once invoked the Axiom of Choice to prove a theorem.[138]

The Axiom of Choice is equivalent to Zorn's lemma. Zorn's lemma states «If *S* is any nonempty partially or-

138 Arguments from physics may not help. Here is Bryce DeWitt reviewing Stephen Hawking and G.F.R. Ellis using the Axiom of Choice; see Bryce DeWitt «Much-Esteemed Theory,» *Science* 182, no. 4113 (November 1973): 706, «The book also contains one failure to distinguish between mathematics and physics that is actually serious. This is in the proof of the main theorem of chapter 7, that given a set of Cauchy data on a smooth spacelike hypersurface there exists a unique maximal development therefrom of Einstein's empty-space equations. The proof, essentially due to Choquet-Bruhat and Geroch, makes use of the axiom of choice, in the guise of Zorn's lemma. Now mathematicians may use this axiom if they wish, but it has no place in physics. Physicists are already stretching things, from an operational standpoint, in using the axiom of infinity. It is not a question here of resurrecting an old and out-of-date mathematical controversy. The simple fact is that the axiom of choice never is really needed except when dealing with sets and relations in non-constructive ways. Many remarkable and beautiful theorems can be proved only with its aid. But its irrelevance to physics should be evident from the fact that its denial, as Paul Cohen has shown us, is equally consistent with the other axioms of set theory. And these other axioms suffice for the constructions of the real numbers, Hilbert spaces, C* algebras, and pseudo-Riemannian manifolds–that is, of all the paraphernalia of theoretical physics. In «proving» the global Cauchy development theorem with the aid of Zorn's lemma what one is actually doing is assuming that a «choice function» exists for every set of developments extending a given Cauchy development. This, of course, is begging the question. The physicist's job is not done until he can show, by an explicit algorithm or construction, how one could in principle always select a member from every such set of developments. Failing that he has proved nothing.»

dered set in which every chain has an upper bound, then *S* has a maximal element.»[139] This amounts to the existence of a maximal element. In this chapter we will discuss the problems for assuming Zorn's lemma (⇔ Axiom of Choice) in cybernetics and economics. One consequence is that: «The Axiom of Choice implies every vector space has a basis.»[140] For the case of Zorn's lemma implying every vector space has a basis, refer to the press release for the *Hermetic Definition* art show which has the proof.[141] Standard economics is premised on the notion of fungibility (what Karl Marx has called exchange-value). But we are not throwing the baby out with the bathwater, purely on the existence of a basis. Surely, a Linear Algebra student would say the existence of a basis for a vector space is not much to assume. Oswald Wiener has criticized the cybernetic paradigm we live in after the research of Norbert Wiener, going so far to say, «science and barbarism go very well together»:

> Because if one thing is clear, it's that science and barbarism go very well together. The scientists needn't necessarily be barbarians themselves, they need only shut their eyes or want to live nice lives or have a voice within the structures of power. The stagnation isn't just in the social or aesthetic fields, but is also present in the natural sciences. In physics there has been no new idea that has really pushed things forwards for a hundred years. This is perhaps putting it very crudely, but thousands of the best researchers would cut off their own right arm to have an idea that amounted to a comparable scientific advance as quantum physics a hundred years ago. All kinds of things are going on, but, in the best cybernetic tradition, the simulation apparatuses are reconciling the contradictions between the theories of physics—it's known as a «handshake»—and innovation is being thwarted.
>
> Many of the simulation techniques used by physics are successful: weather forecasting, for example, has become a lot better, but they cannot explain why. The method is to calculate on different levels that are incompatible with each other. These different levels are then glued together by a form of creative accounting. The glue they use is the same cybernetic kind that I make fun of in *the improvement of central europe*. It's a pure invention, a fiction, but nevertheless it works.[142]

What exactly are these contradictions which are presupposed by cybernetics? The Axiom of Choice supposes a choice function exists to choose an element, an infinite amount of times. This can be thought of as a decision. Because of the halting problem, there is a limit to when a computer can halt by itself and can instead remain in a loop forever. In my stage manifesto,[143] I articulated the basis for which contemporary artists live in a paradigm of an «image economy.» This is a completely codified language

139 «Zorn's Lemma,» Wolfram MathWorld, accessed March 20, 2022, https://mathworld.wolfram.com/ZornsLemma.html.

140 Kevin Barnum, «The Axiom of Choice and its implications» (Unpublished manuscript, University of Chicago, 2013), 5, http://math.uchicago.edu/~may/REU2014/REUPapers/Barnum.pdf.

141 «Hermetic Definition,» Emily Harvey Foundation, accessed July 28, 2021, https://www.emilyharveyfoundation.org/newyork/PDF/EmilyHarveyFoundation/0116.pdf.

142 Oswald Wiener, «Oswald Wiener: ‹Science and Barbarism Go Very Well Together›,» interviewed by Hans-Christian Dany, trans. Nathaniel McBride, *Spike Art Magazine* 42, (January 2015).

143 Eric Schmid, «Stage Manifesto,» What Pipeline, accessed July 28, 2021, http://whatpipeline.com/exhibitions/33kavitabschmid/stage.pdf.

of visuality, the image-object of desire. This is a means of organizing perception according to codification. Similarly, economics supposes that the topology of the space exists on such notions as the «utility of a product» or even a person's performance (key performance indicator). But does this convergence exist for all objects in the realm of econometrics? Do we presuppose that infinite profit is possible? The heart of the matter is Simondon's critique of Norbert Wiener:

> Simondon criticized Norbert Wiener's theory of cybernetics, arguing that «Right from the start, [cybernetics], has accepted what all theory of technology must refuse: a classification of technological objects conducted by means of established criteria and following genera and species.»[144] Simondon aimed to overcome the shortcomings of cybernetics by developing a «general phenomenology» of machines.[145]

We wish to critique the object-oriented epistemology of genus and species, replacing it instead with a Simondonian/Thomian/Piagetian «genetic epistemology,» which has no individuated atom, «the individual,» as its metaphysics. Piaget's developmental psychology was premised on the cognitive invariants of child development, these principles had a topological basis (conceiving that a cup is half empty or full). My problem with cybernetics is a philosophical one formulated in response to its creation of virtual objects

144 Gilbert Simondon, *Du mode d'existence des objets techniques* (Paris: Aubier, 1989), 48.

145 «Gilbert Simondon,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Gilbert_Simondon.

which can be compared and sorted against one another (think of the website Hot or Not). ‹Total orders›, if Zorn's lemma is assumed, create hierarchy where there is a maximal element for the objects in the computer program. In finite cases, Zorn's lemma is unnecessary, but in infinite cases, a spanning tree does not necessarily exist.

In the Java language, which is object-oriented:

> [T]here are several existing methods that already sort objects from any class like `Collections.sort(List<T> list)`. However, Java needs to know the comparison rules between two objects. So when you define a new class and want the objects of your class to be sortable, you have to implement the `Comparable` and redefine the `compareTo(Object obj)` method. [...]
>
> Sometimes we may want to change the ordering of a collection of objects from the same class. We may want to order descending or ascending order. We may want to sort by `name` or by `address.`
>
> We need to create a class for each way of ordering. It has to implement the `Comparator` interface.
>
> Since Java 5.0, the `Comparator` interface is generic; that means when you implement it, you can specify what type of objects your comparator can compare.[146]

In the Java programming language, one needs to use an interface such as `Comparable` or `Comparator`in order to sort objects:

146 «Comparing Objects,» Wikibooks, accessed July 28, 2021, https://en.wikibooks.org/wiki/Java_Programming/Comparing_Objects.

[P]ublic interface `Comparator<T>`

A comparison function, which imposes a *total ordering* on some collection of objects. Comparators can be passed to a sort method (such as `Collections.sort` or `Arrays.sort`) to allow precise control over the sort order. Comparators can also be used to control the order of certain data structures (such as `sorted sets` or `sorted maps`), or to provide an ordering for collections of objects that don't have a `natural ordering`.

The ordering imposed by a comparator `c` on a set of elements `S` is said to be *consistent with equals* if and only if `c.compare(e1, e2)==0` has the same boolean value as `e1.equals(e2)` for every `e1` and `e2` in `S`. [...]

For the mathematically inclined, the *relation* that defines the *imposed ordering* that a given comparator `c` imposes on a given set of objects `S` is:

`{(x, y) such that c.compare(x, y) <= 0}`.

The *quotient* for this total order is:

`{(x, y) such that c.compare(x, y) == 0}`.

It follows immediately from the contract for `compare` that the quotient is an *equivalence relation* on `S`, and that the imposed ordering is a *total order* on `S`. When we say that the ordering imposed by `c` on `S` is *consistent with equals*, we mean that the quotient for the ordering is the equivalence relation defined by the objects' `equals(Object)` method(s):

`{(x, y) such that x.equals(y)}`.

Unlike `Comparable`, a comparator may optionally permit comparison of null arguments, while maintaining the requirements for an equivalence relation.[147]

147 «Interface Comparator<T>,» Java™ Platform, Standard Edition 8 API Specification, accessed July 28, 2021, https://docs.oracle.com/javase/8/docs/api/java/util/Comparator.html.

So the `Comparator` imposes a total order on the objects. In the case of finite elements, a maximal element can easily be identified. We are interested in the case of infinitely many elements where Zorn's lemma (Axiom of Choice) needs to be invoked. One can refer to the mathematical definition of a ‹total order›, which one would learn about in a Discrete Mathematics class.[148] The problem with object-oriented programming is thus the case in which a computer needs to sort an infinite amount of objects. The Axiom of Dependent Choice can be invoked for a countable (infinite) chain.[149] But for the case of uncountable chains, Zorn's lemma is needed. If one were to have virtual objects that were as uncountable as the real number line, one would need to use Zorn's lemma:

> The set of Java programs (or the set of programs in any language) is countably infinite, because you can list them according to the number of characters they contain.
>
> But since there are an uncountably infinite number of functions from $\mathbb{N}$ to $\mathbb{N}$, then there are some functions which cannot be written in Java, or any programming language. Essentially, this means there are some functions which cannot be computed!
>
> Not to be confusing, but there are an uncountably infinite number of functions from $\mathbb{N}$ to $\mathbb{N}$ that *cannot* be computed, so there are ‹more› functions that cannot be computed than ones that can be.[150]

148 Eric W. Weisstein, «Totally Ordered Set,» Wolfram MathWorld, accessed July 28, 2021, https://mathworld.wolfram.com/TotallyOrderedSet.html.

149 «A ‹countable Zorn's lemma›?,» Mathematics Stack Exchange, accessed July 28, 2021, https://math.stackexchange.com/questions/2670231/a-countable-zorns-lemma.

150 «What are some interesting examples of uncountable sets?,» Quora, accessed July 28, 2021, https://www.quora.com/What-are-some-interesting-examples-of-uncountable-sets.

The Riemann sphere in non-Euclidean geometry makes use of 1-point compactification, where infinity is compactified into a point. These spaces are compact. Do economics and cybernetics suppose «infinite wealth» or use models that require «compactified spaces?» Thom addresses the question of ‹infinity› (in aesthetics):

> The problem of aesthetics is a difficult one. I've written about it somewhat, but I must confess that the elaboration of a satisfactory theory would be extremely difficult. I've the impression that at the root of ‹the aesthetic› one finds ‹the sacred›. What is the sacred? It was this question that led me to my theory of *pregnances* and *saliences*. The original idea is that all behavior, starting with that of animals, is controlled by the fact that when the animal perceives a form in its presence, reactions of attraction and repulsion are released with regard to that form, whether they be visual, auditory, olfactory, and so on. In even the most rudimentary cases, one finds these reactions of attraction and repulsion.
>
> I believe that the sense of the sacred in human beings is characterized by the fact that this axis of attraction/repulsion can, in some sense, become self-referencing through being compactified by a point at infinity. This point at infinity is precisely what we call the sacred. Stated differently, a sense of the sacred is aroused every time we find ourselves in the presence of a form which appears to be endowed with infinite power, and which is simultaneously attractive and repulsive. As these two infinities are in opposition, one becomes immobilized relative to this form: Its fascination causes motion to cease. Because such a situation is intolerable for very long, certain accommodations emerge, which relax this paralysis through the phenomenon of sacralization.[151]

We can ask if economics and cybernetics suppose this «point at infinity» in order to create a value system based upon an ideality. Thom is generalizing this «point at infinity» to questions of metaphysics, turning 1-point compactification into a question about aesthetics, «attractions and repulsions» based upon the ideal point which is sacred; we could call it godly. This theory of salience vs. pregnance is defined in his book *Semio Physics*, where he makes the category of pregnances rigorous. My aesthetic attraction or repulsion is based on some ideal form of beauty. The method of 1-point compactification is used in the Riemann sphere. As discussed before, the cross ratio is invariant under the actions of Möbius transformations on the Riemann sphere (non-Euclidean geometry). Most non-Euclidean models suppose some absolute boundary which represents «infinity,» but it is outside of the space. For example, in hyperbolic geometry using the Poincaré half-plane model, the *x*-axis is outside of the space.

If one were to follow Diedrich Diederichsen in *Kai Kein Respekt*, an artist could hypothesize a homofuturism/homosociality that is an anti-systematic transcendental without the «given» or «thesis» based on a domestic phenomenology of friendship or «twin speak.» A non-communication based on intimacy («soulmates?»). Deleuze had said about control societies:

151 René Thom, «René Thom: Interviews with Emile Noël,» ed. Peter Tsatsanis, trans. Roy Lisker (Unpublished translation, Ferment Magazine, 2010), 95–96, https://www.fermentmagazine.org/Stories/Thom/Predire.pdf.

> Maybe speech and communication have been corrupted. They're thoroughly permeated by money—and not by accident but by their very nature. We've got to hijack speech. Creating has always been something different from communicating. The key thing may be to create vacuoles of noncommunication, circuit breakers, so we can elude control.[152]

Deleuze reiterates Pierre Klossowski's *Living Currency*, which is a psychic individuation (pulsion), or Georges Bataille's *Story of the Eye*, which lays out a theory of the sexual. For Klossowski, in *Sade, My Neighbor*, there is no Godhead («headless») in republican society. Therefore, these questions about the sacred or divine beauty are meaningless. But what is the real material correlate before money and commoditization of the thing, before currency? The best solution to this question of the ineffable Real is to be found in the aesthetic work of Gerald Donald, Dieter Roth, Theo Parrish, Robert Filliou, Hermann Nitsch, Dominik Steiger, Dorothy Iannone, André Thomkins, Isa Genzken and Jutta Koether. For example, listen to Parrish's *Soul Control* or *Command Your Soul* or Donald's Dopplereffekt or ARPANET.

In the works of these artists, we perceive examples of the sorts of aesthetic approach to the ineffable Real which support the intuitionistic logic of art as a topos, navigating the locale of our neuroplasticity to the globality of extrinsic structural invariants. Utilizing the gluing axiom, the pragmaticist maxim of multivalent «underdeterminations of the sign»[153] and their corresponding representations forming collectively a univalent foundation, hypothetically derived. The «universal as bimodalization»[154] of phenomenology and physics. The ‹manifest image› of human affairs and the ‹scientific image› of mathematics.

152 Gilles Deleuze and Antonio Negri, «Control and Becoming: Gilles Deleuze In Conversation with Antonio Negri,» trans. M. Joughin, Generation Online, accessed July 28, 2021, http://www.generation-online.org/p/fpdeleuze3.htm.

153 Fernando Zalamea, *Synthetic Philosophy of Contemporary Mathematics* (Falmouth/New York: Urbanomic/Sequence Press, 2012), 118.

154 Reza Negarestani, «Where is the Concept?,» in *When Site Lost the Plot*, ed. Robin Mackay (Falmouth: Urbanomic, 2013), 230.

XIII. Addendum: why de-ontologized metaphysics? Computationalism (e.g., homotopy type theory) or topos theory?

My reply to Tim Pierson's criticism of my *Prolegomenon* through the Derridean critique of structuralism is the following excerpt of a conversation between Reza Negarestani and Robin Mackay on the topic of Rudolf Carnap:

> [Reza Negarestani] Basically, even though my background is *actually* engineering, this whole relevance of engineering with regard to philosophy only came extremely late to me—I was already thinking about it, but in quite a naive sense. It came to me by way of the late work of Rudolf Carnap—who is, by the way, a hidden figure throughout the [*Intelligence and Spirit*]. Carnap began as a logical empiricist. He was a positivist—his 1928 book *The Logical Structure of the World* is a monumental work of logical positivism. However, by 1934, Carnap had fundamentally betrayed the original theses of the Vienna Circle and its vision of positivism. He had moved towards a fundamentally different vision which he continued to refine towards the end of his life. Essentially, Carnap's main idea, what bothers him—what you called ‹ontological cartoons›—is that philosophy, in the traditional sense of big ontological and metaphysical questions, is always, as he puts it, ‹the opiate of the educated›.
>
> So we say: *What is life? What is intelligence? What is mind? What is justice? What is the good?* Big questions that don't just excite graduate students, but also humans in general! And then, under the auspices of such questions, we come up with such tantalising answers: *If we could only do this, we will have answered all of our questions!* Big ideas beget practical illusions

> of grandeur. But Carnap instead thinks about what he calls ‹conceptual engineering›. Carnap's ‹conceptual engineering› is the idea that these overarching upper concepts are all *vague concepts*—what he calls *explicandum*, in the sense that these concepts mean different things in different contexts for different language-using agents. In fact, many incommensurable questions can be posed under the apparently unifying facade of these concepts. So he proposes a process, or an ideal of an engineering process, that he calls *explication*, and which would turn these vague concepts into more refined concepts which are called the *explicata*.
>
> [Robin Mackay] Yet Carnap thinks that there are, even so, real philosophical problems, correct? This is not the same programme as Wittgenstein's dissolution of ‹pseudo-problems› by treating them as mere problems of language.
>
> [R.N.] The Vienna Circle Carnap comes from a Wittgensteinian position, but after *The Logical Syntax of Language*, after a fever that leads him to sever his last links with the Wittgensteinian strings, Carnap turns against both his earlier logical empiricist commitments and his Wittgensteinian influences. Why? In a sense, for both thinkers, the question of language is the question of structure. And if one posits something outside of the dimension of structure, one ends up peddling those pseudo-problematic nasties about immediate knowledge of the world that are the hallmarks of precritical philosophy. In fact, even logical empiricism, Carnap's earlier position, is fundamentally against the kind of naive empiricism that Sellars admonishes as an instance of the myth of the categorial given. But then why is the later Carnap the epitome of anti-Wittgensteinianism? The answer lies in how they understand language. For

Wittgenstein, the limits of language are the limits of the world and existence. For Carnap, after attending the Gödel seminars, language is the ocean of meta-languages and meta-logics, the unbound ocean of possibilities. To see the limits of language, one does not step into the extra-linguistic, but must adopt a new meta-logical position with regard to language, inhabit a new more expansive language. Even though Wittgenstein is apparently anti-Kantian, he endorses the main thesis of Kant regarding the subordination of language to the intuitive, especially in his picturing theory of language. Carnap unshackles the vision of language from this Kantian metaphysical cage. For Carnap, language is not about the world, or anything extra-linguistic. Anyone who says otherwise will have to pay a metaphysical high price. One can say that Carnap in fact breaks away from the metaphysical dualities of mind or language and the world by reinventing their classical problems on the level of theory as a system of object constitution.

[R.M.] This is something that is covered in detail in the book, where language plays an important role—indeed, it is hailed as the *Dasein of Geist!*— but where you anticipate artificial languages that could outstrip natural language both syntactically and semantically: the domain of artificial general language.

[R.N.] Yes, language generally understood—i.e., beyond the existing scope of the ordinary natural languages—is the dimension of structure, and structure is a matter of worldbuilding. We can only expand our representations by expanding the structure, our resources for world-representation are indebted to our resources of world-building, toying around with the possibilities of structure, diversifying it and enriching it. Of course, this raises the question of how such free play can

be related back to our rudimentary experiences so as to avert the danger of naïve idealism or logicism in the vein of *Aufbau*. This is the question of the so-called ‹protocol sentences› and the possibilities of a true physicalistic scientific language which I don't have time to elaborate here. Essentially, the mature Carnap thinks like Poincaré: How can we stop being the ‹victims of our particular habituations›? How can we diverge from our so-called habituated facts of experience by constructing new experience-constituting languages or paradigms such that we can habituate our experiences to new objective ways? This is the revival of the Enlightenment question. Carnap admitted that the Enlightenment paradigm has been corrupted, it has become a recipe for conformity to the order of *is*. But the real ambition of the Enlightenment as he understood it is to move from the order of *is* to the order of what *should be* or what *might be*. The principle of tolerance and the idea of diversifying the formal artificial languages are instances of moving from what is the case (in the order of appearances) to what might actually be the case.

We usually think of Carnap as that positivistic or radical conventionalist guy, someone who thinks rules alone are sufficient. But no, to the extent that he thinks rules alone are not sufficient, he is not conventionalist; and to the extent that he revises his thoughts about how elementary experiences relate to linguistic sentences (of a broader linguistic-logical domain), he is not that logical empiricist guy. In any case, the late Carnap is against Wittgenstein. His position is something between Hegel's vision of language as the Dasein of Geist, Poincaré's idea that new structures make new experiences just as Riemannian geometry opens up new spaces for the observer, and Helmholtz's emphasis on perception and sensory processing, without ever eliding the distinctions between such views or expanding the conclusions reached by one insight to another.

> [R.M.] So, for Carnap, the refining of ‹big› philosophical questions into engineering problems is quite different to a dismissal of them as ‹pseudo-problems›.[155]

Hellman's structure without ontology would support their claim. In response to Hunter Hunt-Hendrix's critique of unsystematicity refer to Lorenz Puntel's *Structure and Being*, Joseph Sneed's *The Logical Structure of Mathematical Physics*, Wolfgang Stegmüller's *The Structure and Dynamics of Theories* and Wolfgang Balzer, C. Ulises Moulines and Sneed's *An Architectonic for Science*. Thank you to Reza Negarestani for making the public aware of these works. Thank you Tim Pierson for sending contemporary research on mathematical linguistics which employs Lambek pregroups,[156] DisCoCat[157] or fibrational linguistics.[158] This research is only the beginning for accounting for ordinary language and possibly poetry.

The recent interest in homotopy type theory within continental philosophy ignores the «wild heart of mathematics.» Homotopy type theory is concerned with foundational issues. The ‹computational trilogy›[159]—computation

155 Reza Negarestani and Robin Mackay, «Reengineering Philosophy,» edited and expanded version of Reza Negarestani's conversation with Robin Mackay at the launch of Negarestani's book *Intelligence and Spirit* in New York in November 2018, https://www.urbanomic.com/document/reengineering-philosophy/.

156 Joachim Lambek, «Type Grammar Revisited,» in *Logical Aspects of Computational Linguistics*, eds. Alain Lecomte, François Lamarche, Guy Perrier (Cham: Springer Nature, 1999): 1–27.

157 Bob Coecke, Konstantinos Meichanetzidis, «Meaning updating of density matrices» (Unpublished manuscript, Oxford University, 2020), https://arxiv.org/pdf/2001.00862.pdf.

158 Fabrizio Genovese, Fosco Loregian, Caterina Puca, «Fibrational linguistics I: first concepts» (Unpublished manuscript, 2022), https://arxiv.org/pdf/2201.01136.pdf.

159 «Computational trilogy,» nLab, accessed March 20, 2022, https://ncatlab.org/nlab/show/computational+trilogy.

and programming languages, intuitionistic logic and type theory, category and topos theory—forgoes Peter Scholze's recent work on unification.[160] Intuitionistic type theory is necessarily related to functional programming languages such as Agda and Coq. Applied category theory and type theory necessarily miss out on the future of mathematics due to an inability for creative acts at the highest level of abstraction. Scholze proposes the following:

> How to do algebra when rings/modules/groups carry a topology? This question comes up rather universally, and many solutions have been found so that in any particular situation it is usually clear what to do. Here are some sample situations of interest:
>
> (i) Representations of topological groups like $GL_n(\mathbb{R})$, $GL_n(\mathbb{Q}_p)$, possibly on Banach spaces or other types of topological vector spaces, or more generally topological abelian groups.
>
> (ii) Continuous group cohomology: If G is a topological group acting on a topological abelian group M, there are continuous group cohomology groups $H^i_{cont}(G, M)$ defined via an explicit cochain complex of continuous cochains.
>
> (iii) Algebraic (or rather analytic) geometry over topological fields K such as $\mathbb{R}$ or $\mathbb{Q}_p$.[161]

160 «Masterclass in Condensed Mathematics,» University of Copenhagen, accessed March 21, 2022, https://www.math.ku.dk/english/calendar/events/condensed-mathematics/, «This theory holds the promise of vastly expanding the reach of geometrical and analytical methods in a broad framework that, in addition to p-adicgeometry, includes complex geometry and analysis. Such a unification is entirely new and may well supply the geometrical underpinning to make it possible to attack some of the most important conjectures in mathematics.»

161 Peter Scholze and Dustin Clausen «Lectures on Condensed Mathematics,» (Lecture notes, University of Bonn, 2019), 6, https://www.math.uni-bonn.de/people/scholze/Condensed.pdf.

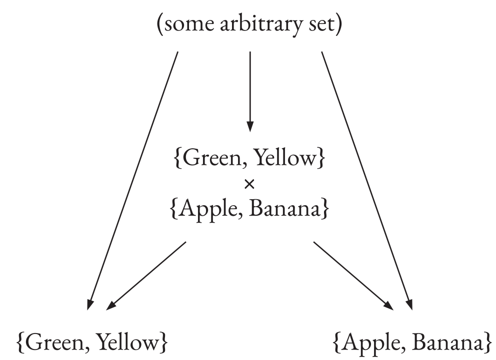
Afterword one (Arrow and substance)

The story of the 20th century as the optimism of science and reason hitting the brick wall of the limits of language and shattering into countless discursive shards is too well-known a tale to go over once again. It's neither sufficient nor even desirable to retread Immanuel Kant or (God forbid) René Descartes in an endless chasing of one's own tail in the hopes of one day directly grasping the Real, to positively articulate that final a priori which one can supposedly privilege above all others once and for all; nor is it any more constructive to dismiss any notion of objectivity or realism in a fit of facile solipsism (or worse, accuse the postmodernists of doing nothing but this), as if such narcissism amounted to anything more than the most childish way possible of claiming special access to unadulterated reality. We've all since learned from (the latter works of) Ludwig Wittgenstein that the Real must be approached on its own terms by deflating the pretenses that block one's ability to further reason; that is, to simply play the language games regardless of where they may lead and learn to move with more finesse in order to better sidestep superfluous semantic baggage.

This method is at the heart of Eric's treatise, where the genetic epistemology of Jean Piaget, the individuation of Gilbert Simondon, the theories of difference of Gilles Deleuze, and the ‹non-philosophy› of François Laruelle dovetail with the synthetic approaches of emerging areas of mathematics such as category and type theory, to sketch out a kind of idealism by which one can play out new tactics for addressing the

Real through a rigorous construction of the universal where the active craft of epistemology replaces the endless regress of fundamentally constipated questions that refuse to further develop and differentiate their affordances. Or, to put it another way, rather than naively positing a static a priori backdrop as either a set of building blocks or a stage upon which everything else fundamentally acts, one effectively converges on an ideal through the construction of universalities rather than merely uncovering a passive a priori.

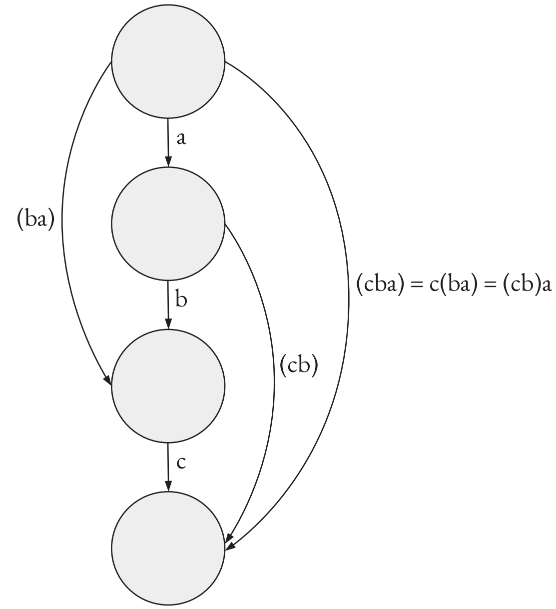
The roots of this kind of objectivity go at least as far back as Kant's synthetic a priori, but this very concept was not fully articulated as a constructive taming of contingency until Deleuze, influenced by Baruch Spinoza's isomorphism linking reason and freedom of action, revised Kant's passive synthesis, by which experience is framed through some automatically granted intuition, into an active synthesis by which abduction is achieved through the interfacing of ideas with one another as part and parcel of acting in the world. From these interfaces that riff on one another in the manner of branches that grow from earlier branches of a tree (forgive the arborescent metaphor), one finds undeniable knowledge in any resulting ‹invariants›: those patterns whose nature are not contingent on the angle from which one approaches them; i.e., that which simply does not vary depending on this or that viewpoint.



Such invariants, from the point of view of category theory, come in the form of ‹universal constructions›: unique factorizations that as a result of their uniqueness come to represent an idea in a way that goes beyond the naive Saussurean notion of «standing for» or «pointing to» some referent. To take a simple example, consider the product of two sets, say, {Green, Yellow} and {Apple, Banana}. In set theory, you'd describe it as all possible pairs between these sets: {Green Apple, Green Banana, Yellow Apple, Yellow Banana}. But now consider this in terms of what this says about affordances: if you take any set and give it two functions—one that goes to the first set, and another that goes to the second set—there exists one and only one function that allows you to factor them, such that each of these functions is a composite of a single function going to the product and the respective projection from that product. One now no longer needs to think about the ‹contents› of the product because it exists by virtue of this universal relationship that is not subject to change.

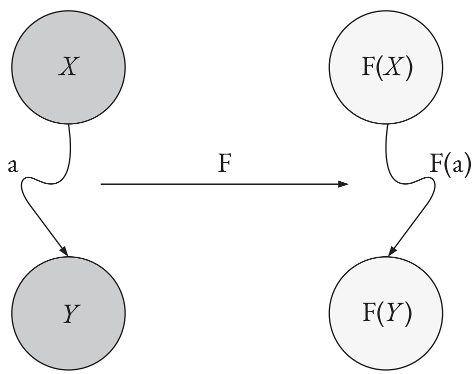
In addition, one also learns that the specific ‹elements› of the set do not matter: if the elements were different but the relationships between arrows (functions) were the same, it would be no different than having the same elements but with different names (and indeed, in the simple example of sets, any set with four elements could serve the exact same function as the product in the example!). In category theory, all such universal constructions therefore exist «up to isomorphism,» where any two things that are isomorphic are things that are different in name only. By finding such isomorphisms, one further deflates excess ontological gestures in favor of the only operationally relevant thing: the pattern, a set, at least in category theory, no longer represents a collection of anything in the usual sense, but simply plays a role in a system defined by other sets.

This lack of distinguishability between the ‹content› of any two such objects beyond their place in a larger order of connections, a notion that reverberates Spinoza's panpsychist ontology, renders all such content a matter of interpretation. By «interpretation,» I do not mean some naive Cartesian pretense of what the viewer may or may not ‹experience› independently of an external world, but a hermeneutics defined by the rigorous investigation of the relationships by which an entity is defined.



In category theory, such a process is formalized by situating objects and their connections in some category, what Deleuze would refer to as an «order of generality,» such that these connections, expressed as little more than arrows on a diagram traveling between nodes, can be trivially composed into a larger arrow should the front-end of one meet the rear-end of another. A category also keeps things purely spatial, free of issues of timing, by imposing the rule that should you compose a chain of three or more arrows into a single one, it does not matter which part of the chain you compose first.

With these rules in place, the possible interpretations of any given object amount to the sum total of the ways in which one can translate an object, each such translation expressed as a ‹functor›: a function (call it *F*) that maps objects and arrows in one category to objects and arrows in another (or even the same) category in a way that preserves the ‹structure› of the connections, insofar that if an arrow *a* goes between objects *X* and *Y*, then the arrow *F(a)* must go between objects *F(X)* and *F(Y)*. This is really just the formal essence of an analogy: when we say one thing is like another, we mean that they're not exactly the same but that there are relationships within them that mirror one another.



Of course, a metaphor rarely captures every last relationship, and functors are themselves no different, if two arrows have the same origin and destination, the functor can simplify things by mapping them to the same arrow so long as it doesn't interfere with the logical consistency of the rest of the functor.

Every such functor can in this way be seen as a special kind of X-ray: by taking an object and situating it within a new order of connections, one attributes to it an anatomy relevant to the new situation at hand. In fact, this is not some mere poetic flight of fancy but an unequivocally provable foundation of the entire theory insofar that its most fundamental theorem, the Yoneda Lemma (a.k.a. ‹よ›), states that in any case where an anatomy is *explicitly* assigned to an object in any such interpretation (that is, in more technical terms, should a functor *F* translate an object *X* into an actual set), a one-to-one correspondence exists between the elements of *F(X)* and all the different ways in which one could naturally derive the interpretation *F* from *X*'s order of connections.

What do I mean by «naturally?» In simple terms, I mean only using the arrows you already have to get from one interpretation to another. Formally, I'm referring to a ‹natural transformation›, which is a collection of arrows that allows one to travel between functors without ever explicitly invoking said functors, but for the sake of brevity I only want to stress that according to よ, every representation is a strict manifestation of what one can do with existing connections, and that Descartes's Daemon is therefore effectively exorcized. Of course, a given functor might not map something to a set, and as such might not actually provide a representation for よ to work with, in which case it falls outside of any such jurisdiction, as one can only talk about internal ‹properties› to begin with if there are such things to talk about at all.

よ is therefore first and foremost an exercise in ontological deflation, insofar that it simply identifies precisely what one can speak of rather than positing any kind of extensionally defined semantics. The richness of categories, as well as that of their ontological cousins such as toposes and sheaves, becomes most apparent when one drops said semantics by looking at the possible interpretations that such things corroborate and deriving a universal construction that serves as their common denominator, after which one can look at the construction itself, rather than any of the interpretations it speaks on behalf of, as a new kind of matter in its own right.

One can see a geometric version of this in the construction of ‹sheaves› which constitutes a special way of essentially gluing together topological spaces in both a consistent and unambiguous way. Without getting into painful amounts of detail, one can imagine a set of various maps of different regions, many of which might be geographically overlapping but nonetheless have differences in their respective language or logic or even in their content. One may for example be a political map and another a topographical map, or you may even see a mountain on the edge of one map and not on another simply because of irregularities in how people draw them, and you'd have to decide what content you're going

to put in the overlapping area if you want to combine both maps into one.

If one thinks of all these maps, as well as all possible ways one can cut and paste these maps, respectively as objects and arrows belonging to a single category, one can think of each such act of gluing as itself being a kind of interpretation. Of course, there may be many different ways to glue any two maps together that is logically consistent, resulting in an explosion of different ways, such that on a local level, any two maps are consistent with each other but don't necessarily give you a consistent picture when you zoom out and look at all of them at once. Such gluings could nonetheless be seen as functors, but such a functor becomes a ‹sheaf› when one identifies a single set of maps, that when put together, becomes a single map that is logically consistent with all of the other possible gluings.

One may object in my example that maybe you should go look and see if there really is a mountain there, but rather than get pedantic about an example meant to merely illustrate the point, I'll simply point out that there are a great many problems of much more complexity where there is no clear demarcation between the map and the territory that one can simply take for granted. In those cases one overcomes this impasse not through an endless slough of skepticism and negation but through the creation of interfaces that allow conflicting claims and concepts to operate on the same level as one another.

In this sense, each «map» can be thought of more as an order of connections (or at least part of one), with sheaves being a construction that reconciles such a hodgepodge of terrains into a region in which denizens of every such realm can interact on a single playing field. This enabling of communication through a deflationist method evokes echoes of Spinoza's own deflationary accounts of God and causality, but he was nonetheless perhaps too eager to declare all substance to simply be One on account of the fact that two substances by definition cannot interact with one another. A more intuitionist view of causality, one in which not all propositions are *necessarily* true or false at all times, would suggest a universe where the possibility of interaction between two things is not a given but a contingent product of an ongoing labor of sociality that defines the very nature of intelligence and an entity must define itself as an individual before one can even speak of it as either one or many.

This leap from contingency to unequivocal syntactic entailment is one in which the universal is not a given, either as a detached set of rules or an inscrutable oneness by which any idea of free will is rendered moot, but a fundamentally uncertain journey from bricolage to empiricism to reason, by which tinkering within one's local neighborhood is contextualized by its burgeoning interactions with an unavoidable outside and ultimately subsumed into a universalism of our own making. Thus avoiding the idle mysticism that Reza Negarestani mercilessly (and correctly) pans in his aforementioned essay on the «localization of the concept.»

It's not the case, however, that this process of universalization is merely some naive Popperian process by which one chooses a hypothesis that «fits» (according to what's

simplest maybe? Whatever that means...), only to tear it down and start over the nanosecond that some vaguely defined amount of counterevidence invalidates it. Science comes into crisis, according to Thomas Kuhn, not necessarily because of either the success or failure of a theory to predict what it sets out to, but because of a drying up of new avenues of exploration: it doesn't matter how correct (or incorrect) you are if it makes it impossible to ask new questions. In such cases, one needs to find new questions by putting together the pieces they have in new ways.

In doing so, one can find a passage to something even more global than before through the simple composition of existing means. This idea is in many ways akin to sheaves if one is to imagine an unsanitized hodgepodge of experimental data conforming to varying assumptions and standards as the countless maps that one must find a way to fully reconcile, but the idea of finding new global ideas not necessarily from existing representations but simply from the moves one can make in the current game they're playing is more clearly exemplified by a concept from topos theory known as the ‹subobject classifier›: a pattern of arrows composed into a function that defines membership in a collection according to whether or not some element is accepted or rejected by this function. Just like such a subobject classifier might be used to establish whether or not a given element belongs to a set, one could just as easily imagine this as the composition of the myriad practices and rules of a scientific paradigm into a criterion by which experimental hypotheses are either corroborated or invalidated.

Is this nonetheless just a long-winded way of describing the process of finding some generalization and then running with it, and if so how does this sidestep the ever-nagging Quine-Duhem hypothesis without falling back on questionable rules of thumb like Occam's razor? If one were simply talking about a delimited language that represents observations and hypotheses as simple propositions this would indeed be a problem, but these ideas of the universal are far more than simple pattern matching: they are descriptions of how one begins enmeshed in the ongoing processes of their situation and reconciles them into a richer environment that affords more agency.

In doing so, one doesn't merely economize by compressing more information into fewer bits of memory but completely reconceptualizes the situation by constructing an entirely new game. In other words, individuals and multitudes alike compose their observations and practices into entirely new intuitions, which are real *theories* rather than models insofar that, instead of merely functioning as congruent simplifications of a larger body of information, they manifest as newfound haecceities that, in their immanence to an entirely new way of seeing and doing things, serve as the axiomatic basis of a new space altogether. That is, they bring about knowledge that wasn't merely unknown but conceptually inconceivable in any previous time. To put it another way, one is not merely declaring a new set of rules, but instantiating a new geometry; a viscerality never before felt in which one moves in entirely new ways. Along these same lines, the relatively new field of homotopy type theory may yet prove philosophically fruitful insofar that it expresses logical propositions and rules not through discrete syllogisms but instead through the spatial properties of geometric paths and their capacity, or lack thereof, to smoothly change shape and mimic or replace one another, taking logic yet another step further away from the notion of generalizing abstraction and towards the idea of the reconciliation of affect. But I digress.

Intuition, therefore, communes with the Real not through some correspondence between causality and some observing mind (a definition which could never go beyond an indefinite series of inexplicable «lucky guesses»), but rather by simply unfolding into new articulations that cannot in any way be anticipated or reduced. In other words, the Real is that which is never directly grasped by reason but is nonetheless accessed by it insofar that one uses reason to overcome impasses and further express their conatus, itself a part of this same Real, in a way that always exceeds any extant mediation.

—Alexander Boland

Afterword two

I was 16 years old when I started experimenting with psychedelics. I had two notable experiences, one that was positive and affirming, and one that deeply crushed my mental state. I did mushrooms on a summer day in the park with three of my friends from high school. It was a bright blue-skied day and I ate a whole handful. Within an hour, the sounds of bird calls and people laughing and squeaks from swing sets formed chains of reverberation, sound and peoples faces ricocheting wildly. It was fascinating. And then trees would spiral. The sky was a network of fractals, and where the trees met the sky there was a glassy, prism-like buffer. As we walked through the woods of the park, I'd see repeating shapes, the trees would curl into triangular spires all while retaining their basic volume. There was a sense that the living phenomena, people, trees, grass, even inorganic sights like the sky, was shuffling all around, and the utterances we made, our giggles, every step you took had some part in shaping these flows. I had never before felt at once liberation and responsibility so imbricated, the infinitesimal gestures of my consciousness directly affecting the anatomy of the world and vice versa. Instead of the appearance of this precious relationship giving cause for anxiety it felt overwhelmingly true and fulfilling, like holding a newborn baby. It was a rare moment of experiencing unity. It convinced me, if only for 12 hours or so, that there is an underlying shape to consciousness that everyone shares, that if individuals could perceive this system of geometrical

structures and feedback that precedes and contains our reality, maybe we could resolve our alienated states of mind.

The second experience I had on psychedelics was a disturbing antithesis to the first. I took acid with my girlfriend and three other people from school. We sat in the same park where I'd taken mushrooms months earlier. It was winter now and the trees were swallowed by darkness. The acid made itself known in my head as the shapes in the dark began to twitch and the little of my friend's faces I could see showed subtle expressions of disappointment. A strange sadness was developing in me. We decided to Uber back to my mom's apartment because it was getting so cold out. We packed into a car, and as we neared the Manhattan bridge, the driver's GPS map glitched, sending us in the wrong direction. I remember looking at the map spiraling out of control, flickering around and shooting our avatar into different regions, and at the same time funneling into traffic on the Manhattan Bridge, and a bizarre hallucination overcame me where I couldn't help but picture us in a gigantic, tower of Babel-like parking garage structure that spiraled up into the sky, that every material and body was linked but not organically, rather systematically under the command of some indifferent, artificial power. The driver looking at the GPS for direction made me think of what the GPS itself was looking to for direction, and the endless and infinitely granular chain of information that determined our location. The idea of a totalizing dependence on infrastructure, on signs, on the surveillance on the earth, was deeply troubling to me. I felt like I could read malicious intentions of my friends, everyone a feeder on everyone else, a hierarchy of leeches. I felt profoundly detached from my ego in the same way I had on mushrooms, like I was seeing how my existence was a vital part of a network much larger than what meets the sober eye, only acid was the depressive inverse of what I had experienced on mushrooms. The world was interconnected but as a means towards total control, constriction, whereas before I had seen this interlocking of existence as a proof of peace, security, and generative harmonies between individuals.

It's hard to say what the psychedelic experience of the world really amounts to, if there's anything concrete enough to extrapolate into a theory of consciousness or the fundaments of life. Part of anyone's experience on psychedelics has to reckon with the fact that we grow up with a pre-programmed cultural understanding of what the value and feel of such an experience is like through the Hollywoodization of psychedelic history, which undoubtedly factors in to how one examines their own experience on these drugs. Nonetheless, from these two different instantiations of experiencing a perceptual zone that, in a state of tripping, appears to operate around the typical world, I was able to conclude on the existence of my own intuition, which is to say I became aware of how I was intending what I was seeing as much as it was determined before me. I was reminded of these experiences while reading Eric's work because what I believe he is interested in is elucidating fundamental systems of consciousness; he is a champion of intuition as much as he is a serious mathematician. It's important for me that the experiences that psychedelics offer don't remain in the vacuum

of the experience itself. It's not the awareness of different orders of reality that should be the end goal; it's how we retroactively think about these experiences and utilize them to reflect on conventional epistemological systems. And then hopefully, through Eric's work, we have the possibility of going somewhere perceptually further without the need to enlighten or fuck up our heads.

—Laszlo Horvath

Afterword three

Modern philosophy is faced with the unaccountable imperative to take nothing for granted, especially its own nature, its task, and the nature of tasks and imperatives as such (also known as ‹agency›). Despite this imperative, which is often associated with the term ‹immanence›, and which fuses an intellectual ideal of conceptual autonomy with a moral-political injunction to act non-ideologically, philosophers tend to be driven by goals that they don't or can't account for in their work, if they're even consciously aware of them. This is so much the case, especially with the ongoing implosion of academia within the ever-clenching grip of the Society of Control, that it could be argued (and this is my personal view) that philosophy's imperative of immanence is simply impossible to achieve. Non-ideological thought, somehow perched away from the philosopher's emotional and practical incentives, to say nothing of her conceptual limitations and biases, simply can't take place—because these incentives are preformatted by Capital before the pen touches the page.

Eric is simultaneously an artist, a philosopher and a musician, who is seriously concerned with the living horizon of each of these domains. This earnest and thorough interdisciplinary approach is what I value most about his project, because in my view the synthesis of these three activities—together with the cultural breaches that the synthesis inevitably produces—perhaps traces a path of liberation, potentially escaping the snare I just described (that no one of the three stands a serious chance of tracing this path on its own).

However, the *way* these disciplines are differentiated from one another and then put in coordination is a key concern, and I think Eric and I see this question very differently. Here I'll try to make sense of Eric's philosophical project and its relation to his vision for art, to the degree that I understand either of them, and to assess its success on its own terms as well as on mine. In my view, a square and thorough consideration of the history of philosophy and the philosophy of history presents a challenge to the conception of time he defends, and leads to a very different project of 21st century gesamtkunstwerk, which I see as more fertile because, given that it is founded upon the resurrection of Jesus Christ, it is animated by the power of God.

The apparent primary goal of Eric's treatise is to define something called ‹the Real›, with the help of a patchwork of theoretical materials drawn from contemporary mathematics, mathematical physics, ‹neurophysics› and continental theory. Other topics in the text, especially those dealing with genesis and manifestation, seem relevant to Eric's conception of total art, which refines the spirit of Fluxus for the contemporary moment under the heading of ‹minor rationalism›. It's not totally clear to me how these two areas relate for Eric—maybe pure math is discursively defining the Real while art-as-politics makes contact with it in some other way. But in any case, to understand the stakes of the question, it would help to start with a short detour through the history of modern philosophy.

The Real was a popular term for 20th century French post-structuralism, where it was assigned various nonidentical but overlapping meanings for, say, Lacan, Badiou and Laruelle, all of which resist clear meaning. Moving backwards a century, more familiar names that gesture towards the same topic within German Idealism, again with overlapping and conflicting meanings, were ‹the Absolute›, ‹the Unconditioned› and ‹the Universal›. It's less well-known that the term has archaic and mystical roots: ‹the Real› is the best translation for the Islamic term ‹al-Haqq›, which also carries significations like ‹the ultimate truth›, ‹that which unilaterally determines all things›, and ‹that which beckons, yet shipwrecks, all attempts at symbolization›.

The caesura distinguishing French post-structuralism and German Idealism from medieval Abrahamic theology (which is to say, differentiating modern philosophy from its «pre-critical» ancestor) is, of course, the Kantian transcendental turn, forced upon the enterprise of philosophy by the success of mathematical physics at the beginning of the scientific revolution. Immanuel Kant saw himself as shrinking a humiliated metaphysics, whose basic presuppositions had been shattered, to an enumeration of antinomial paradoxes about God, ‹the Soul› and ‹the World›, about which nothing further can be said (by philosophy). He then turned his attention primarily to a speculative transcendental psychology that would build upon the logic of his day to enumerate the categories and schemata of the great collective mind through which we dimly scry a sliver of the otherwise undefinable; the condition of possibility of judgment and experience as such (intuition, sensation, imagination, understanding, and reason; being, negation, necessity, contingency and

so forth). In essence, Kant picked up Aristotle's discarded system of categories, fused it with Aristotle's conception of the soul, posited this entity as the form of experience as such, and named it ‹the transcendental subject›, giving it an epistemological rather than ontological valence.

Hegel immediately re-ontologized it. In Kant's system, there are two eternally unknowable abysses on either side of the transcendental subject: on one side, the world; on the other side, the soul. Hegel refused to grant any «reality» to these two unknowable abysses, and rejected the thought that the subject was eternal, or its antinomies and contradictions impassable. Instead, he envisioned a dynamically evolving subject, lured by the *apparently unknowable* (but in fact only unknowable from the perspective of a given horizon) towards contradictions that cause society and the world to fundamentally evolve; uprooting, reshaping or reformulating axioms and fundamental questions in the process. In essence, he expropriated Kant's antinomies, schemata and categories to re-initiate classical metaphysics. But this time, unlike Athenian philosophy and the Abrahamic traditions, for whom God stands apart from a world whose forms are fundamentally static, in Hegel's system metaphysics and logic are fused with philosophy of nature and a Gnostic-Christian theory of history, such that the noumenon itself manifests a series of cultural/technical horizons of intelligibility which subtractively cascade, by way of determinate negation, towards a future disembodied collective world governed by Intellectual Intuition. At the end of the process, Absolute Knowing, the distinction between fact and value is ultimately dissolved. The culmination is a dematerialized post-human society where the God of Aristotle, who creates what he thinks in the same stroke, is finally born.

Let us call this series of paradigms and the tragic labor of its unfolding «transcendental time,» a paradoxical temporal vector across with the transcendental subject somehow traces its eternal history. It's with this temporalized version of the transcendental that the modern notion of the Real begins to emerge, from two directions. From the philosophical direction, as soon as the transcendental subject is conceived as something that evolves, we arrive at a fundamental distinction between the identification of the abyssal transcendental «as such,» before any theoretical account of its structure is given, and the content of this theory itself, whatever it is. At the same time, we see a world-historical analogue, a disjunction between the space of historical time and the series of horizons of intelligibility that successively occupy it, viewed more as cultural epistemes than philosophical theories. In both cases, there is now a theoretical cleft between the ‹matter› of the transcendental (which is the Real) and its ‹form› (the current reigning paradigm, be it cultural or explicitly philosophical).

With this development, modern philosophical materialism is born. The materialisms of Friedrich Nietzsche and Karl Marx, for all their differences, share in common a denigration of the formal dimension of the transcendental, be it conventional morality and legal structures or philosophy that aims at eternal ideals like Truth and Justice (and in doing so, supposedly, serve to bolster convention rather than question it). Both accuse rationalist philosophy of deceptively supporting oppressive social structures, and endorse the non-conceptual substrate—will to power, productive force and so on. Both then propose (especially Nietzsche) that philosophy attempts to think, for the first time, the Real-in-itself itself, with no presuppositions about morality, reason or consciousness, and to develop a set of categories that deliberately eschews any notion of cognition or representation. To escape these strictures, to see the ‹back of the mirror› of thought, Nietzsche conceives of the Real using, instead of logic, materials from evolutionary biology, and natural science generally, as well as anecdotal subjective accounts of the mysterious process of creative expression.

This move exacerbates a problem that has faced the philosophy of the transcendental since Kant. Another key feature of the Real is that neither exact science nor intuitive experience can say anything definitively true about it, given that it is neither subjective nor objective, thus outside of the legitimate domain of inner or outer knowledge. This is precisely *why* philosophy still has a role (rather than being made entirely irrelevant by science), because its tendency to nebulously formulate and answer questions at the same time uniquely suits it for topics like this. Thus the philosophy of the transcendental is inherently speculative in both method and theory. Kant was already explicit about this, though his recommendation can be carried out in a number of ways. One avenue, the idealist one, dominates the Anglophone analytic tradition—the «juridical» approach described by Jay Rosenberg, which retains the idea that philosophy is giving a reasonable account, painting a picture of a transcendental psyche which it has no power to alter.

But the more radical materialist approach, of which Nietzsche is the best exemplar, seeks to eschew the subject as well as the individual at all costs, conceiving of philosophy as a decision, an action, an almost violent gesture. The philosophy of the transcendental ends up participating in its evolution, lending it an unlikely historicity, of which philosophers became conscious during the 20th century—philosophy becomes a legislative Act. Gilles Deleuze's *Difference and Repetition* elaborates a proposal that theoretical resources like dynamical systems theory might be better suited for speculatively defining transcendental temporality, as a «fractured I» through which «it» speaks, rather than «representational» logic. Badiou's *Being and Event* might seem like an opposite approach, given that it focuses on mathematics—arguing that in essence, an architectonic of philosophical mathemes updated to incorporate Georg Cantor's insights, which Hegel had been born too early to make use of, would suffice. But both of them see their philosophy systems as contingent subjective decisions, contributions to the chaos of becoming (this is not widely understood, despite both thinkers clearly stating that it is so).

A side effect, if not a goal, of this approach is, in eschewing representation, to obfuscate the distinction between new, exciting subjectivities and formal inventions which are Good versus ones that are Evil—because there is no absolute reference point to establish moral value. This was given the

name «the ethics of the Real.» This attitude has become enormously popular in theory-oriented fine art and experimental music scenes, as well as anarcho-communist political projects over the past 50 years; but in our time, it's in crisis. The ethics of the Real, refusing reason, morality and mind, is forced either into a «left-wing» anarcho-communist impotence and irrelevance, or into a «right-wing» social disaffection that refuses any conception of universal justice, compassion or belief in democracy or egalitarianism.

In recent years, it has become clear to some philosophers that neither of these choices is acceptable, and neither depicts reality for what it is. What is needed, according to this new awareness, is a «new rationalism.» One that does not sneak in the old conventional bigotries or conformist tendencies with which most associate the term ‹Reason›, but which nevertheless seeks a Universal touchstone accessible only to the intellect. This is the context for examining Eric's approach to the Real, which seems to make a point of disqualifying any notion of a materialist or vital flux at the heart of being, and to endorse mathematical reasoning as the key to unlocking the secrets of the Absolute.

Influenced by the Laruellean theme that the One can't be thought (it can only be thought-according-to), the text involves a nonsystematic collection of themes relating to foundations of mathematics, ontogenesis, and the metaphysics of time. He endorses Reza Negarestani's concept of navigation as an epistemology (which also perhaps becomes an artistic and political injunction), and seems to be employing it as a structuring principle for the text: the way to *know* something is to give accounts of it at vastly different scales, to subject it to different conditions and extremes, discover its intrinsic thresholds, and to refine one's method of investigation and models based on feedback.

The most extensive discussion involves considering set theory, category theory and topos theory as candidates to encompass all of mathematics. While I don't understand these fields well enough to fully grasp what's at stake, Eric seems to be seeking to determine which field is most congruent with a deflationary metaphysics—akin to Quine's famous universal quantifier, the upside-down A, which says «for all *x*» without implying that any instances of *x* do or could exist. It seems that deflationary metaphysics is attractive because it makes it possible to rigorously define a supremely general and abstract field, situated between arithmetic and geometry, maybe even between subject and object, without (unrigorously) presupposing the reality of a mystical or vital ontological substrate to the world science can know.

His discussion of time continues the attack on materialist ontology. He rejects existentialist ontologies like those of Bergson, Heidegger and Deleuze, and affirms a theory of time that seems to draw from general relativity, real analysis and number theory. This section ought to be more rigorous by Eric's own criteria. It's more of a gesture than an argument, and could stand to acknowledge just how great a gap lies between time as a mathematical-physical parameter and the philosophical question of its essence, as well as the plurality of conceptions of time available to math and science. Even if we choose to hand the question of time off to the

scientists, the scientific community itself is unable to agree whether physical time really flows or whether it collapses into space, or on how to handle the disjunction between general relativity, which is symmetrical, and thermodynamics, which is asymmetrical.

A different set of concerns are addressed, drawing on Deleuze, Simondon and Petitot, that seem to bear more directly on the human-world bond and practices like art, philosophy and politics. A key point here seems to be that spatial intuition, with its elemental and embryonic quality, deserves to be affirmed as an authentic site of and mode of access to truth, perhaps because it is not subject to the corrupting power that ideology inevitably wields over the higher forms of cognition like linguistic representation and judgement.

The specifics of some of his suggestions are thought provoking, like the idea that Jean Petitot's neurophysics might define the ‹synthetic a priori›, but I'll pass over them and get to my critique. First of all, there's the topic of immanence with which this text began. Eric seems to separate philosophy's task into two unrelated segments—on the one hand, the task is to clarify the work that contemporary math and science are doing and to draw philosophical conclusions from it. On the other, it seems to exhort a certain type of practice—but there's no explicit relation between the former and the latter. The respective essences of philosophy and politics seem to be taken for granted, where it might be a more powerful move to call both into question. One could wish for a synthesis of analytic and continental thought that would live up to philosophy's highest ambition and account for conventional pre-philosophical belief that democracy and egalitarianism are important ingredients in a thriving polis. These are the norms that the neorationalist turn is supposed to provide (theoretical computer science being a main resource for grounding them), as I understand it, but for Eric, despite all the math, they're still nowhere to be found.

A second thing that leaves me a little unsatisfied is that the ‹minor rationalism› that Eric endorses has quite a lot in common with the ideals of anti-rationalist anarcho-communist and Fluxus-inspired practices. Aside from an increased degree of erudition, which is laudable in itself, it's not completely clear to me what this new theory adds to, or changes, about this approach. I don't disagree with it—surely it's correct that the margins between discursive regimes and power centers are the site of greatest potential for authenticity and originality, and it continues to be dangerous and difficult to maintain faith in this approach. He uses the idea of 1-point compactification in the Riemannian sphere to suggest, I think, that, unlike worldly practices which *do* suppose a hypothetical infinity, authentic artistic interventions do not, and thereby make actual contact with the Real, in some way; I take it that this must be the «same» Real that homotopy type theory is able to define, I think, but if so, I'd like to know why and how.[1]

1 Eric's response to this question, in a subsequent conversation, was that the two Reals in question are *not* the same, but that they have an «analogical» relationship. The need to import a Thomist category-like analogy to close this gap is another example of the difficulties that arise from any attempt to outsource metaphysics to a restricted domain like mathematics. Presumably the hope would be for homotopy type theory to solve this problem on its own.

In proposing that practice as such retain this abject, negative character, Eric seems to level the distinction between meaning-making activities—at least between art and music, if not philosophy. There is a danger of creating a situation in which, as Hegel's famous critique of Schelling's absolute goes, «all cows are black at night,» where praxis takes the form of disturbing lightning flashes that lack aesthetic or diachronic structure, which react to heteronomous (rather than autonomous) forces, and which simply fade away after a brief moment of insight. But I'd like to think that in our age, Craftsmanship, Glory and Beauty should be revisited as crucial aesthetic modes.

This artistic objection seems connected to my main philosophical objection—which is that, just like Alain Badiou and François Laruelle, Eric's conception of the Real is, despite appearances to the contrary, ultimately materialist—a subtractive and deflationary materialism which denies philosophy access to any ontological or pre-ontological noumenon, but materialism all the same, because the object of his thought is just that: an object, not a subject (or an addressee). Maybe it's an extremely abstract and general one, qualified (or stripped of qualities) in manifold ways that deny its specificity, but still, an object: a dead thing, in the third person.

The problem with this materialism is best exemplified by his conception of time. It follows from Eric's identification of time as such with its objective incarnation—whether it's the spacetime of relativity, the iteration of pure number, or the arrow of thermodynamics (which he does not mention)—that physical time collapses into eternity and is no time at all. Eric's claim reminds me of Ray Brassier's thesis in *Nihil Unbound,* but this is a position that Ray himself has recanted in recent years.

Eric is correct to claim that time is not a Bergsonian libidinal flux, but sustained rigorous philosophical dialectic inevitably shows that neither is time any particular regime of mathematics or mathematical physics, nor is our perception of time «merely an illusion.» The way to see this is to use modern tools of thought to re-trace Plato's steps in his allegory of the cave. Obviously there's no space for an argument here, but the dialectical and phenomenological journey traced by the rationalist-mystic philosopher J.N. Findlay's *Transcendence of the Cave* is a good 20th century example of a fully rationalist rejection of materialism.

After examining the contradiction at various levels of reality and knowledge, we arrive at the insight that the physical world as we know it exists only *because of its own eschatological horizon*. Time is a world-historical horizon which began with the resurrection of Christ, passed through the scientific revolution and the fundamental trauma it created for philosophy, and will continue for a time as a series of objective-subjective world-culture formations with immanent contradictions (one of which is the current episteme governed by scientific rationality) guided by three stars: ‹the True›, ‹the Good› and ‹the Beautiful›. In the end, God—a single, loving mind with unique personhood—is born. While this formulation may appear naive according to conventional taste, this is because of bias and prejudice,

or perhaps worse, systemic, soul-debilitating contemporary ideology—in any case, not because of Reason.

Once we accept that time is eschatological, we open the door to true autonomy in the Kantian sense: a thought and project that is motivated by its own inner aims, rather than reacting against outer ones. Philosophy's task becomes to build a conception of art-as-politics animated by a rationalist and critical vision of the kingdom of heaven to come, and to begin to instantiate that vision in a thorough, sustained and imbricated way. The importance of this cannot be overstated—not because any particular conception of heaven will really come to pass, but because to be animated by one in a polyvalent way is to unlock the ability to truly create autonomously, to be driven literally by the desire for heaven rather than by the desire for status in a particular scene, money, influence after one's death, or whatever it is.

Contemporary gesamtkunstwerk must combine Fluxus and Cologne with more traditionalist visions like those of William Blake or Richard Wagner, to arrive at a precise multimodal conception of heaven, elaborated as concept, image, narrative and sound, along with a concrete account of how exactly the humanist practices of art, science and politics are to be related and combined so as to use available materials to achieve it, as well as a practice of conducting these experiments, assessing their consequences and revising one's conception of the end and means, and one's provisional account of world history.

Only such a vision and such a process for iteratively refining/achieving it has the power to lure cultural-libidinal activity away from the ideological forces that govern every cultural silo of our time—the art world and academia, as much as the internet and the culture industry—and towards the kingdom on the other side of the wall.

—Hunter Hunt-Hendrix

Afterword four (Acid Hegel in K-space?)[1]

The understanding of an object via the extraction of invariants allows for an intrinsic definition of an object outside of any extrinsically imposed framework, which is the great benefit of the group theoretical generalization of space (e.g., Riemannian manifolds), and of category theory and homotopy type theory as unifying frameworks for contemporary mathematics. However, the invariants should not be hypostatized as the essence of the object and the variations overlooked as mere noise. The way in which objects vary are often critical to their definition: it is the variations that reveal the invariants. There is a dialectical reciprocity between variance and invariance, and this is related to the complementarity between continuity and discontinuity. In particular, the morphology of an object is defined both by the local continuity of its regular points and those critical points where there is a discontinuity or singularity. The singularity is a variation in the continuity or invariance between one regular point and the next, but it is a topological invariant in the continuous variation of the object's morphology.

To understand a hidden causal process, or ‹black box›, merely observing it will not be helpful. In order to infer the causal structure or process governing its behavior it is necessary to perturb the object in various ways—rotate it, shake it, subject it to noise—the outcome can then be observed, hypotheses formed, and further tests devised. Just as topologi-

1 «Acid Hegel in K-Space?» was first published in December 2021 as an extract from Inigo Wilkins's book *Irreversible Noise* on the Urbanomic website, https://www.urbanomic.com/document/acid-hegel/.

cal transformations may reveal an object's essential geometry, causal transformations reveal essential causal structure, and perceptual variations reveal essential qualitative attributes. The exploratory ascertainment of invariants and variations yield a systematic knowledge of the object. The variations and invariants defining the morphological eidetics of an object are not fixed and context-free characteristics but are defined relative to a scale of analysis, level of abstraction, and interpretative situation, so they should really both be understood as the way an object covaries under specific conditions.

Immanuel Kant's distinction between phenomenon and noumenon highlights a problem that is of a different order than the spatio-temporal perspectivalism that sound studies seems fixated on, which is that «the sublation of the empirical perspectivism of experience is still limited by transcendental perspectivism.»[2] Departing from a strictly Kantian understanding we may define the transcendental framework as those conditions that structure the possibility of cognition, those enabling constraints that define the human as a suffering, thinking, and acting person. This is a more Sellarsian perspective in which the transcendental is not fixed but open to change. Constraints are both hindering and empowering to varying degrees, indeed they facilitate by restricting since without constraints there is just entropy.[3] The

2 Gabriel Catren, «What is Ensoundment?,» in *Florian Hecker, Formulations*, eds. Susanne Gaensheimer, Robin Mackay and Miguel Wandschneider (Cologne: Koenig Books, 2016).

3 The notion of constraint here is based on the complex systems account as developed by thinkers like Alicia Juarrerro and Terrence Deacon, and should be distinguished from its understanding in the ecological theory of perception, where constraint has a narrower definition as the reciprocal of affordances, both being structured by the ‹effectivities› of the organism (i.e., its possibilities for action).

transcendental conditioning factors operate at many levels, including physical, chemical, and biological constraints, as well as cultural, linguistic, and economic constraints. The former are ‹natural› while the latter are artifactually constructed and in some sense ‹non-natural› or normative. There is a certain universality to some constraints but many are of course diversely distributed over populations, and highly uneven.

The whole history of the artifactual elaboration of mind, by which various natural and normative constraints have been overcome or transformed,[4] can be understood as a process of incremental alterations and sudden shifts in transcendental perspective. Gabriel Catren explains that since humans are able to formulate problems that cannot be answered within the language in which they are posed, and to construct new formalisms that breach this apparent transcendental circumscription, there are no fundamental limitations to the capacity to vary the transcendental perspective. He gives as an example the progression from natural numbers to rational numbers, real numbers, and complex numbers.[5] We could also think of the invention of various audio technologies (e.g., phonography, sonar, time-stretching, granular synthesis) and musical techniques (e.g., syncopation, electro-acoustic manipulation of concrete sound, free jazz, DJing) as transforming the conditions of possibility of sound and

4 This understanding comes from various talks given by Reza Negarestani. Also see Glass Bead, «Logic Gate: The Politics of the Artifactual Mind,» *Glass Bead* 1, https://www.glass-bead.org.

5 Gabriel Catren, «A Plea for Narcissus,» talk given at the conference *Continental and Analytic Kantianism: The Legacy of Kant in Meillassoux's and Sellars' Realism* held at University College Dublin, June 15–17, 2016.

listening. Catren calls the possibility space of transcendental perspectives K-space, and argues that just as knowledge of the phenomenon can be gained by kinaesthetic modifications of our empirical standpoint so it is possible to asymptotically grasp the ‹phenoumenon› by moving in K-space.

The concept of the phenoumenon is intended as a replacement for the strict opposition between the phenomenal and noumenal in Kant's philosophy.[6] Rather than the thing-for-us and the thing-in-itself the phenoumenon is a relative absolute around which unfolds a sheaf of empirical and transcendental perspectives. My argument is that it is necessary to articulate the relationship between the naturalistic and the normative, between sense and sensibility, by integrating rather than either fusing or refusing (or worse, con-fusing) their quasi-autonomous explanatory-descriptive powers. In this way we can open our ears to the phenoumenon of noise. This may involve processes of the ‹electro-acoustic sheafification› of sound material,[7] it also requires imaginative and kinaesthetic variations, the discovery and invention of new evidence and hypotheses, the development of technologies, the creation of concepts, and the social-interactive transformation of norms. Following the understanding of the psychedelic as the process of self-realization that a mind undergoes when unhooked from the ‹natural attitude› of perception, Catren describes motion in K-space as a ‹phenoumenodelic› operation. Acid Hegel, if you like.

6 Gabriel Catren, «What is Ensoundment?,» in *Florian Hecker, Formulations*, eds. Susanne Gaensheimer, Robin Mackay and Miguel Wandschneider (Cologne: Koenig Books, 2016).

7 Fernando Zalamea, «Electro-acoustic Sheafification,» in booklet accompanying Eric Frye's LP *On Small Differences in Sensation* (Cejero, 2016).

While theories such as Predictive Coding Theory and cognitive morphodynamics have to some degree «furnished» the black box of cognition and perception with representational content that is no longer grounded on symbol manipulation, phenoumenodelic psychonauts may further perturb its hidden kaleidoscopic interior, inducing unimaginable torsions in the space of social interaction, and unleashing a cascade of symmetry breaking transformations that alter the naturalistic and normative conditions of possibility for thought. That is, we move beyond the description of what cognition is and unfold what it can be and ought to be (i.e., its unbound naturalistic-technological capacities and normative-ethical significance). Speculatively propelled from the gravity well of any cognitive terrestrial ground, this is the irreversible noise that shatters all expectations and blasts out the boundaries of parochially restricted listening subjectivity: the phantastically improbable sound that sends the dancefloor whirling into the outer cosmos, convulsed by polyrhythmic paroxysms and engulfed in an ultrachromatic blaze. Phenoumenon goes boom!

—Inigo Wilkins

Afterword five (Notes on arrows)

How could one reconcile the proposition of univalent foundations in mathematics and the traces of activity marked by the proper name of Dieter Roth? Could the formal inventions of the former help elaborate the field of the latter? The *Prolegomenon* begins to establish some compelling coordinates by which one could begin to approach such a question. Could one carry such formalizing work forward? As a text with foundational aspirations, what would the status of such work be?

The desire for foundational narratives should be accompanied by a warning: Beware the phantasm of totality. The dream of the unified field in physics—prior to any value it may accumulate along the way—is, as Jacques Lacan would say, a dream. And its dreamer wishes to remain asleep.

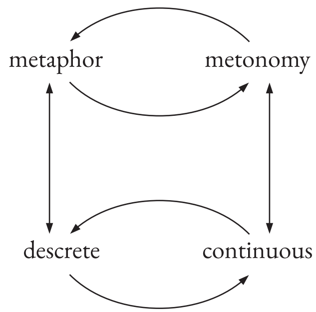
And yet one does create abstractions along the way in anything one does; what differentiates the formal devices used by practicing psychoanalysts and type theorists? A provisional answer seems to be, it depends. For Lacan, there are a number of formal gadgets—the four discourses, the clinical categories, the trefoil—that are capable of modeling invariants in analytic practice. They are in fact still being elaborated in the contemporary clinic. Their epistemological status appears as equal parts foundational formalism and jazz-standard. The bathos is not pejorative. By the 1950s Lacan had concluded that psychoanalysis should not seek to be a science in the sense of the search for the foreclosure of a field. His analysis and expeditions into abstraction for-

go global coherence and remain locally tied to the material of the utterances in the clinic: locally in flux, subject to revision and further elaboration. Jacques-Alain Miller, for example, warns the reader against too much enthusiasm, in the seminar on anxiety, for the many «golden» formulations that present themselves as the lure of the final word on the theory of anxiety. Some of these formulations are later revised (by Lacan and others), other formulations just hang in seeming contradiction. This fact poses little theoretical problem for the audience of this discourse, the analysts, as they are primarily occupied with differential diagnostics for the discourse of their analysands. Global coherence or global incoherence, is not ground for the rejection of a model that may afford clinical value, so some abstractions are allowed to persist with some points of invariance and some points of variance. So, how could one walk the line, in practice, between local models and something that may only ever be the fantasy of global unification?

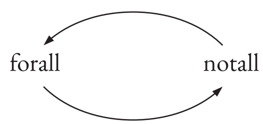
The central insight of Nobuo Yoneda and Alexander Grothendieck was to bracket objects and deal with potential structures of interrelations. Intuitively, this is already quite close to the treatment of a number of Lacanian constructions: Of some things, one can only ever observe the effects and what the thing is, so to speak, *is* beside the point. The *objet petit a* and <the Real> are perhaps the most well known, but there are many and I would go as far as to say the entirety of analytic practice exists in mapping the echoes of such objects in the discourses of analysands. It's also worth noting that there are analogues in physics and, as was the subject of the presentation of the World Association of Psychoanalysis some years ago in New York, the effect of inaccessible objects, in any field, is invariably one of the hallmarks of the speaking subject. The theories of physicists can tell you about the form of the *objet petit a* for their authors.

So, the initial epistemological smell test seems to pass, but can one practically do category theory or type theory in the field broadly classified humanities? There have been at least a couple attempts in linguistics: ‹distributional semantics› and ‹categorical grammars›. These approaches seek to formalize the way that words can be structured using categorical machinery; that in each case, the signification of individual words is elided since one really only cares about the ways words or their underlaying formalism can be related without a notion of concrete signification. To some extent they are category-theoretic models of signification, though they do not take ‹signification› in the full sense of its structuring function for the speaking being. A similar, though not explicitly categorical approach is used in machine learning techniques to learn representations of words in natural language processing: substitute lists of words with lists of objects in the category of finite vector spaces and optimize the vectors with respect to some criterion that intuitively requires capturing the semantics of the corpus at hand. (Corpus! Let's mark in passing that fantasy of unification has deep roots.) Variants of the linear-algebraic approach via vector spaces have been known for some time; Jacques Roubaud derived a set of sestinas from Petrarch using eigenvectors in the 1960s.

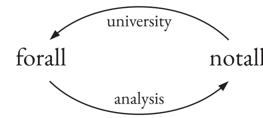
Is it possible to do something similar without substituting, as a model, the known mathematical objects for words? Such a practice would, by default, be the inheritor of the problematics established at the end of ‹structuralism› in the circle surrounding *Cahiers pour l'Analyse*. An initial cause for optimism: if there is a global, only in as much as you can transit between locals, then the global is constituted only by one's ability to change perspective via localities. So, global «meta-languages» would always seem to be just out of reach, beyond the local horizons. There's certainly a tantalizing number of such localities. (As many as there are speaking beings?) For example, the opposition of metaphor and metonymy in some sub-fields of linguistics after Roman Jakobson bears striking similarity to the discreet and the continuous in Fernando Zalamea's work on the philosophy of contemporary mathematics: both appear as fundamental, but don't immediately appear to give the primary account of the other. Could the choice of some categorical structure for the arrows yield any new information about a given locality? Provided the choice of abstract machinery did yield something of interest, what would the status of such a structure be?



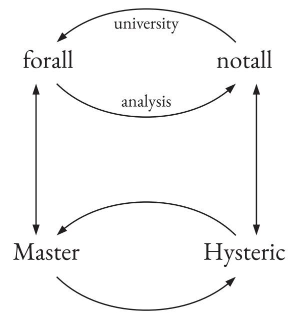
This question is meant as a sounding board for navigating the phantasmagoric abyss. It could be argued that one of primal dualities in Lacan can be stated as the ‹notall› (lack; inexistence; barred subject) and the ‹forall› (One; being; semblant).



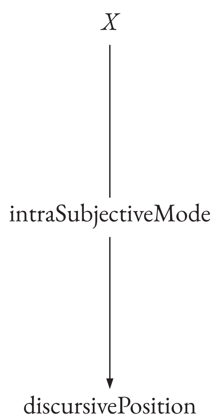
If one considers these two terms as ‹dual categories› or perhaps ‹adjoint functors›, one could exemplify concrete instances of the morphisms that perhaps obey some notion of ‹duality› or ‹*adjoint*ness›.



One could also work out additional morphisms to, say, a subset of the four discourses.



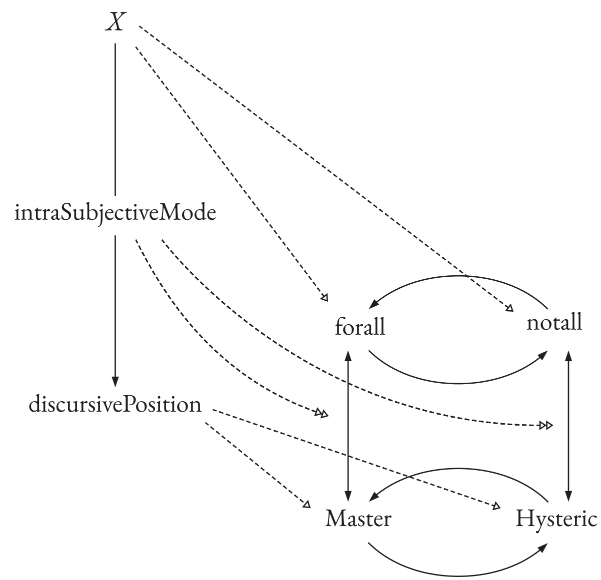
For such a diagram to commute (i.e., to be shown to be valid without respect to the particular chain of arrows chosen) there must be a universal construction that comprehends the transit from the ‹forall›/‹notall› pair to the four discourses. We could call this movement of situating something along the lines of «the specification of an intra-subjective modality.» It's a functor that takes some X and puts it in a discursive position.



One then has a precise presentation of the arrows between the discourses and the fundamental adjunction.

Modulo the fuzzy formalism of signifiers, what exactly have we done? A diagram has a precise formalization in category theory: it is a functor. It takes objects and morphisms

from one category and puts them in the category of diagrams. The diagram would thus appear to be on the side of the *forall*: the diagram's elaboration, at least in one aspect, entails the subject-position of *mastery*.



However, one of the nice techniques of category theory is that as soon as you have some categorical formulation, you can take other constructions and just try them out to see if your particular category admits those structures. Such a dynamic would constitute the obscure parts of the diagram:

nearly any arrow, with or without a name, could be extended according to the algebra. In this sense of the initial blankness of the page, it's my current belief that the diagram is also on the side of the ‹notall›.

Gilles Châtelet writes about the vitalism of drawing diagrams; he does not like the diagrams that appear in mathematical textbooks. The finality with which they appear, typeset and formatted, presents them unambiguously as dead. He instead praises the crude, initial gestures scrawled on whatever one has handy at the time. Gilles Deleuze takes considerable inspiration from this notion of abstraction as «leaping,» with all its connotations of the unknown place one will land.

On the side of mathematics, whose fields are worked out with unmatched rigor, the forgoing might smack a bit of the laxity of non-mathematical fields. To respond to this, one should point out the origin myth of Thales who needed an Other of language to constitute the notion of ‹proof›. Even in homotopy type theory, you must invoke the Other in order to give substance to the sign of proof: To make a proof, one must construct the universe of possible *witnesses* to a proposition. Such a witness, even codified in a proof assistant, does not escape the tether to the poor, fleshy bodies condemned to language.

The fundamental disagreement between Alain Badiou and Gilles Deleuze was that the latter was unable to understand how the former's mathematical ontology could *not be* a metaphor. The details of this disagreement are beyond the scope of these notes, but it seems like there are two general

options: if we're doing something like ontology, it is on the side of mathematics divested of linguistic signifiers. If one is doing something else, then one is substituting mathematical formalism and assuming a relative transparency of the linguistic signifiers. So either mathematics needs to lay in a purified subspace of language, shielded from the differing and deferral of the linguistic signifier, à la Badiou, or mathematics is the initial structure embedded into the hat of language from which is produced, the mathematical rabbit.

Does a third option exist? Can one do category theory in the way the negative theologians did theology? Such a theory could not be fuzzy enough. It would have to commit totally to the obscurity of the signifier: The theory could not reduce God to a countable series of negations of, or inductions on, the Worm; it would have to form a categorical treatment of the entirety of the text by Pseudo-Dionysus as well as all texts that came prior in «the christian tradition.» It would have to be a categorical «reading,» which, as has been argued by many, for a long time, would also be a proper type of writing.

—Tim Pierson

Postscript (Who is the subject of improvisation?)

Improvisation is probably the most contradictory practice of art production, since it initially reveals the framework of activity as seemingly unstable and with fluid contours. Furthermore, it constantly tries to break down pre-established and rigid constructions.

In order to answer the question «Who is the subject of improvisation?» we need to get a notion of this subject.

What we understand here by the subject is an agent which has the capacity to act, to change its conditions and context. The subject must be a form of self-determination. This means that the subject has the capacity to change the framework of action.

Usually, the improvising individuals are considered the subject of improvisation, but the self-determination and changes in improvisation are also co-created by the action and must also be understood as part of the subject.

In contrast to the assumption that the individual as an improviser underlies her autonomous function as a subject of a very liberal ideology, improvisation problematizes the separation of subject and object.

For this reason, we cannot consider the individual alone as the subject of improvisation.

For an improvisation to be successful, it must also disturb our taken-for-granted perception of the individual, since the idea of the «individual» is in large part a precondition of the framework.

In the capitalist, socio-normative structures that currently shape us, the subject/object relation and framework, in both a core and broader perspective, is determined by our engagement in capitalist relations.

Seen in this way, it is problematic to assume the proposition that improvisation can change its own framework for itself. In order to become a subject, it is imperative that improvisation actively undermine and erode capitalist presuppositions (for example, the liberal idea of the individual as subject).

—Mattin

Prolegomenon to a Treatise
Eric Schmid

Fore- and afterwords: Alexander Boland, Will Fraser, Rocco Gangle, Laszlo Horvath, Hunter Hunt-Hendrix, Tim Pierson, Michael Stumpf, Connor Tomaka, Inigo Wilkins, Fernando Zalamea
Postscript: Mattin

Editors: Ben Green, Bauer Verlag
Copy editing mathematics: William Chin
Proofreading: Luke Schumacher
Diagrams and lithography: Felix Hofmann-Wissner
Design: Bauer Verlag
Print: Beltz Grafische Betriebe
Reihe editors: Leslie Bauer, Richard Eß

Acknowledgements: Thank you Ben Green for editing the present publication, William Chin for double checking the math and Luke Schumacher for proofreading. Thanks to the publishers and series's editors Leslie Bauer and Richard Eß. Thanks to my mom and stepdad for their support during the pandemic and encouraging me to go back to school for mathematics. Thanks to Fernando Zalamea for encouraging me to do graduate school, Rocco Gangle for his support of my philosophical project and Colin McLarty for his interest in the *Prolegomenon*. Thanks to Ben and Richard for being understanding of my difference. The author and the publishers would furthermore like to thank Felix Hofmann-Wissner for remodeling the diagrams and lithography, and to Alexander Boland, Will Fraser, Rocco Gangle, Laszlo Horvath, Hunter Hunt-Hendrix, Tim Pierson, Michael Stumpf, Connor Tomaka, Inigo Wilkins, Fernando Zalamea and Mattin for their contributions.

Printed with financial support by the City of Frankfurt Department of Culture.

www.bauerverlag.eu
infoororder@bauerverlag.eu

List of images: Fig. 1: Fernando Zalamea, *Synthetic Philosophy of Contemporary Mathematics* (Falmouth/New York: Urbanomic/Sequence Press, 2012),

118–119, photo: Eric Schmid; all mathematical diagrams remodeled by Felix Hofmann-Wissner based on the following source images: Fig. 2: Ryszard Paweł Kostecki, «An Introduction to Topos Theory,» (Unpublished manuscript: University of Warsaw, Institute of Theoretical Physics, 2011), 28; Fig. 3–5: «Talk:Sheaf (mathematics),» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki?title=Talk:Sheaf_(mathematics)#Some_visualization; Fig. 6: E. C. Zeeman, Catastrophe Theory: Selected Papers, 1972-1977, (Boston: Addison-Wesley, 1977): 66; Fig. 7: «Metastability,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Metastability; Fig. 8: «Spacetime diagram,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Spacetime_diagram; Fig. 9–10: «Special and General Relativity,» Stanford University, accessed July 28, 2021 (content removed, March 19, 2022), https://web.stanford.edu/~oas/SI/SRGR/notes/SRGRLect3_2015.pdf; Fig. 11: «Spacetime,» Wikipedia, accessed July 28, 2021, https://en.wikipedia.org/wiki/Spacetime; Fig. 12: Benjamin Kimia, «On the role of medial geometry in human vision,» in *Special Issue: Neurogeometry and Visual Perception*, eds. Jean Petitot and Jean Lorenceau, *Journal of Physiology-Paris* 97, 2–3 (2003): 155–190; Fig. 13–14: Tai Sing Lee, «Computations in the early visual cortex,» in *Special Issue: Neurogeometry and Visual Perception*, eds. Jean Petitot and Jean Lorenceau, *Journal of Physiology-Paris* 97, 2–3 (2003): 121–139; Fig. 15: «How would you describe hyperbolic space and elliptic space?,» Quora, accessed July 28, 2021, https://www.quora.com/How-would-you-describe-hyperbolic-space-and-elliptic-space.